RATING THE RATERS: ARE REPUTATION CONCERNS POWERFUL ENOUGH TO DISCIPLINE RATING AGENCIES?[*,†]

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ABSTRACT

Credit Rating Agencies (CRAs) are accused of bearing a strong responsibility in the subprime crisis, by having been too lax in the ratings of some structured products. Many commentators explain this behaviour by a conflict of interest generated by the new business model of CRAs, which collect most of their income from the issuers rather than from the investors. In response to this accusation, CRAs argue that such an attitude would be too dangerous for them, since their reputation is at stake. The objective of this article is to examine the validity of this argument within a formal model: are reputation concerns sufficient to discipline rating agencies?

We show that the reputation argument only works when a large fraction of the CRA income comes from other sources than rating complex products. By contrast when the fraction of the CRA income that comes from rating complex products becomes large, the CRA is always too lax with a positive probability and inflates ratings with probability one when its reputation is good enough. This implies the possibility of what we call reputation cycles, ultimately resulting in confidence crises where a single default provokes a complete loss of reputation by the CRA.

We analyze the policy implications of our findings and advocate for a change in the business model of CRAs. Rather than closely regulating ratings, that would reveal a formidable task for regulators, we argue in favor of a new business model, that we call the platform-pays model.

1 Introduction

Credit Rating Agencies (CRAs) are accused of bearing a strong responsibility in the subprime crisis, by having been too lax in the ratings of some structured products. For example the title of the article by Mason and Rosner (2007) is self explanatory: "How misapplied bond ratings cause mortgage backed securities and C.D.O. market disruptions". The authors of this paper argue in particular that CRAs under-evaluated some parameters that were crucial in the assessment of risks such as those measuring correlation between defaults of large obligors. Many commentators explain this behaviour by a fundamental conflict of interest generated by the new business model of CRAs, which collect most of their income from the issuers rather than from the investors, as was the case when John Moody started his company (and the ratings industry) in 1905. Since the development of photocopying machines in the early seventies, the cost of reproducing the books of ratings sold to investors has decreased dramatically, threatening the viability of the investors-pay model, where fees are collected from issuers. With the exception of Egan Jones, which does not have a significant market share of the ratings industry, all CRAs now get the bulk of their revenue from issuers. As some commentators have put it: "It is as if the referee was paid by one of the teams". Another change is due to regulation: since the creation, in 1975, of the status of Nationally Recognized Statistical Ratings Organizations (NRSROs) the stakes associated with obtaining a good rating for issuers have increased considerably. This has been reinforced recently by the official recognition of the ratings provided by NRSROs in the computation of regulatory capital requirements for commercial banks in the Basel II accord. Finally, it is clear that CRAs have plaid a crucial role in the fantastic development of structured finance products in the recent years, and that they have benefited a lot from this development. For example Moody's net income has grown from \$159 m in 2000 to \$705 m in 2006.

In response to the accusation of having been deliberately too lax, CRAs essentially argue that such an attitude would be too dangerous for them, since their reputation is at stake. Although there are important differences between the ratings industry and the accounting industry, a parallel is often made with the rapid fall of the accounting firm Arthur Andersen following its implication in the Enron scandal. CRAs argue that they cannot afford to be the next Arthur Andersen. The objective of this article is to examine rigorously the validity of this argument: are reputation concerns sufficient to discipline rating agencies?

There is a very large literature on reputation models in game theory, but, somewhat surprisingly, there are few applications of these models to financial markets. Our model builds on one of the few exceptions, namely Benabou and Laroque (1992), who study how a financial guru can build a reputation and ultimately cash on it by manipulating market prices on one direction and trading in the opposite direction. We adapt this model to the rating industry by considering a financial market where, at discrete dates, new firms want to issue a security for financing some investment project with a cost normalized to unity. The project quality is a priori unknown (even to issuers). It is good with probability λ, or "bad" with probability $(1-\lambda)$. We assume that good firms should be financed but that without prior knowledge on the quality of the project, no financing should take place. A (monopoly) CRA observes the project quality and communicates a rating to the market. No issue takes place if rating is bad or denied. The CRA can be of two types: either it is fully committed to always tell the truth or it is opportunistic (i.e. chooses the rating that maximizes the expected present value of its profits). The reputation of the CRA is measured by the probability q that investors assess to the CRA being committed. The strategy of an opportunistic CRAs is described by the probability $x(q)$ that a CRA of reputation q will be too lax (i.e. will give a good rating to a bad security). This influences the accuracy $a(q)$ of ratings, i.e. the probability that investors attach to a good rating to mean a good project, given the reputation of the CRA. A Markov Perfect Equilibrium (MPE) is a couple of functions (x, a) that are simultaneously rational from the respective viewpoints of CRAs and investors.

We first show (Proposition 1) that, when the fraction of the CRA income that comes from other sources (than rating complex products) is large enough, there is a unique MPE where an opportunistic CRA always tells the truth. In this case, reputation is a good disciplining device for CRAs. By contrast when the fraction of the CRA income that comes from rating complex products becomes large, there is a unique MPE where the CRA is always too lax with some probability and lies with probability one when its reputation is good enough. This implies the possibility of what we call reputation cycles, consisting of several phases. First, starting from a situation where investors are not very trustful, issuing volumes are low and credit spreads are high, the CRA tries to increase its reputation by being very strict. Then investors become more optimistic, the reputation of the CRA increases, spreads decrease and issuing volume increases. But this is precisely when CRAs become more lax and risk of default increases. Ultimately there is a default, which provokes a crisis of confidence: the opportunistic CRA is detected, its reputation brutally falls down, spreads become high again and issuing volumes decrease dramatically. Thus in this case, it may take a long time to detect an opportunistic CRA and the conflict of interest is not solved by reputation concerns: Reputation building creates confidence cycles.

We analyze the policy implications of our findings and advocate for a change in the business model of CRAs. Rather than closely regulating ratings, that would reveal a formidable task for regulators, we argue in favor of a new business model, that we call the platform-pays model, that would be applicable for all the securities that are publicly traded on a exchange or that use a centralized clearing house.

Related Literature. With the exception of Kuhner (2001), who only considers a one-period model, the theoretical literature on rating agencies has not considered yet the question of reputation building by a strategic CRA. Indeed, most of the literature on CRAs considers non-strategic agencies. For example Skreta and Veldkamp (2008) assume that each CRA observes a noisy signal (rating) on the asset quality. They show that if issuers can choose amoung multiple ratings which rating to disclose to the market (what is called "shopping for ratings") then an increase in the complexity of securities (noise in signal) produces ratings inflation. Increasing competition among agencies does not solve this problem since it enlarges the range of ratings. Boot, Milbourn and Schmeits (2006) show that credit ratings provide a "focal point" for firms and investors, that can serve as a coordinating mechanism in situations where multiple equilibria are possible. On the contrary, Carlson and Hale (2006) establish that introducing a CRA can bring multiple equilibria to a market that otherwise would have possessed a unique equilibrium.

The (meager) literature on strategic CRAs only considers one-period models. Lizzeri (1999) and Faure-Grimaud, Peyrache and Quesada (2007) take a mechanism design approach and consider a model where a CRA commits to a fee and a disclosure rule and choose them so as to maximize expected profits. Lizzeri (1999) models non-contingent fee and rating concerning the (possibly negative) value of an object. He shows that at equilibrium a monopoly CRA will only reveal whether the object has positive or negative value (if all objects have positive values then the CRA discloses no information). Doing so, the CRA can capture a large share of the surplus. When introducing competition *à la* Bertrand CRAs make zero profit and fully disclose information. Faure-Grimaud, Peyrache and Quesada (2007) consider contingent fee and ex-post efficient (renegotiation-proof) contracts. They show that at equilibrium a monopoly CRA will fully disclose information. When introducing competition *à la* Bertrand CRAs make zero profit and fully disclose information on firms that have values higher than the CRAs' marginal observation cost (firms with lower values reject the contract as they would be worth less than the fee paid to obtain rating). They also show that full disclosure is not robust when introducing the possibility for ownership (disclosure rights) to the firms. According to Kuhner (2001), rating agencies are more likely to reveal their private information if their rating can not become self-fulfilling from an ex-post point of view. This is sustained by experimental studies (Sanela and Niessen, 2007).

We are thus the first to tackle explicitly the issue of reputation building by a strategic CRA in a dynamic model: are reputation concerns sufficient to guarantee the truthful behavior of a strategic CRA? We build on two important papers that have modeled reputation building on financial markets: Benabou and Laroque (1992) study the behavior of a guru, who can influence the formation of beliefs by uninformed investors, while Diamond (1989) explains why new firms with insufficient reputational capital have to borrow from banks before they can issue direct debt on financial markets.

