Search and matching frictions and optimal monetary policy*

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Abstract

A recent literature has merged the New Keynesian and the search and matching frameworks, which has allowed the former to analyze the joint dynamics of unemployment and inflation. This paper analyzes optimal monetary policy in this kind of hybrid framework. I show that zero inflation is optimal when all wages are Nash bargained in every period and the economy’s steady state is efficient. In the more realistic case in which nominal wage bargaining is staggered, a case against price stability arises: in response to real shocks, the central bank should use price inflation so as to avoid excessive unemployment volatility and excessive dispersion in hiring rates. For a plausible calibration, the welfare loss under the zero inflation policy is about three times as large as under the optimal policy.

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1 Introduction

The search and matching paradigm has become a powerful tool for the analysis of unemployment and the labor market.¹ It is able to accommodate a wide range of labor market policies and analyze their long-run effect on unemployment and wages. When incorporated into otherwise standard real business cycle (RBC) models, it has been shown to improve significantly their empirical performance.² More importantly, it allows to analyze the cyclical behaviour of unemployment, vacancies and job flows, important phenomena which general equilibrium models based on Walrasian labor markets are not designed to address.

Parallel to this literature, the New Keynesian model has emerged as the standard model of the monetary transmission mechanism. In its simplest version, the New Keynesian model incorporates monopolistic competition and staggered price setting into the standard RBC model. Because it is based on optimizing behaviour, it allows for rigorous welfare analysis of alternative monetary policy rules. This, together with its simplicity, has allowed the model to become the workhorse for the analysis of optimal monetary policy.³ However, its assumption of Walrasian labor markets (inherited from the RBC model) means that it is unable to say anything about unemployment. This is somewhat surprising, given that central banks have traditionally been concerned with the joint dynamics of unemployment and inflation, i.e. the Phillips curve.

The last few years have witnessed the integration of both frameworks.⁴ This literature however has focused on the positive implications of this integration, i.e. how search and matching frictions improve the empirical performance of the New Keynesian model. In this paper, I address a different but equally important question within this hybrid framework: the analysis of optimal monetary policy.

With this aim, I construct a model economy where the presence of search frictions in the labor market prevents some unemployed workers from finding jobs and some vacancies from being filled. The flow of meetings between jobseekers and vacancies is given by the so-called matching function. Aggregate hiring by firms determines the dynamics of unemployment. Inside each firm, the management and the workers bargain over wages and hours per worker. Finally, prices are set in a staggered fashion.

First I analyze the benchmark case in which all wages in the economy are Nash-bargained

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¹For a simple analysis of the search and matching model, see Pissarides (2000, Ch. 1).
in every period. This is what I call the flexible-wage equilibrium. I will show that in this case, and provided the economy’s steady state is efficient, the central bank can implement the efficient equilibrium by keeping the price level constant. This way, it can eliminate the distortionary effects of price staggering, because those firms that cannot reset prices would not want to change them anyway. This result can be thought of as a ‘case for price stability’, in the sense that it requires price level constancy even in response to real shocks.\(^5\)

Perfect wage flexibility however is not a realistic assumption. In most industrialized economies, nominal wages typically remain fixed for several periods and adjust in a staggered fashion (see Taylor, 1999, and the references therein). Therefore, I also analyze the more relevant case of staggered nominal wage bargaining, i.e. in each period only a fraction of firms renegotiate nominal wages with their employees.\(^6\) In response to real shocks, the failure of some nominal wages to adjust creates two kinds of distortions. First, the resulting rigidity in the average real wage translates into inefficient aggregate job creation and therefore inefficient unemployment fluctuations. Second, the ensuing wage dispersion across firms leads to inefficient dispersion in hiring rates. The zero inflation policy is no longer optimal for the following reason. By controlling the inflation rate, the central bank has leverage over the real value of nominal wages. Under the optimal policy commitment, the central bank uses price inflation so as to bring real wages closer to their flexible-wage levels. This reduces the two distortions arising from nominal wage staggering. First, the greater flexibility in real wages reduces the distortion in the unemployment path. Second, since actual wages are closer to their flexible-wage targets, nominal wages in renegotiating firms adjust by less, which reduces wage dispersion. For a reasonable calibration of the model, the welfare loss under the zero inflation policy is approximately three times as large as under the optimal commitment. This result suggests the existence of a case against strict price stability as the only goal of monetary policy.

Previous research in the New Keynesian tradition emphasized the existence of a similar case against price stability in the presence of both price and wage staggering (see Erceg et al., 2000). Relative to this line of research, the current framework offers an important theoretical advantage. As is well known, (New) Keynesian models of wage stickiness are subject to the following criticism due to Barro (1977). Given the ongoing nature of most employment relationships, we would expect employers and employees to neutralize any distortionary effects of wage stickiness. Therefore, the case against price stability motivated by wage stickiness would

\(^5\)Goodfriend and King (2001) and Woodford (2003, Ch. 6) emphasized the existence of a case for price stability in the presence of perfectly competitive labor markets.

\(^6\)Gertler and Trigari (2006) have introduced staggered bargaining of real wages in a RBC framework with search and matching frictions, with the aim of reconciling the smooth cyclical behavior of real wages with the high volatility of labor market activity in the US.
be based on the imposition of arbitrary inefficiencies on existing jobs. By introducing search frictions, I can analyze the distortionary effects of nominal wage staggering in a way that respects the private efficiency of employment relationships. On the one hand, search frictions create a bargaining set between employer and employee; even if the nominal wage is sticky, as long as it remains inside this bargaining set it does not affect the continuity of the match. On the other hand, hours per employee are determined efficiently, i.e. by maximizing the joint surplus of the match, which is itself independent of the wage. Therefore, while nominal wage staggering distorts the rate at which employment relationships are formed, it does not distort existing relationships. In other words, the case against price stability presented here is immune to Barro’s critique.

Search frictions and the efficiency of employment relationships also have important policy implications that differ from the standard New Keynesian analysis. In the Erceg et al. (2000) model of monopolistic competition in both labor and goods markets, closing the output gap (i.e. the gap between actual output and its flexible-price-and-wage level) is nearly optimal for any degree of price and wage staggering. This result stems from the symmetry between labor and goods markets, and the existence of similar inefficiencies in both markets. Search frictions introduce an explicit distinction between employment and hours per employee. The same frictions imply that in the short run firms adjust output by adjusting hours per employee; therefore, conditional on the employment stock, closing the output gap is equivalent to stabilizing hours per worker. Under bilateral efficiency, hours per worker may be distorted by price staggering but not by wage staggering. As a result, the policy that eliminates the distortionary effects of price staggering (zero inflation) is the same as the policy that stabilizes output. Therefore, conditional on the employment stock, closing the output gap is just as suboptimal as the zero inflation policy.

In independent work, other authors have analyzed optimal monetary policy in New Keynesian models with search and matching frictions. A closely related paper is Blanchard and Gali (2006). They present a simple integration of both frameworks that allows for analytical solutions. They find that real wage rigidity creates a case against price stability. An important difference between our papers is how we deviate from the flexible wage benchmark. Instead of considering staggered nominal wage bargaining, Blanchard and Gali assume that real wages follow a weighted average of the Nash real wage and a constant real wage. The latter is an example of a real wage norm, in Hall’s (2005) terminology. An advantage of assuming staggered nominal wage contracts is that the fraction of unchanged wages can be calibrated to match the

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7Hall (2005) first emphasized the privately efficient nature of wage stickiness in this framework.
8Woodford (2003, Ch. 6) first noticed this implication of the Erceg et al. (2000) model.
average duration of nominal wage contracts, whereas a partial adjustment coefficient in a wage equation must be calibrated indirectly at best. More importantly, the real-wage-norm specification may lead to very different policy implications. Once I introduce the latter specification in my model (replacing staggered nominal wages), zero inflation becomes nearly optimal for any degree of real wage rigidity. The reason is that, when real wages are a weighted average of the Nash real wage and a constant real wage (or last period’s real wage), the central bank loses most of its leverage over real wages and thus its ability to close the gap between actual and Nash real wages. It then finds it optimal to focus on stabilizing inflation.

