Risk and Ambiguity in Models of Business Cycles

Dave Backus, Axelle Ferriere, and Stan Zin

Carnegie-Rochester-NYU Conference

April 25, 2014
The “Great Recession” and its aftermath

Real Gross Domestic Product
Percentage change from previous peak, Seasonally Adjusted

Source: U.S. Bureau of Economic Analysis

Quarters from previous peak
The “Great Recession” and its aftermath

Real Personal Consumption Expenditures
Percentage change from previous peak, Seasonally Adjusted

Cooley-Rupert Economic Snapshot; www.econsnapshot.com
U.S. Bureau of Economic Analysis

1973 cycle
1981 cycle
1990 cycle
2001 cycle
Current cycle

Quarters from previous peak

Backus, Ferriere, & Zin (NYU)
The “Great Recession” and its aftermath

Real Private Nonresidential Fixed Investment
Percentage change from previous peak, Seasonally Adjusted

- 1973 cycle
- 1981 cycle
- 1990 cycle
- 2001 cycle
- Current cycle

Quarters from previous peak

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Backus, Ferriere, & Zin (NYU) | Risk & Ambiguity
What happened?

- What we see
  - Magnitude: deeper recession than usual
  - Persistence: longer recovery — maybe slower, too

- Like Kydland-Prescott with productivity shocks?
  - Relative magnitudes look right
  - Comovements look right, too
  - But... measured productivity didn’t fall very much
Great Recession

From Wikipedia, the free encyclopedia

This article is about the global economic downturn during the early 21st century. For background on financial market events dating from 2007, see financial crisis of 2007–08.

The Great Recession[^1][^2][^3][^4] (also referred to as the Lesser Depression[^6], the Long Recession[^6], or the global recession of 2009[^7][^8]) was a global economic decline in the late 2000s. According to aggregated national data, a worldwide recession began in Q3-2008 and ended in Q1-2009. It is widely believed that the severity and length of this recession was the direct consequence of an increase in macroeconomic uncertainty.

It is related to a liquidity crisis, commonly being dated to have started when several central banks had to step in with liquidity lending to the interbank lending market on 9 August 2007. This was a response to a situation where BNP Paribas temporarily had to block money withdrawals from three hedge funds—citing a "complete evaporation of liquidity".[^9] The bursting of the U.S. housing bubble[^10] where the median price for real estate home sales in US started to decline after its peak in July 2006,[^11] had caused the values of securities tied to U.S. real estate pricing to plummet, which damaged financial institutions globally—to a degree ultimately resulting in the subsequent interbank credit crisis.[^12][^15] The first sign...
What we do

- Take a streamlined business cycle model
- Ask: How does *uncertainty* affect the *dynamics* of output, consumption, and investment?
  - Magnitude: Does uncertainty magnify fluctuations?
  - Persistence: Can it reduce the speed of recovery?
Modeling ingredients

- Streamlined **business cycle model**
  - Recursive preferences
  - Unit root in productivity
  - Fixed labor supply

- With fluctuations in **uncertainty**
  - *Risk* (stochastic volatility)
  - *Ambiguity* (unobservable long-term growth)
Preview of results

Fluctuation in uncertainty have **limited impact**

- **Persistence**
  - Separation property: internal *dynamics independent of risk and risk aversion*
  - Persistence must be in the shock

- **Magnitude**
  - Impact typically small, but magnified by *risk aversion*

Business cycle properties governed by IES
Risk and uncertainty

- **Recursive references**

\[
U_t = V[c_t, \mu_t(U_{t+1})] = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \\
\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}
\]

\(V, \mu_t\) homogeneous of degree one, \(RA = 1 - \alpha, IES \equiv \sigma = 1/(1 - \rho)\)

- **Stochastic structure** of productivity \(a_t\)

\[
\log g_t = \log(a_t/a_{t-1}) = \log g + e^\top x_t \text{ ("productivity growth")}
\]
\[
x_{t+1} = Ax_t + v_t^{1/2} B w_{1t+1} \text{ ("news")}
\]
\[
v_{t+1} = (1 - \varphi_v)v + \varphi_v v_t + \tau w_{2t+1} \text{ ("risk")}
\]
\((w_{1t}, w_{2t}) = \text{iid standard normals}\)
Scaling

- **Bellman equation**

  \[
  J(k_t, x_t, v_t, a_t) = \max_{c_t} \mathbb{V}\{c_t, \mu_t[ J(k_{t+1}, x_{t+1}, v_{t+1}, a_{t+1})]\}
  \]

  s.t. \quad k_{t+1} = f(k_t, a_t n) - c_t

- **Assume f \ hd1:** \( f(k, an) = k^\omega (an)^{1-\omega} + (1 - \delta)k \)