Our paper is also related to the literature on reputation building through communication. Since the seminal papers of Kreps and Wilson (1982) and Milgrom and Roberts (1982) it is standard to model reputation through the introduction of a committed type (for an overview, see, e.g., Mailath and Samuelson, 2006). Some papers consider informed players that care about their reputation not because others players will treat them differently, but simply because they want their advice to be believed in the future. Sobel (1985) considers a sender-receiver finitely repeated game with perfect monitoring where both players are long-run. The sender can be of two types: he either has identical preferences to the receiver (he is a "friend") or has completely opposed preferences (he is an "enemy"). Instead of considering a discount factor as usual the author considers that the importance of the date's play varies stochastically over time. Exploiting what happens in the last period by backward induction he shows that the enemy's equilibrium strategy is such that probability of lying increases in the importance of the date's play. Benabou and Laroque (1992) consider a sender-receiver infinitely repeated game with imperfect monitoring (sender with noisy private information) where the receiver is short-run. The sender can be of two types: either committed to tell the truth or has completely opposed preferences to the receiver ("enemy"). Benabou and Laroque show that the sender's ability to manipulate information is limited only in the long run. Moreover if different senders follow one another then learning about the sender's type remains incomplete even in the long run, leaving a constant scope for manipulation. Morris (2001) adapts Benabou and Laroque's model to other sender's preferences. The sender can be of two types: either he is a "friend" or he is a "bad" type that wants as high an action as possible. Just as the bad type sometimes has an incentive to tell the truth (despite a short-run incentive to lie) in order to enhance its reputation, so the good type may have an incentive to lie (despite a short-run incentive to tell the truth) in order to enhance its reputation. In the infinite-horizon model, he shows that the good type does not necessarily have an incentive to tell the truth and that for at least some discount rates for the bad type and utility functions for the decision maker, there is not a truth-telling equilibrium. In particular, even if the good type is arbitrarily patient and the bad type is arbitrarily impatient, an informative equilibrium may not exist.

2 The Basic Framework

The model is an infinite succession of elementary periods of fixed duration. At each period $t = 0, 1, \dots$ a cashless firm wants to issue a security for financing a complex investment project of a size normalized to 1. The project quality is *a priori* unknown, including to the issuer himself.[1] It can be good with probability λ, or bad with probability $(1 - \lambda)$. Good projects always return $X > 1$ (we normalize payoffs so that there is no discounting within periods). Bad projects always return 0. Investors are risk neutral and competitive. We assume $\lambda X < 1$: no investment takes place if investors are completely uninformed about the quality of each project. A Credit Rating Agency (CRA) perfectly[2] observes the quality of each project (at a cost normalized to zero) and communicates a rating (good or bad) to the market. No issue takes place if the rating is bad.

The CRA is a long-run player with discount factor $\delta \in (0, 1)$. His payoff in the infinite horizon game is the discounted sum of stage game payoffs. Issuers and investors are short-run players, each of whom only plays once.

The CRA can be of two types: committed to tell the truth or opportunistic (in which case he wants to maximize its long-run payoff). We assume that investors and issuers observe the same information namely whether past projects have been financed, and whether they have succeeded. Given the uncertainty concerning the CRA type, this information can be summarized by a single state variable, namely the posterior probability that investors and issuers assign to the event that the CRA is truthful. This probability is denoted q, and measures the reputation of the CRA. A (stationary) Markov[3] strategy for the opportunistic CRA is a mapping

$$x : [0, 1] \longmapsto [0, 1],$$

where $x(q)$ is the probability that an opportunistic CRA will "lie",[4] i.e. give a good rating to a bad project, when the reputation of the CRA is q. The truthful CRA is committed to play the strategy consisting in always giving good (resp. bad) rating to a good (resp. bad) project.

Investors and issuers' behavior is described by the Markov belief function

$$p : [0, 1] \longmapsto [0, 1],$$

[1] This assumption fits well the case of structured products, for which CRAs have played an important advisory role vis a vis issuers, concerning in particular the level of credit enhancement need to obtain a good rating.

[2] The case of imperfect monitoring by the CRA is considered in Section 4.

[3] Stationary Markov strategies ignore all of the details of a history except the current state. While restricting opportunistic CRA to such strategies may rule out some equilibria, any equilibrium under this restriction will still be an equilibrium without it.

[4] In principle, the CRA could also "lie" by giving a bad rating to a good project, but this never happens at equilibrium.

where $p(q)$ is the probability investors and issuers assign to a successful investment, given the reputation q of the CRA.

If $p(q) \geq \frac{1}{X}$, the (primary) market equilibrium is characterized by the zero profit condition

$$p(q)R(q) = 1, \tag{1}$$

where $R(q) \in [0, X]$ is the nominal return promised to investors (and only paid in case of success)[5]. When $p(q) < \frac{1}{X}$, no issue takes place.

Fees are transaction based: they are only paid if the issue takes place[6]. When the opportunistic CRA plays the strategy x, the probability that a bad project obtains a bad rating is a function of the CRA's reputation. This probability is called the *perceived accuracy of the rating* and is described by the function

$$a(q, x) \equiv 1 - (1 - q)x(q).$$

Note that $a(q, x) \in [q, 1]$ as $x \in [0; 1]$. It is easy to see that the probability of success $p(q)$ is an increasing function of the rating's accuracy: $p(q) = \frac{\lambda}{1-(1-\lambda)a(q,x)}$. Thus when $a \leq a^* \equiv \frac{1-\lambda X}{1-\lambda}$, $p(q) < \frac{1}{X}$ and no issue takes place. We assume that rating fee $I(a)$ is also a function of the perceived rating accuracy a:

$$I : [0, 1] \longmapsto [0; I(1)].$$

We assume that $I(a)$ is equal to zero whenever $a \leq \frac{1-\lambda X}{1-\lambda} \equiv a^*$ (i.e., $p(q) \leq \frac{1}{X}$) and strictly increasing elsewhere. In addition to this revenue $I(a)$ (received when an issue takes place) the CRA also earns a revenue $i_0 \geq 0$ coming from other activities (e.g., rating simple products such as corporate bonds, or offering credit risk management tools and services to assess and model risk). We suppose that this revenue is constant for $q > 0$, and is lost foreover[7] when $q = 0$. Formally, this revenue equals $i_0 \mathbf{1}_{\{q>0\}}$, with $i_0 \geq 0$ and $\mathbf{1}_{\{A\}}$ is the function that takes value 1 if event A is realized and 0 otherwise.

At the end of each period the players observe one of three possible outcomes: *Success* (S) when a good project is financed; *Failure* (F) when a bad project is financed; or *No financing* (N). We denote by q^z, the posterior probability that the CRA is truthful, given a realized outcome $z \in \{S, F, N\}$ and a prior probability q. If an opportunistic CRA gives a good rating to a bad project with probability $x(q) \in [0; 1]$, then posterior beliefs are

$$\psi(q|S) \equiv q^S = q, \tag{2}$$

$$\psi(q|F) \equiv q^F = 0, \tag{3}$$

and if $a(q, x) \neq 0$

$$\psi(q|N) \equiv q^N = \frac{q}{a(q, x)}. \tag{4}$$

[5] Since issuers are cashless they have to finance their investment plus the rating fee. Thus the size of the issue equals the size of the investment plus the fee issuers have to pay for rating.

[6] If rating fees were paid even when the rating is bad (in which case there is no issuance of security) there would no gain from lying for the CRA, and the conflict of interest would disappear.

[7] The idea is that if the CRA has been caught lying about the quality of complex financial investments, investors consider that it might also lie about that of other financial products such as corporate bonds.

These updated beliefs are obtained by using Bayes rule whenever possible. If $a(q,x)=1$ (that is if $q=1$ or $x(q)=0$) then Bayes rule does not apply when the outcome F is observed. Since a truthful CRA is committed to tell the truth it is legitimate to consider that failure (which is only possible when a bad project has been financed after receiving a good rating) is due to an opportunistic CRA: $q^F=0$. Similarly, if $x(0)=1$ then the outcome N is a zero probability event and (4) is not well-defined. In that case, we shall assume that $q^N=0$. In all other cases Bayes rule applies.

Figure I depicts the decision tree in the perfect monitoring case when well rated projects are financed.

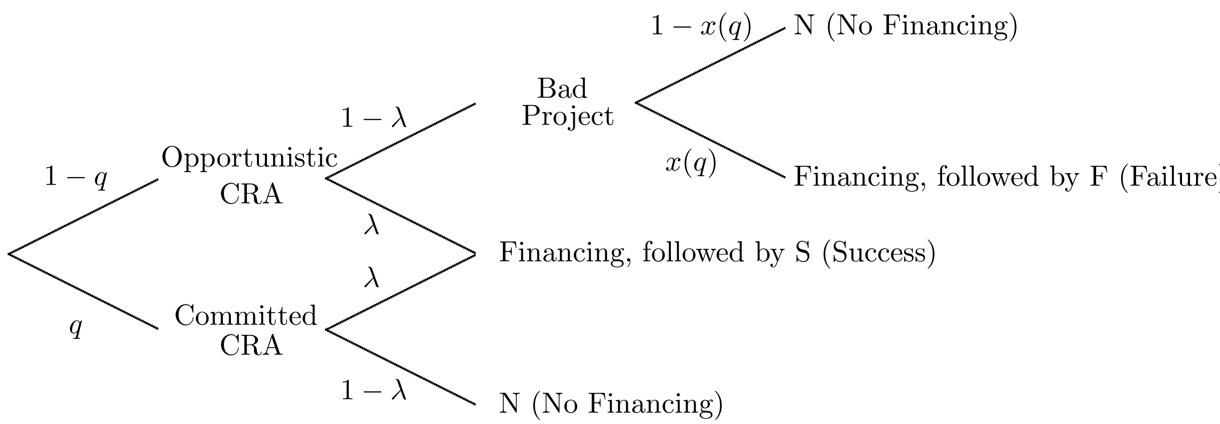

Figure I

Decision tree in the perfect monitoring case.