Faia (2006a) analyzes how the size of the steady state distortions (monopolistic competition and search externalities) affects the optimal variability of the inflation rate in response to shocks, in a model where price stickiness is introduced by assuming quadratic costs of price adjustment. Her analysis complements the one presented here, which is constrained to the case of an efficient steady state. She finds that optimal inflation volatility is U-shaped with respect to workers’ bargaining power. Finally, Faia (2006b) analyzes optimal Taylor rules in a similar kind of model, finding that a rule that responds both to unemployment and inflation performs best.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 obtains the efficient equilibrium in this economy. Section 4 proves that zero inflation is optimal when wages are flexible and the steady state is efficient. Section 5 introduces staggered bargaining of nominal wages. Section 6 casts the monetary policy problem in a linear-quadratic representation, which facilitates the understanding of the central bank’s stabilization goals and trade-offs. The model is then calibrated and simulated both under the zero inflation policy and the optimal commitment. The implications of targeting the output gap, as well as replacing staggered nominal wages with a real wage norm, are discussed in section 7. Section 8 concludes.

2 The model

The following model is a dynamic, stochastic, general equilibrium model of an economy characterized by search and matching frictions in the labor market. I consider explicitly both margins of labor: employment and hours per employee. Nominal goods prices are set in a staggered fashion. Regarding nominal wage bargaining, I will consider both the case of period-by-period renegotiation and the case of staggered bargaining.

On this point, see also Gertler and Trigari (2006).
2.1 The matching function

In this economy, the presence of search frictions in the labor market prevents some jobseekers from finding jobs and some vacant positions from being filled in each period. When a firm finds a suitable job applicant, we say that a match has been formed. The number of matches formed in period $t$ is a given by a matching function,

$$m(v_t, u_t),$$

where $v_t$ is the total number of vacancies posted by firms and $u_t$ is the total number of unemployed workers. Normalizing the labor force to 1, $u_t$ also represents the unemployment rate. The matching function is strictly increasing and strictly concave in both arguments. Assuming constant returns to scale, the matching rate for unemployed workers, or job-finding rate, is given by

$$p(\theta_t) \equiv m(\theta_t, 1),$$

where $\theta_t \equiv v_t/u_t$ is an indicator of labor market tightness.\textsuperscript{10} The rate $p(\theta)$ is increasing in $\theta$: the tighter the labor market, the easier it is for unemployed workers to find jobs. Similarly, the matching rate for vacancies is given by

$$q(\theta_t) \equiv m\left(1, \frac{1}{\theta_t}\right).$$

The rate $q(\theta)$ is decreasing in $\theta$: the tighter the labor market, the more difficult it is for firms to fill vacant positions. Notice that $p(\theta_t) = \theta_t q(\theta_t)$.

2.2 Households

In the presence of unemployment risk, we may observe differences in consumption levels between employed and unemployed consumers. However, under the assumption of perfect insurance markets, consumption is equalized across consumers. This is equivalent to assuming the existence of a large representative household, as in Merz (1995). In this household, a fraction $n_t = 1 - u_t$ of its members are employed. The remaining fraction $u_t$ search for jobs. All members pool their income so as to ensure equal consumption across members. The household’s

\textsuperscript{10}See Petrongolo and Pissarides (2001) for empirical evidence of constant returns to scale in the matching function for several industrialized economies.
welfare criterion is given by
\[ H_t = u(c_t) - \int_0^1 n_{it} [v(h_{it}) + b] di + \beta E_t H_{t+1}, \]
where
\[ c_t \equiv \left( \int_0^1 \frac{\gamma - 1}{c_{jt}^\gamma} dj \right)^{\frac{1}{\gamma-1}} \]
is the Dixit-Stiglitz basket of final goods (with \( \gamma > 1 \), \( n_{it} \) and \( h_{it} \) are the number of workers and hours per worker respectively in firm \( i \in [0, 1] \), and \( b \) is fixed work disutility (e.g. time spent commuting). The function \( u(\cdot) \) is strictly increasing and strictly concave, and \( v(\cdot) \) is strictly increasing and strictly convex. From cost minimization, nominal spending in final goods is given by \( P_t c_t \), where
\[ P_t \equiv \left( \int_0^1 P_{jt}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \]
is the Dixit-Stiglitz price index. The household faces the following budget constraint,
\[ (1 + i_{t-1}) B_{t-1} + \int_0^1 n_{it} W_{it} di + P_t r_t \bar{k} + \Pi_t \geq P_t c_t + B_t, \]
where \( i_{t-1} \) is the nominal interest rate between periods \( t - 1 \) and \( t \), \( B_{t-1} \) are holdings of one-period nominal bonds, \( W_{it} \) is the nominal wage in firm \( i \), \( r_t \) is the real rental rate on the household’s fixed stock of physical capital, \( \bar{k} \), and \( \Pi_t \) are nominal dividends from the firm sector. The household chooses \( c_t \) and \( B_t \) to maximize welfare subject to its budget constraint. The resulting first order conditions can be combined into the standard consumption Euler equation,
\[ u'(c_t) = \beta(1 + i_t) E_t \left[ \frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]. \]
Employed members separate from their jobs at the exogenous rate \( \lambda \), whereas unemployed members find jobs at the rate \( p(\theta_t) \). Therefore, the household’s employment rate evolves according to the following law of motion,
\[ n_{t+1} = (1 - \lambda)n_t + p(\theta_t)(1 - n_t). \]
Equation (1), together with \( n_t = 1 - u_t \) and \( \theta_t = v_t / u_t \), describe the relationship between unemployment and vacancies, also known as the Beveridge curve. It is convenient at this point to obtain the value enjoyed by the household from the marginal job in firm \( i \). Using the budget
constraint to substitute for $c_t$ in the welfare criterion, I obtain

$$
\frac{\partial H_t}{\partial n_{it}} = u'(c_t) \frac{W_{it}}{P_t} - v(h_t) - b - p(\theta_i) \beta \int_0^1 \frac{v_{It}}{v_I} E_t \frac{\partial H_{t+1}}{\partial n_{It+1}} dI + (1 - \lambda) \beta E_t \frac{\partial H_{t+1}}{\partial n_{It+1}},
$$

(2)

where $p(\theta_i) \frac{v_{It}}{v_I}$ is the probability of being matched to firm $I$ and $1 - \lambda$ is the probability that the worker is not separated from firm $i$. Therefore, the contribution of the worker to the welfare of the household is the real wage (in utility units), minus labor disutility, minus the value this worker would contribute if she searched for another job, plus the future value of the job conditional on non-separation.

### 2.3 Firms

I assume the existence of two types of firms: producers and retailers. Producers use capital and labor to produce a homogenous intermediate good; hiring in this sector is subject to search and matching frictions. Retailers buy the intermediate good from producers, transform it into differentiated final goods and sell them for a price chosen at random intervals. I first analyze the producer problem.