- **Rescaled** Bellman equation \([\tilde{k}_t = k_t / a_t, \tilde{c}_t = c_t / a_t]\)

  \[
  J(\tilde{k}_t, x_t, v_t) = \max_{\tilde{c}_t} \mathbb{V}\{\tilde{c}_t, \mu_t[ g_{t+1} J(\tilde{k}_{t+1}, x_{t+1}, v_{t+1})]\}
  \]

  s.t. \quad g_{t+1} \tilde{k}_{t+1} = f(\tilde{k}_t, n) - \tilde{c}_t

- **Numerical solution**
## Parameter values

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Model is essentially loglinear
Insights from loglinearization I

- Goal: loglinear decision rule for capital

\[ \log \tilde{k}_{t+1} = h_k \log \tilde{k}_t + h_x x_t + h_v v_t - \log g_{t+1} \]

- Dynamic programming version of Campbell (JME, 1994)

- Loglinearization around the **stochastic** steady-state
Insights from loglinearization II

- Loglinearize **capital’s marginal product** and **law of motion**

\[
\log f_{kt} = \lambda_r \log \tilde{k}_t + \lambda_0 \\
\log \tilde{k}_{t+1} = \lambda_k \log \tilde{k}_t - \lambda_c \log \tilde{c}_t + \lambda_1 - \log g_{t+1}
\]

where \((\lambda_k, \lambda_c, \lambda_r)\) are steady-state objects.

- Guess **loglinear value function and derivative**

\[
\log J_t = p_k \log \tilde{k}_t + p_x^T x_t + p_v v_t + p_0 \\
\log J_{t}^{\rho^{-1}} J_{k,t} = q_k \log \tilde{k}_t + q_x^T x_t + q_v v_t + q_0
\]
Separation property

Claim

Consider the loglinear approximation of capital’s law of motion,

$$\log \tilde{k}_{t+1} = h_0 + h_k \log \tilde{k}_t + h_x^\top x_t + h_v v_t - \log g_{t+1}$$

If we hold constant the stochastic steady state:

1. $h_k$ is independent of **properties of all shocks and risk aversion**
2. $h_x$ is independent of **properties of uncertainty shocks and risk aversion**

$$h_k = \lambda_k + \sigma \lambda_c (q_k - \lambda_r), \quad h_x^\top = \sigma \lambda_c q_x^\top$$

$$q_k = q_k [\lambda_k + \sigma \lambda_c (q_k - \lambda_r)] + \lambda_r$$

$$q_x = -(\sigma^{-1} + q_k) e^T A [(1 - \sigma q_k \lambda_c) l - A]^{-1}$$
The claim is informative
Risk aversion magnifies uncertainty shocks

Backus, Ferriere, & Zin (NYU)  Risk & Ambiguity
### Business cycles governed by IES

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<td><strong>Risk Aversion</strong></td>
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<td><strong>Standard deviations (%)</strong></td>
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**Intertemporal elasticity of substitution: 0.5**

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**Standard deviations (%)**

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**Correlations with output growth**

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**Risk aversion: 10**
Risk and ambiguity

Divide the state in two:

\[ s_{t} = (s_{1t}, s_{2t}) \]

Ambiguity (Klibanoff, Marinacci, & Mukerji; Ju & Miao)

\[
\begin{align*}
\text{risk} &= p_{1t}(s_{1t}+1|s_{2t}+1, I) \\
\text{ambiguity} &= p_{2t}(s_{2t}+1|I)
\end{align*}
\]

Two-part certainty equivalent

\[
\begin{align*}
\mu_{1t}(U_{t+1}) &= \left[ E_{1t}(\mu_{1t}(U_{t+1})) \right]^{1/\alpha} \\
\mu_{2t}\left[ \mu_{1t}(U_{t+1}) \right] &= \left\{ E_{2t}\left[ \mu_{1t}(U_{t+1}) \right]^{\gamma} \right\}^{1/\gamma}
\end{align*}
\]

\( \alpha \) controls risk aversion, \( \gamma < \alpha \) controls ambiguity aversion

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Risk and ambiguity

- Divide the state in two: \( s_t = (s_{1t}, s_{2t}) \)
- **Ambiguity** (Klibanoff, Marinacci, & Mukerji; Ju & Miao)
  
  \[
  \text{risk} = p_{1t}(s_{1t+1}|s_{2t+1}, \mathcal{I}_t) \\
  \text{ambiguity} = p_{2t}(s_{2t+1}|\mathcal{I}_t)
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- Two-part certainty equivalent

\[
\mu_{1t}(U_{t+1}) = \left[ E_{1t}(U_{t+1}^{\alpha}) \right]^{1/\alpha} \\
\mu_{2t}[\mu_{1t}(U_{t+1})] = \left\{ E_{2t}[\mu_{1t}(U_{t+1})^{\gamma}] \right\}^{1/\gamma}
\]

\( \alpha \) controls risk aversion, \( \gamma < \alpha \) controls ambiguity aversion
Rule of thumb: associate ambiguity with unobservables

Consider three stochastic processes

- $x_t = \text{mean growth rate}$ (not observable)
- $\log g_t = \text{realized growth rate}$ (observable)
- $\nu_t = \text{"stochastic volatility"}$

\[
\log g_t = \log g + x_t + \nu_{t-1}^{1/2} w_{1,t} \]
\[
x_{t+1} = \varphi_x x_t + \nu_{t}^{1/2} w_{2,t+1} \]
\[
\nu_{t+1} = \varphi \tilde{\nu} + (1 - \varphi) \nu_t + \tau w_{3,t+1} \]

Kill learning ($\varphi_x = 0$)

Magnitudes small, separation property holds — as before
## Calibration

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The model is still essentially loglinear.
Learning?