In a *(stationary) Markov perfect equilibrium*, an opportunistic CRA maximizes profits, investors' and issuers' expectations are correct, and investors and issuers rationally update their beliefs:

Definition 1. A *stationary Markov perfect equilibrium* is a triple (x,p,ψ) such that:

(i) $x(q)$ maximizes the intertemporal profit of the CRA for all q;

(ii) $p(q)=\frac{\lambda}{\lambda+(1-\lambda)(1-q)x(q)}$; and

(iii) ψ satisfies (2), (3), and (4). Moreover $q^F=0$ if $a(q,x)=1$ and $q^N=0$ if $x(0)=1$.

In the following we shall simply refer to such a triple as an *equilibrium*. The strategy $x(\cdot)$ of the CRA uniquely determines the equilibrium updating rule that investors and issuers use.

The Bellman equation characterizes the value function $V(\cdot)$ of the opportunistic CRA. When it has reputation q, and investors and issuers update their beliefs according to x^*, the continuation payoff satisfies:

$$V(q)=i_0\mathbf{1}_{\{q>0\}}+\max_{0\leq x\leq 1}\Big[(\lambda+(1-\lambda)x)I(a(q,x^*)) \\ +\delta\{\lambda V(q)+(1-\lambda)xV(0)+(1-\lambda)(1-x)V(q^N)\}\Big]. \quad (5)$$

An equilibrium value function V is thus a fixed point of the Bellman operator T, defined for any continuation value function W as

$$TW(q) = i_0 \mathbf{1}_{\{q>0\}} + \max_{0 \le x \le 1} \Bigg[\{\lambda + (1-\lambda)x\} I(a(q, x^*_W(q))) \\ + \delta \left\{ \lambda W(q) + (1-\lambda) x W(0) + (1-\lambda)(1-x) W\left(\frac{q}{a(q, x^*_W(q))}\right) \right\} \Bigg], \tag{6}$$

where the maximum is attained for $x = x^*_W(q)$. Note that investors and issuers have rational expectations: thus the perceived accuracy of ratings is $a(q, x^*_W(q))$ and does not change if the CRA deviates and chooses $x \neq x^*(q)$.

We shall concentrate on equilibria associated with a value function V that is continuous and nondecreasing in the CRA reputation q. Let C_+ denote the space of such functions on $[0;1]$, endowed with the norm of uniform convergence.

The one-stage deviation principle for infinite horizon games allows us to obtain necessary and sufficient conditions for a strategy to support equilibrium. CRA's strategy x is an equilibrium strategy if and only if for any q, for any deviation x':

$$x(q)I(a(q,x(q)))+\delta\{x(q)V(0)+(1-x(q))V(q^N)\} \ge x'(q)I(a(q,x(q)))+\delta\{x'(q)V(0)+(1-x'(q))V(q^N)\}$$

that is

$$(x(q) - x'(q))I(a(q, x(q))) + \delta(x(q) - x'(q))V(0) \ge \delta(x(q) - x'(q))V(q^N) \tag{7}$$

with q^N satisfying (4). The following property will allow us to simplify this condition.

Property 1. At equilibrium, $V(q = 0) = 0$.

Proof. By (4) if $q = 0$ then $q^N = 0$ and $V(q^N) = V(0)$. According to (7) we have $(x - x')I(a(q, x(q))) \ge 0$ for any x'. Thus either $I(a(0, x(0))) = 0$ or $(x-x') \ge 0$ for any x', that is $x(0) = 1$. Since by assumption investors do not finance the project in the absence of information about project quality, if $q = 0$ and $x(0) = 1$ then $a(0, 1) = 0$ and $I(0) = 0$. In all cases $I(a(0, x(0))) = 0$. Therefore $V(0) = 0$. $\square$

Property 1 is a consequence of our perfect monitoring assumption. If a financed project fails, the reputation of the CRA is lost forever ($q^F = 0$). Given Property 1, we shall restrict our attention in what follows to value functions $V \in C_+$ satisfying $V(0) = 0$. We shall denote C^*_+ the space of such functions. Whenever $V \in C^*_+$ the one-stage deviation principle expressed in (7) can be written as:

$x(q) = 1$ is part of an equilibrium strategy if and only if

$$I(a(q, x(q))) \ge \delta V(q^N); \tag{8}$$

$x(q) = 0$ is part of an equilibrium strategy if and only if

$$I(a(q, x(q))) \le \delta V(q^N); \tag{9}$$

and $x(q) \in (0;1)$ is part of an equilibrium strategy if and only if

$$I(a(q, x(q))) = \delta V(q^N). \tag{10}$$

These conditions (7), (8) and (9) simply express the fact that when choosing an action x, an opportunistic CRA facing a bad project compares its present gain from giving a good rating $I(a(q, x(q))$ (but losing all future gains) to its discounted future gain from giving a bad rating $\delta V(q^N)$.

Whenever $V \in C_+^*$, (5) can be written

$$V(q) = \frac{i_0 \mathbf{1}_{\{q>0\}} + (\lambda + (1-\lambda)x(q))I(a(q,x)) + \delta(1-\lambda)(1-x(q))V(q^N)}{1-\delta\lambda}. \tag{11}$$

As stated in Property 1, at equilibrium we have $V(0) = 0$. $V(q) = 0$ for $a > 0$ may occur if investors do not finance any issuer. For q sufficiently small (i.e., $q \leq \frac{1-\lambda X}{1-\lambda}$ so that $X \leq R(q)$) an equilibrium in which investors do not provide any finance always exists. It relies on the opportunistic CRA's strategy consisting in lying for sure ("babbling" equilibrium). We focus on the more interesting equilibria in which the CRA is "active", i.e. $V(q) > 0$ for any $q > 0$.

Definition 2. An equilibrium is said to be *active* if $V(q) > 0$ for any $q > 0$.

As usual in models with a long-run player, the discipline of the opportunistic player depends on his discount factor δ. What is more original is the importance of the ratio $\frac{i_0}{I(1)}$ of the revenue i_0 obtained by the CRA from other sources, divided by the maximum income $I(1)$ obtained by rating a complex product. If this ratio is larger than $\frac{1}{\delta} - 1 - \lambda$ (which is always true if $\delta > \frac{1}{1+\lambda}$) then telling the truth (*i.e.*, always giving a bad rating to bad projects) is an equilibrium strategy. This is a simple illustration of the folk theorem for repeated games, stating that if the opportunistic player is sufficiently patient then mimicking the behavior of the truthful type is an equilibrium strategy. We show that this constitutes the unique active equilibrium strategy.[8] On the contrary, if the ratio $\frac{i_0}{I(1)}$ is smaller than $\frac{1}{\delta} - 1 - \lambda$ (which implies that $\delta < \frac{1}{1+\lambda}$) then we show that the equilibrium is not truthful. It involves the opportunistic CRA lying (*i.e.*, giving a good rating to bad projects) for sure for q close to 1 and lying with positive probability otherwise is the unique active equilibrium strategy. Thus the conditions for the opportunistic CRA to be disciplined at equilbrium are clear: they all depend on the ratio of incomes $\frac{I_0}{I(1)}$. More specifically we have:

Proposition 1 *If* $\frac{i_0}{I(1)} > \frac{1}{\delta} - (1+\lambda)$*, there exists a unique active equilibrium. In this equilibrium, the opportunistic CRA always tells the truth:* $x^*(q) \equiv 0$.

Proposition 2 *If* $\frac{i_0}{I(1)} < \frac{1}{\delta} - (1+\lambda)$*, there also exists a unique active equilibrium. In this equilibrium, the opportunistic CRA always lies with a positive probability:* $x^*(q) > 0$ *for all* $q > 0$*. Moreover when* q *is close enough to one, the opportunistic CRA lies with probability 1.*

[8] If $\lambda + \frac{i_0}{I(1)} = \frac{1}{\delta} - 1$ then never lying is still an equilibrium strategy but not necessarily unique.

Proof. See the Appendix.

The proof of Proposition 1 proceeds in two parts.

In the first part, we show that if $\frac{i_0}{I(1)} > \frac{1}{\delta} - (1+\lambda)$ then telling the truth is the unique active equilibrium strategy. This is the easy part. The existence relies on the following argument. In a truthful equilibrium ($x(q) \equiv 0$) reputation does not matter. The value function is a constant equal to $\frac{i_0+\lambda I(1)}{1-\delta}$. Hence (9) writes as $I(1) \leq \frac{\delta}{1-\delta}(i_0 + \lambda I(1))$. Or equivalently, $\frac{i_0}{I(1)} \geq \frac{1}{\delta} - (1+\lambda)$. Since $V(q) = V(q^N) = \frac{i_0+\lambda I(1)}{1-\delta} > 0$, the equilibrium is active.

Uniqueness is also easily proven. We simply show that if there is $q > 0$, for which the opportunistic CRA lies with positive probability and $V(q) > 0$, then telling the truth is a profitable deviation. This establishes the uniqueness of the opportunistic CRA's strategy for the short-term game, independently of the continuation value function $W \in C_+^*$. But since both the corresponding equilibrium strategy $x(q, W) = 0$ and the beginning-of-period valuation resulting from the short-term game $V(x(q, W); W, q) = \frac{i_0+\lambda I(1)}{1-\delta}\mathbf{1}_{\{q>0\}}$ do not depend on W, we obtain the uniqueness of equilibrium for the long-term game. As the truthful CRA is committed to a particular strategy and the investors and issuers' equilibrium updating rule are uniquely determined by the CRA's strategy, these together constitute the unique active equilibrium.