#### 2.3.1 Producers

A measure-one continuum of producers produce a homogenous intermediate good and sell it to retailers at the perfectly competitive real price $\varphi_t$. Each firm $i$ rents capital $k_{it}$ in a competitive market with rental price $r_t$. The firm also employs $n_{it}$ workers. Each worker in firm $i$ provides $h_{it}$ hours of work and receives a real wage $\frac{w_{it}}{P_t}$. The latter denotes total, not hourly, compensation. These inputs are transformed into output, $y_{it}$, by means of a constant returns to scale technology,

$$
y_{it} = A_t f(n_{it} h_{it}, k_{it}),
$$

where $A_t$ is a common productivity shock. The log of the latter, $a_t = \ln A_t$, follows an autoregressive process, $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, where $\varepsilon_t^a$ is an iid shock. The firm posts a number $v_{it}$ of vacancies. This comes at a utility cost for the management, given by

$$
\frac{\chi}{1 + \psi} \left( \frac{v_{it}}{n_{it}} \right)^{1+\psi} n_{it},
$$

where $\psi > 0$. Assuming that firms in this sector are sufficiently large, $\lambda$ and $q(\theta_t)$ represent the fraction of workers that are separated from the firm and the fraction of vacancies that are
filled in period \( t \), respectively. New workers do not become productive until the next period, due to the time involved in finding and training them. Therefore, the law of motion of the employment stock in firm \( i \) is given by

\[
n_{it+1} = (1 - \lambda)n_{it} + q(\theta_t)v_{it}.
\]

The firm chooses \( k_{it} \) and \( v_{it} \) to maximize the real present discounted value of profits,

\[
J_{it} = \varphi_tA_{it}(n_{it}h_{it}, k_{it}) - r_itk_{it} - \frac{W_{it}}{P_t}n_{it} - \frac{\chi}{1 + \psi} \left( \frac{v_{it}}{n_{it}} \right)^{1+\psi} n_{it} + E_t\beta_{t,t+1}J_{it+1},
\]

subject to equation (3). \( \beta_{t,t+s} \equiv \beta^s\frac{u'(c_{t+s})}{u'(c_t)} \) is the stochastic discount factor between periods \( t \) and \( t+s \). The first order condition with respect to \( k_{it} \) equalizes the marginal revenue product of capital to its rental rate, \( \varphi_tA_{it}f_2(n_{it}h_{it}, k_{it}) = r_it \). Given constant returns to scale in production, it follows that the capital-labor ratio \( \frac{k_{it}}{n_{it}h_{it}} \) is equalized across firms. This implies that also the marginal product of labor, \( mpl_{it} \equiv A_{it}f_1(n_{it}h_{it}, k_{it}) \), is equalized across firms: \( mpl_{it} = mpl_t \) for all \( i \). The first order condition with respect to vacancies is given by

\[
\frac{\chi z_{it}}{u'(c_t)} = q(\theta_t)E_t\beta_{t,t+1}\frac{\partial J_{it+1}}{\partial n_{it+1}},
\]

where \( z_{it} \equiv \frac{v_{it}}{n_{it}} \) is the vacancy rate. The value of the marginal worker for the firm is given by

\[
\frac{\partial J_{it}}{\partial n_{it}} = \varphi_tmpl_nh_{it} - \frac{W_{it}}{P_t} + \frac{\psi\chi}{1 + \psi} \frac{z_{it}^{1+\psi}}{u'(c_t)} + (1 - \lambda)E_t\beta_{t,t+1}\frac{\partial J_{it+1}}{\partial n_{it+1}}.
\]

The right hand side of equation (5) consists of the marginal revenue product, minus the real wage, plus the saving on hiring costs from having one more worker, plus the continuation value of the job. Combining (4) and (5), I can write the firm’s hiring decision as,

\[
\frac{\chi z_{it}^\psi}{q(\theta_t)} = \beta E_t \left\{ u'(c_{t+1}) \left[ \varphi_{t+1}mpl_{t+1}h_{it+1} - \frac{W_{it+1}}{P_{t+1}} \right] + \frac{\psi\chi}{1 + \psi} z_{it+1}^{1+\psi} + (1 - \lambda)\frac{\chi z_{it+1}^\psi}{q(\theta_{t+1})} \right\}.
\]
to \( h_{it} \), I obtain the following first order condition,

\[
\varphi_t m p_t = \frac{v'(h_{it})}{u'(c_t)}.
\]  

(7)

Therefore, the marginal revenue product of labor is equal to the marginal rate of substitution between consumption and leisure. Notice that hours are independent of the wage, precisely because they are chosen to maximize the joint surplus. Also, since the marginal revenue product is the same for all producers, equation (7) implies that hours are equalized across producers, \( h_{it} = h_t \) for all \( i \).

2.3.2 Retailers

There is a measure-one continuum of monopolistic retailers, each of them producing one differentiated consumption good. Due to imperfect substitutability across consumption goods, each retailer faces the following demand curve for its product,

\[
c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\gamma} c_t.
\]  

(8)

Producing \( c_{jt} \) units of good \( j \) requires the same amount of the intermediate input, which is purchased from producers at the real price \( \varphi_t \). Therefore, \( \varphi_t \) represents the real marginal cost of production for retailers. I follow the Calvo (1983) model of price setting: each period a randomly chosen fraction \( \delta_p \) of firms fail to reset their price. Therefore, when a firm has the chance of changing its price, it maximizes

\[
E_t \sum_{s=0}^{\infty} \delta_p \beta_{t,t+s} \left( \frac{P_{jt}}{P_{t+s}} - \varphi_{t+s} \right) c_{jt+s}
\]

with respect to \( P_{jt} \), subject to \( c_{jt+s} = (P_{jt}/P_{t+s})^{-\gamma} c_{t+s} \). The optimal pricing decision is given by

\[
E_t \sum_{s=0}^{\infty} \delta_p \beta_{t,t+s} P_{t+s}^\gamma c_{t+s} \left( \frac{P_t^*}{P_{t+s}} - \frac{\gamma}{\gamma - 1} \varphi_{t+s} \right) = 0,
\]  

(9)

where \( P_t^* \) is the common price chosen by all price-setters. Therefore, price-setters target a constant mark-up \( \frac{\gamma}{\gamma - 1} > 1 \) over real marginal costs for the expected duration of the price contract. Since price-setters are randomly chosen, the law of motion for the price level is given by

\[
P_t^{1-\gamma} = \delta_p P_{t-1}^{1-\gamma} + (1 - \delta_p) (P_t^*)^{1-\gamma}.
\]  

(10)
2.3.3 Equilibrium in the intermediate good market

Given constant returns to scale in production, I can express aggregate output of the intermediate good as

$$\int_0^1 A_t f(n_t h_t, \tilde{k}_t) dt = A_t n_t f(1, \tilde{k}_t) = A_t f(n_t h_t, \bar{k}),$$

where $\tilde{k}_t$ is the common capital-labor ratio and in the second equality I have used the market clearing condition for capital, $\bar{k} = \int k_{it} dt = \int n_t h_t \tilde{k}_t dt = n_t h_t \bar{k}_t$. In equilibrium, total supply of the intermediate good, $A_t f(n_t h_t, \bar{k})$, must equal total demand by retailers, $\int c_{jt} dj$. Using equation (8), this condition can be written as

$$A_t f(n_t h_t, \bar{k}) = c_t \Delta_t,$$

where

$$\Delta_t = \int_0^1 \left( \frac{P_{jt}}{P_t} \right)^{-\gamma} dj$$

is a measure of price dispersion.

3 The efficient equilibrium

Before analyzing wage determination in the decentralized economy, it is convenient to find the efficient allocation, which will be the benchmark relative to which monetary policy outcomes will be evaluated. I assume that the search frictions in the labor market are a technological constraint on the social planner, just like the production function. Therefore, I want to characterize the constrained-efficient equilibrium of this economy. The social planner chooses the state-contingent path of $c_t$, $h_t$, $v_t$ and $n_t$ to maximize the joint welfare of households and managers,

$$\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) - n_t [v(h_t) + b] - \frac{\chi}{1 + \psi} \left( \frac{v_t}{n_t} \right)^{1+\psi} n_t \right\},$$

subject to the law of motion of employment,

$$n_{t+1} = (1 - \lambda)n_t + m(v_t, 1 - n_t),$$

and the aggregate resource constraint,

$$A_t f(n_t h_t, \bar{k}) = c_t.$$
Notice that, since the social planner avoids any inefficient dispersion in relative prices, the price dispersion term $\Delta_t$ in equation (11) becomes 1. The same holds for vacancy rates in the objective function. Using (13) to substitute for $c_t$ in the objective function, the social planner is left with the choice of $h_t$, $v_t$ and $n_t$. The first order condition with respect to $h_t$ is given by

$$mpl_t = \frac{v'(h_t)}{u'(c_t)},$$

where $mpl_t = A_t f_1(n_t h_t, \bar{k})$ is the marginal product of labor. Therefore, the social planner equalizes the marginal product of labor and the marginal rate of substitution between consumption and leisure. The first order conditions with respect to $v_t$ and $n_{t+1}$ are given by