- Absent stochastic volatility...

\[
\log g_{t+1} = \log g + x_{t+1} + \sigma_1 w_{1,t+1}
\]

\[
x_{t+1} = \varphi x_t + \sigma_2 w_{2,t+1}
\]

\[
\nu_{t+1} = (1 - \varphi_\nu) \bar{\nu} + \varphi_\nu \nu_t + \tau w_{3,t+1}
\]

- Learning stabilizes: **No fluctuations in uncertainty**

\[
\hat{x}_{t+1} = \varphi \frac{\sigma_1^2}{A_t + \sigma_1^2} \hat{x}_t + \varphi \frac{A_t}{A_t + \sigma_1^2} \log (g_t/g)
\]

\[
A_{t+1} = \sigma_2^2 + \frac{\varphi^2 A_t}{A_t + \sigma_1^2} \sigma_1^2
\]
Learning?

- Add stochastic volatility

\[
\log g_{t+1} = \log g + x_{t+1} + v_t^{1/2} w_{1,t+1}
\]
\[
x_{t+1} = \varphi x_t + v_t^{1/2} w_{2,t+1}
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\[
v_{t+1} = (1 - \varphi_v) \bar{v} + \varphi_v v_t + \tau w_{3,t+1}
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- Fluctuating uncertainty
Learning?

- Add stochastic volatility

\[
\log g_{t+1} = \log g + x_{t+1} + \sigma_1 w_{1,t+1}
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v_{t+1} = (1 - \varphi_v) \bar{v} + \varphi_v v_t + \tau w_{3,t+1}
\]

- Fluctuating uncertainty

- But will it break the **separation property**?

\[
\log k_{t+1} = h_k \log k_t + m(x_t, \hat{x}_{t+1}, v_t, A_{t+1})
\]
Summary

- Uncertainty fluctuations have intuitive appeal
- But they add little to standard business cycle model
  - Magnitude: impact is small with common parameter values
  - Persistence: they add nothing to internal dynamics, just the persistence of the shocks themselves
Open questions?

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It is related to a liquidity crisis, commonly being dated to have started when several central banks had to step in with liquidity lending to the interbank lending market on 9 August 2007. This was a response to a situation where BNP Paribas temporarily had to block money withdrawals from three hedge funds—citing a "complete evaporation of liquidity".[^8] The bursting of the U.S. housing bubble,[^10] where the median price for real estate home sales in US started to decline after its...
Open questions

- What are we ambiguous about?
- What extensions hold the most promise?
  - Endogenous uncertainty
    Veldkamp; Fajgelbaum, Schaal, & Taschereau-Dumouchel
  - Idiosyncratic shocks
    Bloom, Floetotto, Jaimovich, Saporta-Eksten, & Terry
  - Financial frictions
    Cooley, Quadrini, & Marimon; Arellano, Bai, & Kehoe
- Other suggestions?
Related work (some of it)

- Recursive business cycles
  - Tallarini; Campanale, Castro, & Clementi; Rubio & Villaverde; Liu & Miao

- Approximation methods
  - Anderson, Hansen, McGrattan, & Sargent; Campbell; Kaltenbrunner and Lochstoer; Malkhozov

- Risk and business cycles
  - Caldara, Fernandez-Villaverde, Rubio-Ramirez, & Wen; Justiniano & Primiceri; Liu & Miao

- Ambiguity and business cycles
  - Klibanoff, Marinacci, & Mukerji; Jahan-Parvar & Miao; Ju & Miao; Ilut & Schneider
Rescaled Bellman equation

\[ J(\tilde{k}_t, x_t, v_t) = \max_{\tilde{c}_t} V\left\{ \tilde{c}_t, \mu_t [g_{t+1} J(\tilde{k}_{t+1}, x_{t+1}, v_{t+1})] \right\} \]

subject to

\[ g_{t+1} \tilde{k}_{t+1} = f(\tilde{k}_t, n) - \tilde{c}_t \]

plus shocks & initial conditions

Let \( K \equiv g_{t+1} \tilde{k}_{t+1} \). Then,

\[ J(\tilde{k}_t, x_t, v_t) = \max_{K} V\left\{ \left( f(\tilde{k}_t, n) - K \right), \mu_t \left[ g_{t+1} J\left( \frac{K}{g_{t+1}}, x_{t+1}, v_{t+1}, 1 \right) \right] \right\} \]

plus shocks & initial conditions
Risk aversion magnifies uncertainty shocks.
Productivity

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Annex