The proof of Proposition 2 is less obvious. We first show that if $\lambda + \frac{i_0}{I(1)} < \frac{1}{\delta} - 1$ and $V \in C_+^*$ then there is an active equilibrium in which the opportunistic CRA lies with positive probability for any $q > 0$ and lies for sure for q close to 1. To prove this, we construct the equilibrium strategy of the opportunistic CRA's. This construction proceeds as follows. When $q = 1$ there is no possible increase in reputation ($q^N = q = 1$). Telling the truth provides a discounted future gain $\frac{\delta[\lambda I(1)+i_0]}{1-\delta}$, while lying provides immediate gain $I(1)$. So when $\frac{i_0}{I(1)}$ is sufficiently small (i.e., lower than $\frac{1}{\delta}-(1+\lambda)$) the present gain from lying is higher than the discounted future gain from telling the truth. Our equilibrium strategy construction starts with $x(1) = 1$ and proceeds by induction: given the equilibrium strategy on the interval $[q_n, q_{n-1}]$, we construct the equilibrium strategy $x(q)$ and the perceived accuracy of ratings $a(q)$ on the interval $[q_{n+1}, q_n]$. We construct a sequence of pairs $(q_n, a_n)_{n\in\mathbb{N}}$, where a_n is the rating accuracy associated with reputation q_n, that is $a_n \equiv 1 - (1 - q_n)x(q_n)$. Observe that q_n and a_n together determine $x(q_n)$. We start this sequence at $q_0 = a_0 = 1$, and for any $n > 0$ we consider (q_n, a_n) such that

$$q_n^N \equiv q_{n-1} \tag{12}$$

and

$$I(a_n) = \delta V(q_n^N). \tag{13}$$

In that manner, q_n corresponds to the reputation from which a rise following a bad rating leads to q_{n-1}. The term a_n corresponds to the rating accuracy for which an opportunistic CRA facing a bad project is indifferent between the present gain from lying and the discounted future gain from telling the truth (as expressed in (10)). By (4)

$$q_n = a_n q_{n-1}. \tag{14}$$

Using (13) and (5) yields

$$V(q_n) = \frac{I(a_n) + i_0}{1 - \delta\lambda}.$$

Using this equation at order $(n-1)$ together with (12) and (13) gives

$$a_n = I^{-1}\left(\alpha\left(I(a_{n-1}) + i_0\right)\right), \tag{15}$$

where $\alpha \equiv \frac{\delta}{1-\delta\lambda}$. Since $I(\cdot)$ is strictly increasing on $[a^*, 1]$ and $\alpha < 1$ the parameters a_n and q_n are both well-defined and strictly decreasing in n. By construction

$$x(q_n) \equiv \frac{1-a_n}{1-q_n} \tag{16}$$

satisfies (10) for any $n \in \mathbb{N}$. At this stage it remains to construct the strategy $x(q)$ for $q \in [q_{n+1}, q_n]$. For any $a \in [a_{n+1}, a_n]$ we define:

$$q(a) \equiv \prod_{k=0}^{n} I^{-1}\left(\frac{I(a) - i_0\frac{\alpha-\alpha^{k+1}}{1-\alpha}}{\alpha^k}\right) \tag{17}$$

and

$$x(q(a)) = \frac{1-a}{1-q(a)}. \tag{18}$$

This is inspired by the fact that by construction we have both, from (14)

$$q_n = \prod_{k=0}^{n} a_{n-k},$$

and from (15)

$$a_n = I^{-1}\left(\alpha^k I(a_{n-k}) + i_0\frac{\alpha-\alpha^{k+1}}{1-\alpha}\right),$$

which is equivalent to

$$a_{n-k} = I^{-1}\left(\frac{I(a_n) - i_0\frac{\alpha-\alpha^{k+1}}{1-\alpha}}{\alpha^k}\right).$$

Then we show that: $q(a) \in [q_{n+1}; q_n]$; $\frac{q(a)}{a} \in [q_n; q_{n-1}]$; and $I(a) = \delta V(\frac{q(a)}{a})$. This establishes the existence. The positive value of $x(q)$ for any $q > 0$ relies in part on the fact that the sequence a_n is strictly decreasing in n with $a_0 = 1$ so (16) is positive for any $n > 0$. Then, we prove that $x(1) = 1$ and exploit the continuity of the value function V on C_+^* to show that $x(q)$ goes to 1 as q goes to 1. The activeness is straightforwardly deduced from $I(a_n) = \alpha^n I(1) + i_0\frac{\alpha-\alpha^{n+1}}{1-\alpha} > 0$ for any $n \in \mathbb{N}$.

Finally, we prove the uniqueness result. We start by fixing a continuation payoff function $W \in C_+^*$ and we show that for any $q \in [0;1]$ there is at most one $x_W^*(q) \in [0,1]$ that realizes the maximum in (6) (temporary equilibrium). Again, as the truthful CRA is committed to a particular strategy and the investors and issuers' equilibrium updating rule are uniquely determined by the CRA's strategy, this proves that there is at most one temporary active equilibrium. Then, we consider the mapping that associates to any end-of-period valuation W the beginning-of-period valuation resulting from the short-term game $T(W) : q \in [0;1] \mapsto V(x_W^*(q); W, q)$. Using a contraction mapping theorem we show that T has a unique fixed point. Consequently, there is a unique active equilibrium.

3 Properties of the non-truthful equilibrium

The properties of the non-truthful equilibrium obtained in Proposition 2 are interesting. The strategic behavior of the opportunistic CRA can be characterized by the family of intervals $[q_{n+1}, q_n], n \geq 0$:

- If the reputation q of the CRA belongs to the top interval $[q_1, q_0]$, its equilibrium strategy is to lie with probability 1 ($x^*(q) \equiv 1$). The accuracy of ratings is $a(q) = q$. Therefore all projects are financed and the reputation of the CRA is lost forever after the first bad project fails (what we call a crisis).
- By induction, formulas (17) and (18) define the accuracy of ratings (the inverse mapping of $a \longmapsto q(a)$) and the probability of lying $q \longmapsto x(q)$, on each interval $[q_{n+1}, q_n]$. After a bad rating, the reputation of the CRA moves to the next interval on the right: $q^N = \frac{q(a)}{a} \in [q_n, q_{n-1}]$. After a good rating, the reputation of the CRA either stays constant (after a success) or is lost forever (after a failure).

The dynamic behavior of the CRA reputation is peculiar: it always increases (weakly) until it is lost forever when a crisis occurs. In the non-truthful equilibrium, the reputation of CRAs behaves like a rating. There are different classes of CRAs with a transition matrix that looks like the following table:

Reputation $\nearrow$	0.91	0.82	...	0.39	0.35	Proba of a crisis	Proba of being lax $x(q)$	spread
0.91	90%					10%	100%	1%
0.82	4.5%	90%				5.5%	55%	1.1%
...			90%					
0.39			8.4%	90%		1.6%	16%	1.1%
0.35				8.5%	90%	1.5%	15%	1.1%
...								

Table 1

This table is generated by the assumption of a piecewise linear function $I(a) = k(a - a^*)$, with parameter values $a^* = 0.9$, $\lambda = 90\%$, $\delta = 0.1$ (the value of k is irrelevant).

The non-truthful equilibrium generates what can be called reputation cycles, as illustrated by the following numerical example, taken from Table 1:

Consider a CRA that starts with a low reputation: $q = 0.35$. It chooses to give a good rating to only 15% of "bad" projects. As a result its reputation increases with proba 8.5% and a crisis only occurs with proba 1.5%. This reputation building phase goes on until the CRA reaches a good reputation, say $q = 0.82$. Then it becomes more lax: gives a good rating to 55% of "bad" projects. But the probability of crisis still only 5.5%. This implies that it may take a long time to detect an opportunistic CRA.

To sum up, reputation cycles can be decomposed into three phases:

- **Phase 1: reputation building:** q which implies that CRAs is very strict: $x^*(q)$ is low, investors are careful (high credit spreads) since perceived rating accuracy $a(q) = 1-(1-q)x^*(q)$ is low.
- **Phase 2: cashing on reputation:** reputation increases, spreads decrease. But the CRA becomes more lax, which implies that the probability of a crisis increases.
- **Phase 3: crisis of confidence:** the opportunistic CRA is detected, the market loses confidence (q brutally falls down to zero), and the market disappears.

The implications of our results are clear:

- Even when monitoring is perfect, it may take a long time to detect an opportunistic CRA.
- The conflict of interest is not solved by reputation concerns.
- Reputation building creates confidence cycles.

The main policy implication is that there is a crucial need for changing the business model of CRAs. Several policy responses are possible. Public supervision of CRAs seems difficult and counterproductive (why have regulators "out-sourced" the monitoring of banks and funds in the first place?). Making CRAs legally liable for their ratings (so far viewed as "opinions") would probably kill the business. Goodhart (2008) proposes another solution, namely to create an independent agency in charge of assessing private ratings.