$$\chi z_t^\psi = m_1(v_t, u_t) \beta E_t \Omega_{t+1},$$

$$\Omega_t = u'(c_t)mpl_t h_t + \frac{\psi \chi}{1 + \psi} z_t^{1+\psi} v(h_t) - b + [1 - \lambda - m_2(v_t, u_t)] \beta E_t \Omega_{t+1},$$

respectively, where $\Omega_{t+1}$ is the Lagrange multiplier on constraint (12). Given constant returns to scale in the matching function, I can write $m_1(v_t, u_t) = \epsilon q(\theta_t)$ and $m_2(v_t, u_t) = (1 - \epsilon) p(\theta_t)$, where $\epsilon \equiv \frac{\partial m}{\partial v} \frac{v}{m}$ is the elasticity of $m$ with respect to vacancies. Combining this with equations (15) and (16), I obtain the following condition for efficient job creation,

$$\chi z_t^\psi = \frac{\beta E_t}{q(\theta_t)} \left\{ \epsilon \left[ u'(c_{t+1}) mpl_{t+1} h_{t+1} + \frac{\psi \chi}{1 + \psi} z_{t+1}^{1+\psi} v(h_{t+1}) - b \right] + [1 - \lambda - (1 - \epsilon) p(\theta_{t+1})] \frac{\chi z_{t+1}^\psi}{q(\theta_{t+1})} \right\}.$$

### 4 Equilibrium with flexible wages

Following most of the search and matching literature, I assume that, when firms reset nominal wages, they do so according to the Nash bargaining solution. That is, firm and worker each receive a constant fraction of the match surplus, which is the sum of firm and worker surplus. Letting $S^f_{it} \equiv \frac{\partial J}{\partial w_{it}}$ denote the firm surplus, I can write (5) as

$$S^f_{it} = \bar{w}_{it} - \frac{W_{it}}{P_t} + (1 - \lambda) E_t \beta_{t,t+1} S^f_{it+1},$$

where

$$\bar{w}_{it} \equiv \varphi_t mpl_t h_t + \frac{\psi \chi z_{it}^{1+\psi}}{1 + \psi u'(c_t)}.$$
is the sum of marginal revenue product and marginal saving on hiring costs. Similarly, let $S^w_{it} = \frac{\partial H_t}{\partial n_t} u(c_t)$ denote the worker surplus in consumption units. I can then express (2) as

$$S^w_{it} = \frac{W_{it}}{P_t} - w_t + (1 - \lambda)E_t \beta_{t,t+1} S^w_{it+1}, \quad (20)$$

where

$$w_t = v(h_t) + b u'(c_t) + p(\theta_t) \int_0^1 \frac{v_{it}}{v_t} E_t \beta_{t,t+1} S^w_{it+1} dI \quad (21)$$

is the sum of labor disutility and the value of searching for other jobs. Let $\xi \in (0,1)$ denote the firm’s bargaining power. Nash bargaining implies that the firm receives a fraction $\xi$ of the joint match surplus,

$$(1 - \xi)S^f_{it} = \xi S^w_{it}. \quad (22)$$

Combining equations (18), (20) and (22), I obtain that the following solution for the real wage,

$$\frac{W_{it}}{P_t} = (1 - \xi)\bar{w}_{it} + \xi w_t. \quad (23)$$

If all wages are renegotiated in every period, then all producers behave in exactly the same way and I can drop the subscript $i$. On the other hand, using equation (22) and the first order condition for vacancies, equation (4), I can write

$$E_t \beta_{t,t+1} S^w_{t+1} = \frac{1 - \xi}{\xi} E_t \beta_{t,t+1} S^f_{t+1} = \frac{1 - \xi}{\xi} \frac{\chi z^\psi_t}{q(\theta_t) u'(c_t)}. \quad (24)$$

Combining this with equations (19), (21) and (23), I can write the real wage as

$$\frac{W_t}{P_t} = (1 - \xi) \left[ \varphi_t mpl_t h_t + \frac{\psi}{1 + \psi} \frac{z^{1+\psi}_t}{u'(c_t)} + \chi \frac{z^\psi_t}{u'(c_t)} \theta_t \right] + \xi \frac{v(h_t) + b}{u'(c_t)}. \quad (25)$$

Using equation (24) to substitute for the real wage in equation (6), I obtain the following job creation condition,

$$\frac{\chi z^\psi_t}{q(\theta_t)} = \beta E_t \left\{ \xi \left[ u'(c_{t+1}) \varphi_{t+1} mpl_{t+1} h_{t+1} + \frac{\psi}{1 + \psi} z^{1+\psi}_{t+1} - v(h_{t+1}) - b \right] + [1 - \lambda - (1 - \xi) p(\theta_{t+1})] \frac{\chi z^\psi_{t+1}}{q(\theta_{t+1})} \right\}. \quad (25)$$

We can now compare the decentralized outcome with the efficient equilibrium. Efficiency requires equations (14) and (17) to hold, as well as the absence of price dispersion, $\Delta_t = 1$. By
equation (10), avoiding price dispersion requires keeping the price level constant. By equation (9), keeping $P_t$ constant requires the central bank to ensure that the real marginal cost equals the inverse of the retailer mark-up in every period: $\{\varphi_t\} = \frac{\gamma - 1}{\gamma} < 1$. Comparing equations (7) and (14), it is clear that this policy would not implement the first best. In principle, the mark-up could be eliminated by assuming that retailer sales are subsidized at the rate $s = \frac{1}{\gamma - 1}$, such that the effective mark-up, $\frac{1}{\gamma - 1} \frac{1}{1+\varepsilon}$, is exactly equal to one. However, imposing $\varphi_{t+1} = 1$ in equation (25) and comparing the resulting expression with equation (17), it follows that job creation is efficient only if the firm’s bargaining power, $\xi$, is equal to the elasticity of the matching function with respect to vacancies, $\epsilon$. In this case, firms internalize the congestions that they create in the labor market in a way that leads them to post the efficient number of vacancies. This is the well-known Hosios condition for efficient job creation (Hosios, 1990). Given that a unit effective mark-up and the Hosios condition jointly imply an efficient steady state, we can summarize this discussion as follows.

**Proposition 1** If all wages are Nash-bargained in every period and the economy’s steady state is efficient, keeping the price level constant implements the efficient allocation.

This result resembles the 'case for price stability' presented by the New Keynesian literature. According to the latter, when labor markets are perfectly competitive and the effective mark-up is one, price level constancy implements the first best even in the face of real shocks. Proposition 1 confirms that the optimality of zero inflation carries over to a search and matching labor-market framework, as long as all wages are renegotiated in every period and the Hosios condition is satisfied.

## 5 Equilibrium with staggered nominal wages

Period-by-period renegotiation of wages is a convenient theoretical benchmark, but is not empirically appealing. In reality, nominal wages remain unchanged for many periods, and most wages are not changed more often than once a year. In addition, there is plenty of evidence that wage changes across firms are not synchronized, at least for the majority of industrialized economies. In order to formalize the staggered nature of nominal wage adjustment, I assume that in every period a randomly chosen fraction $\delta_w$ of firms do not renegotiate nominal wages

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11 See also Rotemberg and Woodford (1997). This subsidy should be financed with lump-sum taxes.
12 See e.g. Goodfriend and King (2001) and Woodford (2003, section 6.3).
13 See e.g. Gottschalk (2004).
14 See Taylor (1999) and the references therein.
with their employees, in analogy with the Calvo model of price-setting.\textsuperscript{15} Therefore, $\delta_w$ is the probability that a firm’s nominal wage does not change in the following period. I assume that workers hired in between contracting periods receive the same wage as continuing workers. As found by Bewley’s (1999) survey of business managers and labor leaders in the US, equity considerations at the firm level often lead the wages of new hires to be linked to the internal pay structure. Bewley observes that this is especially true for the primary sector of the labor market, i.e. jobs that are long-term and full-time, which are precisely the kind of employment relationships considered in this model.

Let the superscript $*$ denote renegotiating firms, and let $W_{it}^*$ be the agreed-upon nominal wage. In renegotiating firms, the surplus derived by the firm can be expressed as

$$S_{it}^f = \bar{w}_{it} - \frac{W_{it}^*}{P_t} + (1 - \lambda)E_t \beta_{t,t+1}[\delta_w S_{it+1}^f] + (1 - \delta_w)S_{it+1}^f],$$  

(26)

where the subscript $it+s$ indicates that firm $i$ has not changed its nominal wage since period $t$. The expression for $S_{it|t-s}^f$ is the same, except for $W_{it}^*$ and $\bar{w}_{it}$ being replaced by $W_{it-s}^*$ and $\bar{w}_{it|t-s}$, respectively. Integrating equation (26) forward, I can write

$$S_{it}^f = E_t \sum_{s=0}^{\infty} \beta_{t,t+s}(1 - \lambda)^s \delta_w^s \left( \bar{w}_{it+s|t} - \frac{W_{it}^*}{P_{t+s}} \right) + (1 - \lambda)(1 - \delta_w)E_t \sum_{s=0}^{\infty} \beta_{t,t+1+s}(1 - \lambda)^s \delta_w^s S_{it+s+1}^f.$$

Similarly, worker surplus in a renegotiating firm can be expressed as

$$S_{it}^w = \frac{W_{it}^*}{P_t} - w_t + (1 - \lambda)E_t \beta_{t,t+1}[\delta_w S_{it+1}^w] + (1 - \delta_w)S_{it+1}^w].$$  

(27)

$$S_{it}^w = E_t \sum_{s=0}^{\infty} \beta_{t,t+s}(1 - \lambda)^s \delta_w^s \left( \frac{W_{it}^*}{P_{t+s}} - w_{t+s} \right)$$

$$(1 - \lambda)(1 - \delta_w)E_t \sum_{s=0}^{\infty} \beta_{t,t+1+s}(1 - \lambda)^s \delta_w^s S_{it+s+1}^w.$$

In renegotiating firms, the Nash bargaining rule applies,

$$(1 - \xi)S_{it}^f = \xi S_{it}^w.$$  

(28)

\textsuperscript{15}Staggered wage bargaining a la Calvo has been first introduced by Gertler and Trigari (2006), in a model that abstracts from nominal variables.
Combining equation (28) with the expressions for $S_{it}^f$ and $S_{it}^w$, I obtain the following expression for the nominal wage agreement,

$$E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1 - \lambda)^s \delta_w \left( \frac{W_{it}^*}{P_{t+s}} - w_{it+s|t} \right) = 0,$$

(29)

where

$$w_{it}^f \equiv (1 - \xi) \bar{w}_{it} + \xi w_t$$

(30)

and $\bar{w}_{it}$ and $w_t$ are given by equations (19) and (21), respectively. The term $w_{it}^f$ represents the real wage that would obtain if all wages were Nash bargained in every period, i.e. if wages were perfectly flexible. For brevity, I will refer to $w_{it}^f$ as the flexible real wage in firm $i$. Notice the analogy between wage agreements, equation (29), and pricing decisions, equation (9): the same way that real marginal costs constitute the target for multiperiod price contracts, firm and worker target the flexible real wage when they agree on a multiperiod nominal wage contract.

I now guess that all renegotiating firms strike the same agreement, $W_{it}^* = W_t^*$. From equation (6), wage symmetry leads to symmetry in vacancy rates, $z_{it}$.\textsuperscript{16} From equations (30) and (19), the path of $w_{it+s}^f$ is the same for all firms that last renegotiated at time $t$. From equation (29), all renegotiating firms choose the same nominal wage, which verifies my guess. Finally, since renegotiating firms are randomly chosen, the average nominal wage across firms, $W_t \equiv \int_0^1 W_{it} di$, evolves according to the following law of motion,

$$W_t = \delta_w W_{t-1} + (1 - \delta_w) W_t^*.$$  

(31)

### 6 Linear-quadratic analysis

I am now ready to analyze optimal monetary policy in the economy with staggered nominal wage bargaining. In the rest of the paper, I will follow the linear-quadratic (LQ) approach to monetary policy analysis pioneered by Rotemberg and Woodford (1997) and extensively applied in Woodford (2003, Ch. 6). This requires obtaining a second order approximation of the representative households’ welfare criterion, as well as a first order approximation of the equilibrium conditions. As is well-known, this method has several advantages. First, the quadratic welfare criterion clarifies what the stabilization goals of the central bank are. Second, a first-order approximation of the equilibrium conditions is enough to evaluate the welfare effects of alternative policies with a degree of accuracy of up to second order. Finally, this

\textsuperscript{16}Hours per worker, $h_{it}$, are equalized by virtue of equation (7).
linear representation of the monetary transmission mechanism makes it easier to understand what are the trade-offs for monetary policy.\footnote{See Woodford and Benigno (2007) for a general discussion of the advantages of the LQ method relative to other approaches.}

\section{The model in log-linear form}

From now onwards, I assume the following functional forms for preferences over consumption and labor, as well as the production and matching technologies,

\[ u(c_t) = c_t^{1-\sigma} \left(1 - \frac{1}{\sigma}\right), \]

\[ v(h_t) = h_t^{1+\eta} \left(1 + \frac{\eta}{1}\right), \]

\[ f(n_t h_t, k) = (n_t h_t)^{\alpha} k^{1-\alpha}, \]

\[ m(v_t, u_t) = v_t^{\epsilon} u_t^{1-\epsilon}. \]

Also, for any variable \( e_{it} \), let \( \hat{e}_{it} \equiv \log(e_{it}/\bar{e}) \) denote the log-deviation from its steady state value, \( \bar{e} \), and let \( \hat{e}_i \equiv \int_0^1 \hat{e}_{it} \text{d}i \) denote its cross-sectional average. I start the approximation by log-linearizing the firm’s job creation condition, equation (6), and rescaling it by \( \bar{u}_t'(c) \) to obtain

\[ \frac{1 + \psi}{\lambda} s_v [(1-e)\hat{\theta}_t + \psi \hat{z}_{it}] = \beta E_t \left\{ \alpha \left( \hat{\varphi}_{t+1} + \frac{mpl_{t+1} + \hat{h}_{t+1}}{\bar{h}} \right) + \psi \left( 1 + \psi \right)s_v \hat{z}_{it+1} - \left( \alpha - s_w \right) \sigma^{-1} \hat{c}_{t+1} - s_w \hat{w}_{it+1} + (1 - \lambda) \frac{1 + \psi}{\lambda} s_v \left[ (1 - e)\hat{\theta}_{t+1} + \psi \hat{z}_{it+1} \right] \right\}, \]

where \( s_v \equiv \frac{\hat{z}^{1+\psi}}{1+\psi} \frac{n}{y} u'(c) \) are vacancy posting costs in consumption units as a fraction of GDP and \( s_w \equiv \frac{nw}{y} \) is the labor share of GDP, both in the steady state.\footnote{In the derivation of (32) I have used the fact that \( \frac{1}{q(\theta)} = \frac{v}{\lambda m} \), where \( q(\theta)v \) is the total number of matches in the steady state. I have also used \( mpL = \alpha \frac{q}{n} \).} In order to aggregate the individual job creation condition, I will make use of the following result.\footnote{The proof of all Lemmas is in Appendix A.}

\textbf{Lemma 1} The vacancy rate of any firm \( i \) admits the following log-linear approximation,

\[ \hat{z}_{it} = \hat{z}_t - \tau_z (\log W_{it} - \log W_t), \]
where

\[ \tau_z = \frac{\beta \delta_w s_w}{(1 - \beta \delta_w) \psi \frac{1+\psi}{\lambda} s_v}. \]

That is, relative vacancy rates are a negative function of relative wages. Intuitively, since the current nominal wage is kept constant in the following period with some probability, firms with a higher current nominal wage expect a lower surplus from new matches and therefore post fewer vacancies.