However the main concern is how to eliminate the conflicts of interest. Our results show that these conflicts of interest are created by the "issuers pay model". Moreover, it is probably impossible to go back to the "investors pay model" (due to free riding, or information leakage). We see as only one possibility to get rid of these conflicts of interest, which is involve a "central platform" (exchange, clearing house, or central depository), as described below.

Our proposal, which we call the platform pays model would work as follows. When a potential issuer wants to apply for credit ratings by a NRSRO, it is required to contact a platform, that could be a clearing house or a central depository. This platform would be completely in control of the ratings process and would also provide record keeping services to the different parties in the securitization operation. The idea is to cut any direct commercial links between issuers and CRAs. The potential issuer would pay a pre-issue fee to the central platform. The central platform would then organize the rating of the pool of loans by one or several NRSROs. The rating fees would be paid by the central platform to the NRSROs. These fees would be independent of the outcome of the rating process and of the fact that the issue finally takes place or not. This would eliminate any perverse incentives for a lax behaviour by CRAs. This would also solve the conflict of interest between issuers and investors, since the central platform's profit maximization depends on appropriately aggregating the interests of the two sides of the market.

4 Imperfect Monitoring

4.1 The Extended Model

In this section, the CRA does not know for sure whether each project will be successful or not. In each period there is again a probability λ that a project is of a good quality. But now, a project of good (resp. bad) quality is successful with probability $p_G \in (0;1]$ (resp. $p_B \in [0;1)$), with $p_G > p_B$. We assume that good firms should be financed (*i.e.*, $p_G X > 1$) but that no financing takes place if investors have no information about projects' quality (*i.e.*, $(\lambda p_G - (1-\lambda)p_B)X < 1$). Now, the threshold a^* below which the CRA's revenue from ratings is equal to zero (i.e., whenever $p(q) \leq \frac{1}{X}$) becomes $a^* \equiv \frac{1-X(\lambda p_G+(1-\lambda)p_B)}{(1-\lambda)(1-p_B X)}$. The posterior beliefs (2) and (3) become

$$\psi(q|S) \equiv q^S = \frac{q}{1+(1-q)x(q)\frac{1-\lambda}{\lambda}\frac{p_B}{p_G}}, \tag{19}$$

and for $p_G < 1$

$$\psi(q|F) \equiv q^F = \frac{q}{1+(1-q)x(q)\frac{1-\lambda}{\lambda}\frac{1-p_B}{1-p_G}}. \tag{20}$$

If $p_G = 1$ then we set $q^F = 0$. Updating is deduced from Bayes rule when it applies and by assuming that a failure is due to an opportunistic CRA otherwise (that is, as under perfect monitoring, when $q = 1$ or $x(q) = 0$). Since the outcomes success and failure are not observed when a project obtains a bad rating the expression of q^N does not change. Observe that again the only way to enhance the reputation is to give a bad rating since $q^S \leq q$, $q^F \leq q$ and $q^N \geq q$.

Figure II depicts the decision tree in the imperfect monitoring case when well rated projects are financed.

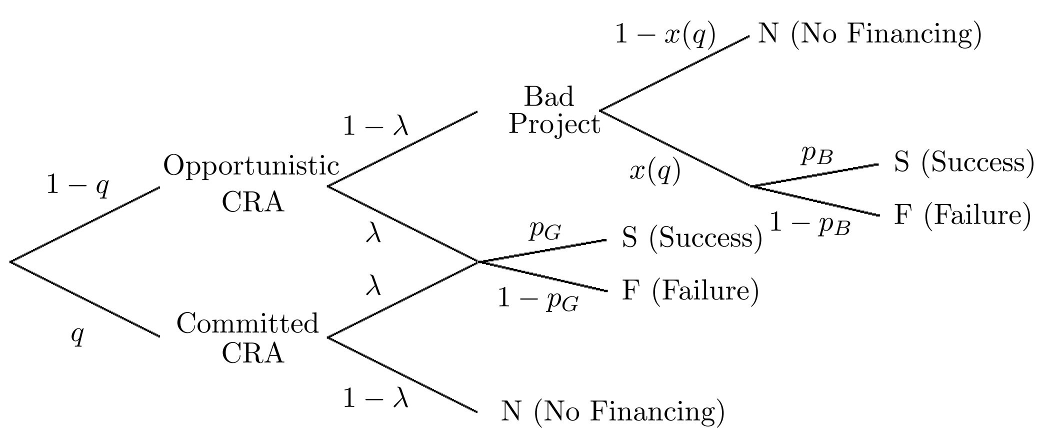

Figure II

Decision tree under imperfect monitoring.

The value function of the opportunistic CRA when playing the strategy x and investors and issuers have posterior q is given by the Bellman equation

$$\begin{aligned} V(q) &= i_0\mathbf{1}_{\{q>0\}} + (\lambda + (1-\lambda)x(q))I(a(q,x)) + \delta\{(\lambda p_G + (1-\lambda)p_B x(q))V(q^S) \\ &\quad +(\lambda(1-p_G) + (1-\lambda)(1-p_B)x(q))V(q^F) + (1-\lambda)(1-x(q))V(q^N)\}. \end{aligned} \tag{21}$$

This Bellman equation expresses that an increase in the probability x of lying (giving a good rating to a bad project) leads to an increase in the present gain and a decrease in the discounted future gain. Indeed in that case the probability to experience a fall in reputation (following a failure or success) increases and the probability to enhance the reputation (following a denied rating) decreases.

According to the one-stage deviation principle an opportunistic CRA's strategy x is an equilibrium strategy if and only if for any q, for any deviation x':

$$(x(q) - x'(q))\left[I(a(q,x(q))) + \delta\{p_B V(q^S) + (1-p_B)V(q^F)\}\right] \geq \delta(x(q) - x'(q))V(q^N) \tag{22}$$

with q^S, q^F and q^N satisfying (19), (20) and (4).

According to the sign of $(x(q) - x'(q))$ the one-stage deviation principle expressed in (22) can be written as:

$x(q) = 1$ is part of an equilibrium strategy if and only if

$$I(a(q,x(q))) + \delta\{p_B V(q^S) + (1-p_B)V(q^F)\} \geq \delta V(q^N); \tag{23}$$

$x(q) = 0$ is part of an equilibrium strategy if and only if

$$I(a(q,x(q))) + \delta\{p_B V(q^S) + (1-p_B)V(q^F)\} \leq \delta V(q^N); \tag{24}$$

and $x(q) \in (0,1)$ is part of an equilibrium strategy if and only if

$$I(a(q,x(q))) + \delta\{p_B V(q^S) + (1-p_B)V(q^F)\} = \delta V(q^N). \tag{25}$$

When choosing an action an opportunistic CRA facing a bad project compare its present gain for giving a good rating $I(a(q,x(q))) + \delta\{p_B V(q^S) + (1-p_B)V(q^F)\}$ to its discounted future gain for giving a bad rating $\delta V(q^N)$.

4.2 A particular case: $p_G = 1$

When $p_G = 1$ the imperfect monitoring only concerns the probability of success for a bad project. Hence $q^F = 0$. This is useful in the expression of (23), (24) and (25) because Property 1 extends to imperfect monitoring.

Property 1'. At equilibrium, $V(q = 0) = 0$.

Proof. By (19), (20) and (4) if $q = 0$ then $q^S = q^F = q^N = 0$. According to (22) we have $(x - x')I(a(q,x(q))) \geq 0$ for any x'. Analogous arguments to the ones used in the proof of Property 1 allow us to conclude. □

When $p_G = 1$, a failure still leads to a permanent loss of reputation, which according reduces the CRA's revenue to zero forever. Again, as shown by the next result, the opportunistic CRA is fully disciplined at equilibrium if and only if the proportions of successful project and exogenous revenue $\lambda + \frac{i_0}{I(1)}$ are large enough. But now the threshold that such proportions have to exceed can be much higher because it is divided by the probability that a bad project fails $1 - p_B$.

Proposition 3 *Assume $p_G = 1$. For any $q > 0$, $x(q) = 0$ is an equilibrium strategy if and only if $\lambda + \frac{i_0}{I(1)} \geq \frac{1-\delta}{\delta(1-p_B)}$. This equilibrium is active.*

Proof. Sufficiency. Let $q > 0$. Let us show that (24) holds. By (20) $q^F = 0$. By (19) and (4), $x(q) = 0$ implies that $q^S = q^N = q$. From (21) and $x(q) = 0$ we get $V(q) = \frac{i_0 + \lambda I(1)}{1-\delta}$. So from $\lambda + \frac{i_0}{I(1)} \geq \frac{1-\delta}{\delta(1-p_B)}$ we obtain $I(1) \leq \frac{\delta(1-p_B)}{1-\delta}(i_0 + \lambda I(1))$ which by Property 1' is (24). Finally, we obtain activeness from $V(q) = \frac{\lambda I(1) + i_0}{1-\delta} > 0$.
Necessity. Let $q > 0$. Using $p_G = 1$, (19), (20), (4) and Property 1' in both the necessary condition for $x(q) = 0$ to be an equilibrium strategy (24) and the expression of $V(q)$ (21) respectively yields to $I(1) \leq \delta(1-p_B)V(q)$ and $V(q) = \frac{i_0 + \lambda I(1)}{1-\delta}$. Introducing this last equality into the previous inequality, and simplifying by $I(1)$ gives $\lambda + \frac{i_0}{I(1)} \geq \frac{1-\delta}{\delta(1-p_B)}$. □

By comparing to the perfect monitoring case, we can see that the likelihood of success for a bad project has a negative effect on the RA's discipline. The higher is p_B the larger must be the proportions of successful project λ and exogenous revenue $\frac{i_0}{I(1)}$ for opportunistic CRA to be well disciplined at equilibrium. Examining this closely, an increase in p_B plays two kind of effects on the CRA's discipline. There is a negative effect: the higher is p_B the higher (resp. lower) is the probability to experience a success (resp. failure) when lying (giving a good rating to a bad project). There is also a positive (resp. another negative) effect: according to (19) (resp. (20)) the higher is p_B the more (resp. less) is the fall in reputation following a success (resp. failure). Proposition 2 simply states that these two negative effects dominate the positive one.