As shown in the proof of Lemma 1, this result allows me to obtain the following aggregate job creation equation,

\[ 1 + \psi \frac{1}{\lambda} s_v[(1-\epsilon)\hat{\theta}_{t+1} + \psi \hat{v}_t] = \beta E_t \left\{ \alpha \left( \hat{\varphi}_{t+1} + \text{mpl}_{t+1} + \hat{h}_{t+1} \right) + \psi \frac{1+\psi}{\lambda} s_v \hat{v}_{t+1} - (\alpha - s_w) \sigma^{-1} \hat{c}_{t+1} \right\}. \]

From \( \theta_t = \frac{n_t}{u_t} \) and \( u_t = 1 - n_t \), I can write, respectively,

\[ \hat{\theta}_t = \hat{v}_t - \hat{u}_t, \]

\[ \hat{u}_t = -\frac{p(\theta)}{\lambda} \hat{n}_t, \]

where in the second equation I have used \( \frac{n}{u} = \frac{p(\theta)}{\lambda} \). Letting \( z_t \equiv \int \frac{n_t}{n} z_idi = \frac{n_t}{n_t} \) denote the average vacancy rate, I obtain

\[ \hat{z}_t = \hat{v}_t - \hat{n}_t. \]

The log-linear approximation of the law of motion of the employment stock, equation (1), is given by

\[ \hat{n}_{t+1} = (1 - \lambda - p)\hat{n}_t + \lambda \hat{\theta}_t, \]

where \( p \equiv p(\theta) \) and I have used the fact that, in the steady state, \( \frac{n}{u} = \lambda \). The marginal product of labor is given by

\[ \text{mpl}_t = a_t - (1 - \alpha)(\hat{n}_t + \hat{h}_t). \]

Log-linearization of the hours decision and the aggregate resource constraint, equations (7) and (11) respectively, yields

\[ \hat{\varphi}_t = \eta \hat{h}_t + \sigma^{-1} \hat{c}_t - \text{mpl}_t, \]

\[ a_t + \alpha(\hat{n}_t + \hat{h}_t) = \hat{c}_t. \]
In equation (40) I have used the fact that the price dispersion term $\hat{\Delta}_t$ is actually a second order term.\(^{20}\)

By definition, real wage inflation is equal to nominal wage inflation minus price inflation,

$$\hat{w}_t = \hat{w}_{t-1} + \pi_{wt} - \pi_t, \quad (41)$$

where $\pi_{wt} \equiv \log(W_t/W_{t-1})$ and $\pi_t \equiv \log(P_t/P_{t-1})$. Log-linearizing equations (9) and (10) and combining them, I obtain the familiar New-Keynesian Phillips curve,\(^{21}\)

$$\pi_t = \kappa_p \hat{\varphi}_t + \beta E_t \pi_{t+1}, \quad (42)$$

where

$$\kappa_p \equiv \frac{(1 - \beta \delta_p)(1 - \delta_p)}{\delta_p}.$$

Solving for wage inflation deserves more attention. I start by log-linearizing the nominal wage agreement, equation (29),

$$E_t \sum_{s=0}^{\infty} \beta^s (1 - \lambda)^s \delta_w \left( \log W^*_t - \log P_{t+s} - \log w - \hat{w}^f_{it+s|t} \right) = 0. \quad (43)$$

From equations (19), (21) and (30), firm $i$’s flexible real wage can be approximated by

$$s \hat{w}^f_{it} = s \sigma^{-1} \hat{c}_t + (1 - \xi) \left[ \alpha \left( \hat{\varphi}_t + \hat{mpl}_t - \sigma^{-1} \hat{c}_t \right) + \psi (1 + \psi) s_v \hat{z}_{it} \right] + \alpha \hat{h}_t + \xi \beta S^w_y \left[ \theta_t + E_t \left( \hat{w}^f_{t+1} - \sigma^{-1} \hat{c}_{t+1} \right) \right], \quad (44)$$

where $S^w_y \equiv n^w_y$ is total worker surplus over GDP in the steady state. Notice that all terms in equation (44) except for $\hat{z}_{it}$ are common to all firms. This implies that, for a firm that has not changed its nominal wage since period $t$, I can write

$$s \hat{w}^f_{it+s|t} = s \hat{w}^f_{it+s} + (1 - \xi) \psi s_v (1 + \psi) (\hat{z}_{it+s|t} - \hat{z}_{t+s}) \quad (45)$$

where in the second equality I have used Lemma 1 to substitute for the firm’s relative vacancy rate. From here, it is relatively straightforward to derive the following result.

\(^{20}\)See Appendix B.

\(^{21}\)The derivation of equation (42) is fairly standard. See e.g. Walsh (2003a, section 5.4).
Lemma 2 Nominal wage inflation is given by

\[ \pi_{wt} = \kappa_w (\hat{w}_t^f - \bar{w}_t) + \beta_\lambda E_t \pi_{wt+1}, \]

where

\[ \kappa_w \equiv \frac{(1 - \beta_\lambda \delta_w)(1 - \delta_w)}{\delta_w} \frac{1}{1 + \phi}, \]

\[ \phi \equiv s_w^{-1}(1 - \xi)\psi s_v(1 + \psi)\tau_z, \]

and \( \beta_\lambda \equiv (1 - \lambda)\beta. \)

Therefore, the gap between the actual and the flexible average real wage is the driving force of wage inflation in this model. The reason is the following. The flexible real wage is the target wage for multiperiod wage contracts. To the extent that actual real wages are e.g. below their target, renegotiating firms will increase their nominal wages, with the resulting positive wage inflation.\(^{22}\) Notice that the slope of equation (46), \( \kappa_w, \) is analogous to that of the price inflation equation (42), \( \kappa_p, \) with two differences. First, the discount factor \( \beta_\lambda \) takes into account the fact that the job match survives in the following period with probability \( 1 - \lambda, \) thus shortening the horizon of the bargaining parties. Second, \( \kappa_w \) is reduced by the presence of \( \phi. \) This parameter represents an index of real rigidity in the sense of Ball and Romer (1990), and it has the effect of slowing the adjustment of nominal wages. To see why, take a firm where the bargaining parties are considering a nominal wage increase. From Lemma 1, a wage increase leads to a fall in the firm’s relative vacancy rate. By equation (45), its own flexible (target) real wage falls relative to the average. This in turn leads the bargaining parties to undo some of the initial increase in the nominal wage. This effect is absent in the case of prices, because the target price (i.e. the marginal cost) does not depend on the firm’s own pricing decision.\(^{23}\)

It only remains to derive the average flexible real wage. The following result will prove useful in that respect.

Lemma 3 The worker surplus and the firm surplus, respectively, admit the following log-linear approximation,

\[ \hat{S}_{it}^w = \hat{S}_{it}^w + \tau_w (\log W_{it} - \log W_t), \]

\(^{22}\)The Erceg et al. (2000) model produces a wage inflation equation similar to (46), with the gap between the marginal rate of substitution between consumption and leisure and the average real wage as the driving force. This is because in their model, the marginal rate of substitution is the target for multiperiod nominal wage contracts.

\(^{23}\)The absence of real rigidity in price setting is due to my two-sector assumption and to retailers’ linear production technology. This is not the case in the standard one-sector New Keynesian model with decreasing returns in production; see e.g. Woodford (2003, Ch. 3).
\[
\hat{S}_{it}^f = \hat{S}_t^f - \tau_f (\log W_{it} - \log W_t),
\]

where
\[
\tau_w = \frac{s_w}{[1 - (1 - \lambda)\beta s_w]Sw},
\]
\[
\tau_f = \frac{s_w + \psi s_v (1 + \psi)\tau_z}{[1 - (1 - \lambda)\beta s_w]Sf},
\]
\[
S_y^w = \frac{snsw}{y} \text{ and } S_y^f = \frac{nsf}{y}.
\]

That is, the surplus enjoyed by workers in firm \(i\) relative to the average is increasing in that firm’s relative wage. The opposite is true for the relative surplus enjoyed by the firm. With Lemma 3 in hand, it is possible to obtain the following expression for the average flexible real wage.

**Lemma 4** The average flexible real wage is given by
\[
s_w \hat{w}_t^f = s_w \sigma^{-1} \hat{c}_t + (1 - \xi) \left[ \alpha (\hat{\theta}_t + \tilde{m}pl - \sigma^{-1} \hat{c}_t) + \psi (1 + \psi) s_v \hat{z}_t \right] + \alpha \hat{h}_t \\
+ (1 - \xi) p \frac{1 + \psi}{\lambda} s_v \left( \hat{\theta}_t + \psi \hat{z}_t - \tau E_t \pi_{w+1} \right),
\]

where \(\tau \equiv (\tau_w + \tau_f) \frac{\delta f}{1 - \delta w}\).

Notice that log-linearization of the real wage in the flexible-wage case, equation (24), would yield exactly equation (47), except for the presence of \(E_t \pi_{w+1}\). With staggered nominal wages, expected wage inflation reduces the average Nash wage relative to the flexible-wage case. To see this, I log-linearize equation (31) in period \(t + 1\),
\[
\log W_{t+1}^* - \log W_{t+1} = \frac{\delta_w}{1 - \delta_w} (\log W_{t+1} - \log W_t),
\]

When wage inflation is positive, both sides of the equation are positive. A job seeker that is matched at time \(t\) to a non-renegotiating firm receives an average negative relative wage in \(t + 1\) (since \(\log W_t < \log W_{t+1}\)) and, from Lemma 3, enjoys a negative relative surplus. If she is matched to a renegotiating firm, from (28) we have \(\hat{S}_{t+1}^w = \hat{S}_{t+1}^f\); but since \(\log W_{t+1}^* > \log W_{t+1}\), from Lemma 3 that firm’s relative surplus is also negative. Either way, wage inflation reduces the expected average surplus for job seekers, relative to the expected average firm surplus (given by \(\hat{\theta}_t + \psi \hat{z}_t\) in equation 47). This reduces the outside option for workers and therefore the Nash wage.
6.2 The quadratic welfare criterion

The second step of the LQ method consists of deriving a second order approximation of the welfare criterion, which will be the objective function in the central bank’s optimal monetary policy problem. At this point, I assume that the steady state of this economy is efficient. As seen in section 4, this requires making the following two assumptions.

**Assumption 1** Retailer sales are subsidized at the rate $\frac{1}{1-\gamma}$, such that the effective mark-up is one.

**Assumption 2** The Hosios condition is satisfied: $\xi = \epsilon$.

The reason for assuming an efficient steady state is that the linear terms in the second order approximation of the welfare criterion cancel out. As shown by Benigno and Woodford (2007), when the steady state is not efficient the LQ method requires obtaining a second order approximation of the equilibrium conditions in order to substitute for the linear terms in the welfare criterion. This is cumbersome even in the standard New Keynesian model. In order to simplify the analysis, I will constrain it to the case of an efficient steady state. As shown in Appendix B, the household’s welfare criterion admits the following second order approximation,

\[
\sum_{t=0}^{\infty} \beta^t \left\{ c_t^{1-\sigma-1} - n_t \left( \frac{h_t^{1+n}}{1+\eta} + b \right) - \frac{\chi}{1+\psi} \int_0^1 \left( \frac{v_t}{n_t} \right)^{1+\psi} n_t \, dt \right\} = -\frac{u'(c)c}{2} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O^3, \tag{48}
\]

where $t.i.p.$ represents terms independent of policy as of date 0, $O^3$ are terms of order third and higher in the size of the exogenous shock, and $L_t$ is a purely quadratic loss function given by

\[
L_t = \lambda_p \pi_t^2 + \lambda_w \pi_{wt}^2 + L_t^{n,h}, \tag{49}
\]

where $\lambda_p = \frac{\gamma}{\rho_p}$, $\lambda_w = (1 + \psi) \psi s_v \tau^2 z^2 \delta_w \frac{\delta_w}{(1-\delta_w)(1-\beta \delta_w)}$ and

\[
L_t^{n,h} = (\sigma^{-1} - 1) \tilde{c}_t^2 + \alpha \left[ (\tilde{n}_t + \tilde{h}_t)^2 + \eta \tilde{h}_t^2 \right] + (1 + \psi) s_v \left[ (1-\epsilon) \theta_t^2 + \psi \tilde{z}_t^2 \right]. \tag{50}
\]

Equation (49) illustrates the central bank’s stabilization objectives. First, when price setting is staggered, inflation causes price dispersion. From equation (11), price dispersion $\Delta_t$ increases the amount of labor effort needed to supply a certain amount of the consumption basket, $c_t$.

\[\text{See Benigno and Woodford (2005).}\]
\[\text{See Faia (2006a) for an analysis of the effect of steady state distortions on optimal monetary policy in a similar framework.}\]
As a result, inflation causes a welfare loss. Second, under staggered nominal wage bargaining, wage inflation causes wage dispersion. From Lemma 1, wage dispersion creates dispersion in hiring rates. Since the utility cost of hiring is convex in hiring rates, dispersion in the latter increases the welfare cost involved in aggregate job creation. Therefore, wage inflation reduces welfare.

Finally, the term \( L^{n,h}_t \) measures the success of monetary policy in stabilizing employment and hours around their efficient paths.\(^{26}\) To see this, notice that the quadratic approximation of the welfare criterion in the social planner problem is given by \( \sum_{t=0}^{\infty} \beta^t L^{n,h}_t \), where all variables in \( L^{n,h}_t \) follow their efficient paths.\(^{27}\) This implies that \( \sum_{t=0}^{\infty} \beta^t L^{n,h}_t \) has a global minimum in the social planner allocation; such a minimum need not be zero (to the extent that the economy fluctuates in the first best equilibrium) but is independent of monetary policy. Therefore, the welfare loss under a given policy is simply the difference between the value of \( \sum_{t=0}^{\infty} \beta^t L_t \) in the decentralized economy and the value of \( \sum_{t=0}^{\infty} \beta^t L^{n,h}_t \) in the social-planner allocation.

### 6.3 Policy trade-offs

It is now possible to illustrate easily the trade-offs that the central bank faces. Assume that, consistent with the case for price stability, the central bank sets \( \pi_t = 0 \) in every period. This requires stabilizing real marginal costs completely: \( \check{\varphi}_t = 0, \forall t \). In response to real shocks, the failure of some nominal wages to adjust creates wage dispersion across firms. As we saw in the previous subsection, wage dispersion creates a welfare loss which is proportional to \( \gamma^2 wt \), up to second order. Therefore, the staggering of nominal wages introduces a \textit{trade-off between price and wage inflation}.\(^{28}\)

Second, the stickiness of some nominal wages introduces a rigidity in the average real wage, which distorts aggregate job creation. To see this, I express the average real wage as \( \hat{w}_t = \hat{w}_t^f + (\hat{w}_t - \hat{w}_t^f) \). Inserting this in equation (33), using (47) to substitute for \( s_w \hat{w}_t^f \), and

\[ \hat{n}_{t+1} = (1 - \lambda - p)\check{n}_t + \lambda \epsilon \left( \hat{v}_t + \frac{n}{u} \check{n}_t \right), \]

respectively. \( \check{\varphi}_t \) and \( \hat{\varphi}_t \) are simply \( \hat{v}_t + \frac{u}{n} \check{n}_t \) and \( \hat{v}_t - \check{n}_t \), respectively.