4.3 The general case: $p_G < 1$

When $p_G < 1$ a failure may come from a good project. So as long as $q > 0$ we have $q^F > 0$. This considerably changes the previous results where good projects were perfectly monitored. Now observing a failure is *on* the equilibrium path of a well disciplined opportunistic RA. Thus, no realization allows to detect any deviation from $x(q) = 0$. Furthermore, for any observed outcome (success, failure or no financing) the updated beliefs will not change (i.e., $q^S = q^F = q^N = q$). Therefore by deviating from such a strategy an opportunistic CRA benefits from an increase in its present gain without any change for its future payoff. There is no more discounted future loss associated with the observation of failure that was *off* the equilibrium path under perfect monitoring and then was followed by a definitive fall of reputation to zero. The next result states that when good projects are imperfectly monitored the opportunistic CRA is never well disciplined at equilibrium. It also states that at equilibrium the opportunistic CRA's strategy consists in giving for sure a good rating to bad project for q close to 1. This is due to the fact that when q is close to 1 then q^S, q^F and q^N are all close to 1. So the CRA's future gain is almost no contingent on its actual strategy and it then has incentive to lie for sure in order to maximize its present revenue.

Proposition 4 *Assume $p_G < 1$. At any equilibrium the opportunistic CRA's strategy consists in lying with positive probability for any q and lying for sure for q close to 1.*

Proof. We proceed in three steps. First, we show that at any equilibrium the opportunistic CRA's strategy consists in lying with positive probability for any q. Second, we show that at equilibrium the opportunistic CRA's strategy consists in giving for sure a good rating to bad project for q close to 1.

Step 1. Let $q \in [0;1]$. Assume *per contra* that $x(q) = 0$ is part of an equilibrium strategy. By (19), (20) and (4) we have $q^S = q^F = q^N = q$. So by (24), $I(1) \leq 0$, a contradiction.

Step 3. From (19), (20), (4) and $V \in C_+$ we have

$$\lim_{q\to 1} V(q^S) = \lim_{q\to 1} V(q^F) = \lim_{q\to 1} V(q^N) = V(1).$$

Therefore

$$\lim_{q\to 1} \left[I(a(q, x(q))) + \delta\{p_B V(q^S) + (1 - p_B)\}V(q^F) - V(q^N) \right] = I(1) > 0.$$

Hence (23) holds while (24) and (25) do not hold. $\square$

5 Appendix

Proof of Proposition 1. We show that if $\lambda + \frac{i_0}{I(1)} > \frac{1}{\delta} - 1$ then telling the truth is the unique active equilibrium strategy.

Existence. Let $q > 0$. Let us show that (9) holds. By (4) $x(q) = 0$ implies that $q^N = q$ so $V(q^N) = V(q)$. Hence by (11) $V(q) = i_0 + \lambda I(1) + \delta V(q)$. So (9) writes as $I(1) \leq \frac{\delta}{1-\delta}(i_0 + \lambda I(1))$. Or equivalently, $\frac{i_0}{I(1)} \geq \frac{1}{\delta} - (1 + \lambda)$. So $x(q) = 0$ is an equilibrium strategy. Furthermore it is active as $V(q) = \frac{\lambda I(1)+i_0}{1-\delta} > 0$.

Uniqueness. Let us begin by showing that for any $q > 0$, $x(q) = 0$ is the unique equilibrium strategy such that $V(q) > 0$. Assume, *per contra*, that there is $q > 0$ such that $V(q) > 0$ and $x(q) > 0$ is part of the equilibrium strategy. Therefore the deviation $x'(q) = 0$ is not profitable. This implies $V(q) \geq \frac{\delta V(q^N)+i_0}{1-\delta\lambda}$. Thus

$$\frac{V(q^N)}{1 - \delta\lambda} \left(\delta + \frac{i_0}{V(q^N)} \right) \leq V(q) \leq V(q^N)$$

where the last inequality comes from the facts that $q \leq q^N$ and $V(\cdot) \in C_+^*$. Since from $V(q) > 0$, we deduce that $\delta + \frac{i_0}{V(q^N)} \leq 1 - \delta\lambda$, that is $\frac{i_0}{\delta V(q^N)} \leq \frac{1}{\delta} - (1 + \lambda)$. Now since $x(q) > 0$ is part of an equilibrium strategy we can use (8) and (10) together with the assumption that $I(\cdot)$ is increasing to obtain $\delta V(q^N) \leq I(a(q, x(q))) \leq I(1)$. Thus, $\frac{i_0}{I(1)} \leq \frac{i_0}{\delta V(q^N)} \leq \frac{1}{\delta} - (1+\lambda)$, a contradiction. This establishes the uniqueness of the opportunistic CRA's strategy for the short-term game for any fixed end-of-period valuation $W \in C_+^*$. But since both the corresponding equilibrium strategy $x(q, W) = 0$ and the beginning-of-period valuation resulting from the short-term game $V(x(q, W); W, q) = \frac{i_0 + \lambda I(1)}{1-\delta} \mathbf{1}_{\{q>0\}}$ do not depend on W, we obtain the uniqueness of the opportunistic CRA's strategy for the long-term game. $\square$

Proof of Proposition 2 Again, we proceed in two parts. In Part 1, we construct an equilibrium that satisfies the stated properties. In Part 2, we prove the uniqueness result.

Part 1: Existence. We proceed in four steps. First, we construct an equilibrium strategy for an opportunistic CRA's. Second, we show that this strategy consists in lying with positive probability for any $q > 0$. Third, we show that it consists in giving for sure a good rating to bad projects for q close to 1. Fourth, we show that this strategy supports an active equilibrium.

Step 1. By assumption $I(\cdot)$ is strictly increasing in a on $[a^*; 1]$, with $I(a^*) = 0$. So the inverse function $I^{-1}(\cdot)$ is well-defined and strictly increasing on $(0; I(1)]$. Let $\alpha \equiv \frac{\delta}{1-\lambda\delta}$. By assumption $\lambda \leq \lambda + \frac{i_0}{I(1)} < \frac{1}{\delta} - 1$, so $\alpha < 1$. We define a sequence (a_n, q_n) as follows:

$$\begin{cases} a_0 \equiv 1 \\ q_0 \equiv 1 \end{cases},$$

and for any $n > 0$,

$$\begin{cases} a_n \equiv I^{-1}\left(\alpha\left(I(a_{n-1}) + i_0\right)\right) \\ q_n \equiv a_n q_{n-1} \end{cases}.$$

The parameters a_n and q_n are both well-defined and strictly decreasing in n. By introduction on n, a_n can be written as

$$a_n = I^{-1}\left(\alpha^n I(1) + i_0 \frac{\alpha - \alpha^{n+1}}{1-\alpha}\right) \tag{26}$$

For any $n \in \mathbb{N}$, for any $a \in [a_{n+1}; a_n]$ let

$$q(a) \equiv \prod_{k=0}^{n} I^{-1}\left(\frac{I(a) - i_0 \frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right). \tag{27}$$

Observe that by construction $q(a) \in [q_{n+1}; q_n]$ and $\frac{q(a)}{a} \in [q_n; q_{n-1}]$. This is because for any $k \in \{0, 1, ..., n\}$ we have for $a \in [a_{n+1}, a_n]$:

$$I^{-1}\left(\frac{I(a_{n+1}) - i_0 \frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right) \leq I^{-1}\left(\frac{I(a) - i_0 \frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right) \leq I^{-1}\left(\frac{I(a_n) - i_0 \frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)$$

.

Now using (26), we get:

$$I^{-1}\left(\alpha^{n+1-k} I(1) + i_0 \frac{\alpha - \alpha^{n+2-k}}{1-\alpha}\right) \leq I^{-1}\left(\frac{I(a) - i_0 \frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right) \leq I^{-1}\left(\alpha^{n-k} I(1) + i_0 \frac{\alpha - \alpha^{n+1-k}}{1-\alpha}\right)$$

from which we obtain

$$\begin{aligned}
q_{n+1} &= \prod_{k=0}^{n} I^{-1}\left(\alpha^{n+1-k}I(1)+i_0\frac{\alpha-\alpha^{n+2-k}}{1-\alpha}\right)\\
&\leq \prod_{k=0}^{n} I^{-1}\left(\frac{I(a)-i_0\frac{\alpha-\alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)=q(a)\\
&\leq \prod_{k=0}^{n} I^{-1}\left(\alpha^{n-k}I(1)+i_0\frac{\alpha-\alpha^{n+1-k}}{1-\alpha}\right)=q_n.
\end{aligned}$$

Similarly:

$$\begin{aligned}
q_{n} &= \prod_{k=1}^{n} I^{-1}\left(\alpha^{n+1-k}I(1)+i_0\frac{\alpha-\alpha^{n+2-k}}{1-\alpha}\right)\\
&\leq \prod_{k=1}^{n} I^{-1}\left(\frac{I(a)-i_0\frac{\alpha-\alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)=\frac{q(a)}{a}\\
&\leq \prod_{k=1}^{n} I^{-1}\left(\alpha^{n-k}I(1)+i_0\frac{\alpha-\alpha^{n+1-k}}{1-\alpha}\right)=q_{n-1}.
\end{aligned}$$

Observe also that at equilibrium due to (4) with this notation when $q(a)<1$ the value function expressed in (11) writes as

$$V(q(a))=\frac{i_0\mathbf{1}_{\{q>0\}}+\left(\lambda+(1-\lambda)\frac{1-a}{1-q(a)}\right)I(a)+\delta(1-\lambda)\left(1-\frac{1-a}{1-q(a)}\right)V\left(\frac{q(a)}{a}\right)}{1-\delta\lambda}. \tag{28}$$

When $q=1$ it is easily checked that $x(q=1)=1$ satisfies (8) (see the explanation preceding Proposition).
We now show that for any a such that $q(a)<1$ the opportunistic CRA's strategy defined by

$$x(q(a))\equiv\frac{1-a}{1-q(a)} \tag{29}$$

satisfies (10). That is, in terms of our actual notation

$$I(a)=\delta V\left(\frac{q(a)}{a}\right),$$

or equivalently using this in (28)

$$V(q(a))=\frac{I(a)+i_0\mathbf{1}_{\{q>0\}}}{1-\delta\lambda}. \tag{30}$$

By induction we now prove that the property

$$P(n): a\in[a_{n+1};a_n] \text{ satisfies (30)}$$

is true for every $n \in \mathbb{N}$. Clearly, $P(0)$ is true as from $q(a) = a$ in (28) we obtain (30).
Let us check that $P(1)$ is true. Let $a \in [a_2; a_1]$. As stated previously $\frac{q(a)}{a} \in [q_1; q_0]$. So using $P(0)$ we get

$$V\left(\frac{q(a)}{a}\right) = \frac{I\left(a'\right) + i_0 \mathbf{1}_{\{q>0\}}}{1 - \delta\lambda},$$

with $a' \in [a_1; a_0]$ such that $q(a') = \frac{q(a)}{a}$. Using the definition of $q(\cdot)$ we have

$$a' = q(a') = \frac{q(a)}{a} = I^{-1}\left(\frac{I(a)}{\alpha} - i_0\right).$$

Therefore

$$V\left(\frac{q(a)}{a}\right) = \frac{I(a') + i_0}{1 - \delta\lambda} = \frac{I(a)}{\alpha(1 - \delta\lambda)} = \frac{I(a)}{\delta}$$

Using this in (28) we obtain (30).
Suppose $P(n)$ holds. Let us show that $P(n+1)$ is true. Let $a \in [a_{n+2}; a_{n+1}]$. By definition

$$q(a) = \prod_{k=0}^{n+1}\left(I^{-1}\left(\frac{I(a) - i_0\frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)\right). \tag{31}$$

Again, as stated previously $\frac{q(a)}{a} \in [q_{n+1}; q_n]$. Using $P(n)$ we then obtain

$$V\left(\frac{q(a)}{a}\right) = \frac{I\left(a'\right) + i_0}{1 - \delta\lambda},$$

with $a' \in [a_{n+1}; a_n]$ such that $q(a') = \frac{q(a)}{a}$. Let us show that

$$I(a') = \frac{I(a)}{\alpha} - i_0. \tag{32}$$

By definition of $q(a')$ we have

$$q(a') = \prod_{k=0}^{n}\left(I^{-1}\left(\frac{I(a') - i_0\frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)\right). \tag{33}$$

Expressing the equality $\frac{q(a)}{a} = q(a')$ with (31) and (33) yields to

$$\prod_{k=1}^{n+1}\left(I^{-1}\left(\frac{I(a) - i_0\frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}\right)\right) = \prod_{k=1}^{n+1}\left(I^{-1}\left(\frac{I(a') - i_0\frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^{k-1}}\right)\right). \tag{34}$$

For any $k \in \{1, ..., n+1\}$, for any $\bar{I} \in [0; I(1)]$ consider the function

$$f_k(\bar{I}) \equiv \frac{\bar{I} - i_0\frac{\alpha - \alpha^{k+1}}{1-\alpha}}{\alpha^k}.$$

Observe that (34) can be written as

$$\prod_{k=1}^{n+1}\left(I^{-1}\left(f_k\left(\frac{I(a)}{\alpha} - i_0\right)\right)\right) = \prod_{k=1}^{n+1}\left(I^{-1}\left(f_k\left(I(a')\right)\right)\right).$$

The functions $I^{-1}(\cdot)$ and $f_k(\cdot)$, for any $k \in \{2, ..., n+1\}$, are strictly increasing. So if $I^{-1}\left(\frac{I(a)}{\alpha} - i_0\right)$ is strictly lower (resp. higher) than $I^{-1}(I(a'))$ then it is also true for $I^{-1}\left(f_k\left(\frac{I(a)}{\alpha} - i_0\right)\right)$ with respect to $I^{-1}(f_k(I(a')))$. Therefore the unique solution to (34) satisfies (32). Now using (32) in $P(n)$ we obtain

$$V\left(\frac{q(a)}{a}\right) = \frac{I(a)}{\alpha(1-\delta\lambda)} = \frac{I(a)}{\delta}.$$

Finally, using this in (28) we obtain (30).

Step 2. Let us show that the strategy constructed in step 1 consists in lying with positive probability for any $q > 0$. As we already stated $x(q = 1) = 1$. Let $q < 1$. By construction, there are $n \in \mathbb{N}$ and $a \in [a_{n+1}; a_n]$ such that $q = q(a)$, with $q(a)$ defined by (27). Since $q(\cdot)$ is strictly increasing in a on $[a^*; 1]$ from $q < 1 = q_0$ we have $a < 1 = a_0$. Hence, from (29) we get $x(q(a)) > 0$.

Step 3. Let us show that the strategy constructed in Step 1 consists in giving for sure a good rating to unsuccessful project for q close to 1. By assumption $\frac{i_0}{I(1)} < \frac{1}{\delta} - (1+\lambda)$, that is $1 > \frac{\delta}{1-\delta\lambda}\left(1 + \frac{i_0}{I(1)}\right)$. Multiplying both sides of the previous inequality by $I(1)$ and then using (11) for $x(q) = 1$ we obtain $I(1) > \delta V(1)$. Since $I(a(\cdot, x(\cdot)))$ is continuous in q, the left-side of the inequality

$$I(a(q, x(q))) > \delta V(1) \geq \delta V(q^N)$$

holds for q sufficiently close to 1. The right-side of this inequality holds for $V \in C_+$. Hence (8) holds, so $x(q) = 1$ for q sufficiently close to 1.

Step 4. To conclude Part A observe that by definition $I(a_n) = \alpha^n I(1) + i_0 \frac{\alpha - \alpha^{n+1}}{1-\alpha} > 0$ for any $n \in \mathbb{N}$, so the equilibrium is such that $V(q) > 0$ for any $q > 0$. Hence it is active.

Part 2: Uniqueness. We shall exploit the following lemma:

Lemma A. Assume $V(\cdot) \in C_+$. At equilibrium, if $x(q)$ is such that $I(a(q, x(q))) = 0$ then $V(q) = 0$.

Proof of Lemma A. From (4) and $V(\cdot) \in C_+$ we get $V(q) \leq V(q^N)$. If $x(q) > 0$ then (8) and (10) imply that $\delta V(q^N) \leq I(a(q, x(q))) = 0$. Thus $V(q) \leq V(q^N) = 0$. If $x(q) = 0$ then from (2)-(4) $q^N = q^S = q \geq q^F$ so $I(a(q^N, x(q^N))) = I(a(q^S, x(q^S))) = I(a(q, x(q))) = 0 \geq I(a(q^F, x(q^F)))$ and $V(q) = 0$. $\square$

At first, fix a value function of future reputation $W \in C_+$ and let us solve the temporary equilibrium. We shall show that for any $q \in [0; 1]$ there is at most one $x^*(W, q) \in [0, 1]$ part of an equilibrium strategy profile such that the associated beginning-of-period valuation $V(W, q)$ is strictly positive. As the truthful CRA is committed to a particular strategy and the investors and issuers' equilibrium updating rule are uniquely determined by the CRA's strategy, this will prove that there is at most one temporary equilibrium such that $V(q) > 0$.
Denote $x^*(W, q)$ as x^*. Since x^* is part of an equilibrium strategy by Property 1 we have $W(0) = 0$. So, the associated beginning-of-period valuation $V(W, q)$ writes as

$$i_0 \mathbf{1}_{\{q>0\}} + (\lambda + (1-\lambda)x^*) I(a(q, x^*)) + \delta\{\lambda W(q) + (1-\lambda)(1-x^*) W(q^N)\} \tag{35}$$

Consider the function

$$\varphi(W,q)[x] \equiv I(a(q,x)) - \delta W(q^N),$$

with q^N satisfying (4). $\varphi(W,q)$ is decreasing in x on $[0,1]$ and strictly decreasing in x on $[0, \frac{\lambda(X-1)}{(1-q)(1-\lambda)}]$. (Indeed, $I(a(q,x))$ is strictly decreasing in x on $[0, \frac{\lambda(X-1)}{(1-q)(1-\lambda)}]$ and is equal to zero elsewhere. W is increasing with q and q^N is increasing with x.) We distinguish three cases.
i) $\varphi(W,q)[x=0] < 0$. By (8), (9) and (10) x^* is then equal to 0. (Observe that $I(a(q,x^*=0)) > 0$ so $V(W,q) > 0$.)
ii) $\varphi(W,q)[x=1] > 0$. By (8), (9) and (10) x^* is then equal to 1. (Observe that $I(a(q,x^*=1)) > \delta W(q^N) \geq 0$ so $V(W,q) > 0$.)
iii) $\varphi(W,q)[x=0] \geq 0 \geq \varphi(W,q)[x=1]$. If $V(W,q) > 0$ then Lemma A implies that $I(a(q,x(q))) > 0$, that is $x^* < \frac{\lambda(X-1)}{(1-q)(1-\lambda)}$. Since $\varphi(W,q)$ is continuous and strictly decreasing on $[0, \frac{\lambda(X-1)}{(1-q)(1-\lambda)}]$ the equation $\varphi(W,q)[x] = 0$ has at most one root x^* that yields to $V(W,q) > 0$.
Now, $\forall W \in C_+$, $\forall q \in [0,1]$, let $V(x^*(W,q);W,q)$ be the beginning-of-period valuation associated with $x^*(W,q)$ a current temporary equilibrium strategy. As previously shown, if $V(x^*(W,q);W,q) > 0$ then $x^*(W,q)$ is unique. $V(x^*(W,q);W,q)$ is then single-valued on $C_+ \times [0,1]$. Furthermore, the continuity of the function $\varphi(W,q)$ in (W,q) implies the continuity of its unique root $x^*(W,q)$ on $[0, \frac{\lambda(X-1)}{(1-q)(1-\lambda)}]$. If $x^*(W,q) \geq \frac{\lambda(X-1)}{(1-q)(1-\lambda)}$ then $I(a(q,x^*(W,q))) = 0$, so by Lemma A $V(x^*(W,q);W,q) = 0$. In turns, it implies that $V(x^*(W,q);W,q)$ is continuous in (W,q) on $C_+ \times [0,1]$. Let T be the mapping that associates to any end-of-period valuation W the beginning-of-period valuation resulting from the short-term game

$$T(W) : q \in [0;1] \mapsto V(x^*(W,q);W,q).$$

T maps C_+ continuously into itself. Let us show that T has a unique fixed point. For this, using a contraction mapping theorem it suffices to show that T is a contraction. According to (4) $q^N = \frac{q}{a}$ whenever $a \neq 0$. Also, if $x=1$ then $a(q,x) = q$ and $q^N = 1$; if $x = 0$ then $a(q,x) = 1$ and $q^N = q$. According to (8), (9) and (10) for any q and W we get

$$a(q,W) \begin{cases} = q \text{ if } I(q) \geq \delta W(1) \\ = 1 \text{ if } I(1) \leq \delta W(q) \\ \in [q,1] \text{ if } I(a) = \delta W(\frac{q}{a}) \end{cases} \tag{36}$$

If $a < 1$ then either $[a = q$ and $x^* = 0]$ or $[a > q$ and $I(a) = \delta W(q^N)]$. In both cases, using (35) we obtain

$$T(W)[q] = i_0 \mathbf{1}_{\{q>0\}} + I(a(q,W)) + \delta\lambda W(q). \tag{37}$$

If $a = 1$ then $x^* = 0$ and (35) gives

$$T(W)[q] = i_0 \mathbf{1}_{\{q>0\}} + \lambda I(1) + \delta W(q).$$

But $a = 1$ never occurs at equilibrium as it requires $I(1) \leq \delta W(q)$ with $W(q, a=1) = \frac{i_0 \mathbf{1}_{\{q>0\}} + \lambda I(1)}{1-\delta}$ so $1 \leq \frac{\delta}{1-\delta}\left(\frac{i_0}{I(1)} + \lambda\right)$ which contradicts $\frac{i_0}{I(1)} < \frac{1}{\delta} - (1+\lambda)$.
From (37) we get

$$W(q) = \frac{i_0 \mathbf{1}_{\{q>0\}} + I(a(q,W))}{1-\delta\lambda} \leq \frac{i_0 \mathbf{1}_{\{q>0\}} + I(1)}{1-\delta\lambda} \leq \frac{i_0 + I(1)}{\delta},$$

as $I(\cdot)$ is increasing and $0 \leq \frac{i_0}{I(1)} < \frac{1}{\delta} - (1+\lambda)$, so $W(\cdot)$ is bounded.
Now, let us establish that T is a contraction. Fix W_1, W_2 and q. As by assumption $0 \leq \frac{i_0}{I(1)} < \frac{1}{\delta} - (1+\lambda)$, so that $\delta(1+\lambda) < 1$ it suffices to show that

$$|(T(W_1) - T(W_2))[q]| \leq \delta(1+\lambda) \|W_1 - W_2\|,$$

where $\|\cdot\|$ is the supremum norm. We use the following lemma.

Lemma B. Let $a_i \equiv a(q, W_i)$, $i = 1, 2$. If (36) holds then $|I(a_1) - I(a_2)| \leq \delta \|W_1 - W_2\|$.

Proof of Lemma B. The case $a_1 = a_2$ is obvious. Assume $a_1 < a_2$. We have already seen that $a_2 = 1$ cannot occur at equilibrium. We distinguish two cases.
If $q < a_1 < a_2 < 1$ then

$$\begin{aligned} |I(a_1) - I(a_2)| &= I(a_2) - I(a_1) = \delta \left[W_2\left(\frac{q}{a_2}\right) - W_1\left(\frac{q}{a_1}\right) \right] \\ &\leq \delta \left[W_2\left(\frac{q}{a_2}\right) - W_1\left(\frac{q}{a_2}\right) \right] \leq \delta \|W_2 - W_1\|, \end{aligned}$$

as we have (36), $I(\cdot)$ and $W(\cdot)$ are increasing and $a_1 < a_2$.
If $q = a_1 < a_2$ then from (36)

$$I(a_1) \geq \delta W_1(1).$$

So

$$\begin{aligned} |I(a_1) - I(a_2)| &= I(a_2) - I(a_1) \leq \delta \left[W_2\left(\frac{q}{a_2}\right) - W_1(1) \right] \\ &\leq \delta \left[W_2(1) - W_1(1) \right] \leq \delta \|W_2 - W_1\|. \qquad \square \end{aligned}$$

Now, applying Lemma B, we obtain

$$\begin{aligned} |(T(W_1) - T(W_2))[q]| &= I(a_1) - I(a_2) + \delta\lambda \left[W_1(q) - W_2(q) \right] \\ &\leq \delta(1+\lambda) \|W_1 - W_2\|. \qquad \square \end{aligned}$$

References

Benabou, R., and G. Laroque (1992): "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility," Quarterly Journal of Economics, 107(3), 921–958.

Boot, A., Milbourn, T., and A. Schmeits (2006): "Credit ratings as coordinating mechanisms. The Review of Financial Studies 19, 81-118.

Carlson, M. and Hale, G. B. (2006) "Rating Agencies and Sovereign Debt Rollover," Topics in Macroeconomics: Vol. 6 : Iss. 2, Article 8.

Diamond, D.W.(1989): "Reputation Acquisition in Debt Markets," Journal of Political Economy, 97 (4), 828–862.

Faure-Grimaud, A., E. Peyrache and L. Quesada (2007): "The Ownership of Ratings," wp, Financial Markets Group, LSE, London.

Goodhart, C. (2008): "The Regulatory Response to the Financial Crisis," FMG Special Papers sp177, Financial Markets Group.

Kuhner, C. (2001): "Financial rating agencies: are they credible? – Insights into the reporting incentives of rating agencies in times of enhanced systemic risk," Schmalenbach Business Review 53, p. 2-26.

Lizzeri, A. (1999): "Information Revelation and Certification Intermediaries," RAND Journal of Economics, The RAND Corporation, vol. 30(2), pages 214-231, Summer.

Mailath, G. J., and L. Samuelson (2006): "Repeated Games and Reputations: Long-Run Relationships," Oxford University Press.

Mason, J. R. and J. Rosner (2007), "Where Did the Risk Go? How Misapplied Bond Ratings Cause Mortgage Backed Securities and Collateralized Debt Obligation Market Disruptions," SSRN Working Paper #1027475.

Morris, S. (2001), "Political Correctness," Journal of Political Economy, Vol 109, No 2, pp. 231-265

Sanela Č.H. and A. Niessen (2007): "How do Self-fulfilling Prophecies affect Financial Ratings? An experimental study," wp

Skreta V. and L. Veldkamp (2008): "Cherry-Picking and Asset Complexity: A Theory of Ratings Inflation," wp.

Sobel, J. (1985): "A Theory of Credibility," Review of Economic Studies, LII, pp. 557-573.