\(^{26}\)Once the paths of \( \hat{n}_t \) and \( \check{n}_t \) are known, \( \hat{c}_t \) and \( \hat{v}_t \) can be derived from equation (40) and

\[^{27}\]Remember that the social planner avoids any dispersion in prices and vacancy rates, such that price and wage inflation are absent in the social planner’s quadratic loss function.

\[^{28}\]This trade-off was originally emphasized by Erceg et al. (2000), but in a framework that violates the efficiency of employment relationships.
imposing $\xi = \epsilon$ and $\hat{\phi}_{t+1} = 0$, I can represent the job creation condition under zero inflation as

$$1 + \psi \frac{1}{\lambda} s_v [(1 - \epsilon) \hat{\theta}_t + \psi \hat{z}_t] = \beta E_t \left\{ \frac{\epsilon}{\lambda} \left[ \alpha \left( \mu_{t+1} \right) \right] + \psi (1 + \psi) s_v \hat{z}_{t+1} \right\}.$$  

Log-linearization of the efficient job creation condition, equation (17), yields

$$1 + \psi \frac{1}{\lambda} s_v [(1 - \epsilon) \hat{\theta}_t + \psi \hat{z}_t] = \beta E_t \left\{ \frac{\epsilon}{\lambda} \left[ \alpha \left( \mu_{t+1} \right) \right] + \psi (1 + \psi) s_v \hat{z}_{t+1} \right\}.$$  

Equations (51) and (52) are the same, except for the last two terms on the right hand side of (51). These terms stem directly from the staggering of nominal wages. The failure of some nominal wages to adjust creates a gap between the actual and the flexible average real wage, which distorts the marginal benefit of posting vacancies. From equation (47), expected wage inflation distorts the path of $\hat{w}_t^f$ itself relative to the flexible-wage case, which further distorts vacancy posting. Since $\hat{w}_t^f - \hat{w}_t$ is the driving force of $\pi_{wt}$ in equation (46), both effects actually work in the same direction. Therefore, job creation is inefficient under the zero inflation policy. Since job creation determines the path of employment, and the latter is just the symmetric of unemployment, it follows that there is a trade-off between inflation and unemployment stabilization.

Finally, combining equations (38), (39) and (40), and imposing $\hat{\phi}_t = 0$, it is possible to solve for the path of $\hat{h}_t$ under zero inflation as a function of the state of the economy,

$$\hat{h}_t^{\pi=0} = \frac{\sigma - 1}{\alpha + \sigma (1 - \alpha) + \eta} a_t - \frac{(1 + \alpha)(1 - \alpha)}{\alpha + \sigma (1 - \alpha) + \eta} \hat{h}_t.$$  

Given that the employment path is distorted under the zero inflation policy, so is the path of hours per worker. It follows that there is a trade-off between inflation and hours stabilization.

How can the central bank improve upon the zero inflation policy? Up to a first order approximation, the distortions created by nominal wage staggering are summarized by the gap between actual and flexible average real wages (see equations 46 and 51). In response to shocks, the central bank should allow for non-zero inflation rates, in order to accelerate the convergence of real wages towards their flexible-wage levels. As actual wages are brought closer to their targets, renegotiating firms will adjust their nominal wages by less, which reduces the extent of wage dispersion in the economy. Also, the smaller gap in real wages reduces the distortion in
job creation, which brings unemployment and hours per worker closer to their efficient paths.

Quantifying the magnitude of these trade-offs is the purpose of section 6.5. Before turning to this, it is now worth to consider the nature of optimal monetary policy in two limiting cases: flexible wages and flexible prices.

### 6.3.1 Flexible wages

If nominal wages are fully flexible, $\delta_w = 0$, then the weight on wage inflation in the loss function, $\lambda_w$, becomes zero and wage inflation is not a concern for the central bank anymore. Equation (46) becomes $\dot{w}_t = \dot{w}_t^f$. The central bank can then set $\dot{\varphi}_t = 0$ and eliminate price inflation. On the other hand, job creation would be determined by (51), with $\ddot{w}_{t+1} - \dot{w}_{t+1}^f = 0$ and $\tau = 0$; this would yield equation (52), which is the social planner’s job creation decision. Employment would follow its efficient path, and so would hours per worker. Therefore, the monetary authority would be able to replicate the first-best allocation. This is just a restatement of Proposition 1, which claims the same result for the model in exact form.

### 6.3.2 Flexible prices

If prices are fully flexible, $\delta_p = 0$, then $\lambda_p$ in the loss function becomes 0 and inflation has no welfare consequences. Furthermore, all retailers set price equal to marginal cost, which implies $\varphi_t = 1$, or $\dot{\varphi}_t = 0$. The central bank can then ensure that $\dot{w}_t = \dot{w}_t^f$ holds at all times by adjusting price inflation in (41) accordingly. From (46), wage inflation and thus wage dispersion are eliminated. Since $\dot{\varphi}_{t+1} = 0$, $\ddot{w}_{t+1} = \dot{w}_{t+1}^f$ and $\pi_{wt+2} = 0$, the job creation condition, equation (51), becomes exactly the same as its social planner counterpart, equation (52). Therefore, when prices are flexible, the monetary authority is also able to replicate the first-best equilibrium.

### 6.4 Calibration

In what follows, I assume a monthly frequency for the model. First, as emphasized by Gertler and Trigari (2006), a monthly calibration is better able to capture the high rate of job finding in the US. Second, monetary policy decisions by the world’s major central banks are made on a monthly basis. Finally, inflation data and forecasts, on which policy decisions are largely

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29 In the flexible price case, it is still possible for the central bank to control the price level through the consumption Euler equation. See Woodford (2003, Ch. 2) for more details.

30 This is the case for the Bank of England, the European Central Bank and the Bank of Japan. The US Federal Reserve holds eight regularly scheduled meetings every year, but more meetings are held when needed.
based, have a monthly frequency too.

As in most of the business cycle literature, I set the intertemporal elasticity of substitution $\sigma$ to 1, a value which is consistent with balanced growth in the neoclassical growth model. Assuming log-utility is also convenient because the social-planner equilibrium becomes invariant to productivity shocks, such that all variables in that equilibrium are always at their steady-state levels.\textsuperscript{31} It follows that all fluctuations in the decentralized economy are inefficient, and that the loss function derived before also measures the welfare loss relative to the first-best allocation.

Regarding the Frisch elasticity of labor supply, $\eta^{-1}$, there is traditionally a conflict between the low values estimated in the micro empirical literature (up to 0.5; see e.g. Card 1994) and the values higher than unity used in the macro literature on the basis of balanced growth considerations (see e.g. Cooley and Prescott, 1995). Therefore, I choose a compromise value of 1. I assume standard values for the discount factor, $\beta = 0.99^2$, and the autorecorrelation of the productivity shock, $\rho_u = 0.95^3$.

For the matching function, I set $\epsilon$ equal to 0.6, following the US evidence in Blanchard and Diamond (1989). As in Gertler and Trigari (2006), I use a monthly separation rate of $\lambda = 0.035$, in order to match the evidence that jobs last for two years and a half. Shimer (2005) calculates a monthly job-finding rate of 0.276 for the US; I thus set $p = 0.30$. From the Beveridge curve, this implies a steady-state unemployment rate of $u = \lambda/(p + \lambda) = 0.10$, which is reasonable if we allow the model unemployment rate to include those individuals registered as inactive that are actively searching for jobs.\textsuperscript{32} The employment rate is $n = 1 - u = 0.90$.

Following Gertler and Trigari (2004), the degree of convexity of hiring costs is set to $\psi = 1$. As in Andolfatto (1996) and Gertler and Trigari (2006), I set hiring costs as a fraction of GDP to $s_v = 0.01$. I now consider the Nash wage, equation (24), in the steady state and rescale it

\textsuperscript{31}The social-planner job creation decision, equation (17) can be written as

$$\frac{Xz^\psi_t}{q(\theta_t)} = \beta E_t \left\{ \epsilon \left[ v'(h_{t+1})h_{t+1} + \frac{\psi X}{1 + \psi} z_{t+1}^1 + v(h_{t+1}) - b \right] + [1 - \lambda - (1 - \epsilon)p(\theta_{t+1})] \frac{Xz^\psi_{t+1}}{q(\theta_{t+1})} \right\},$$

where I have used equation (14) to substitute for $u'(c_{t+1})mpl_{t+1}$. It follows that efficient job creation is driven only by fluctuations in hours per worker. Under log-utility equation (14) becomes

$$v'(h_t) = \frac{A_t f_1(n_t h_t, \bar{k})}{c_t},$$

where $c_t = A_t f(n_t h_t, \bar{k})$. It follows that shocks to $A_t$ have no effect on $h_t$ and therefore on any of the other variables.

\textsuperscript{32}The evidence shows that flows from inactivity to employment account for a large fraction of total flows to employment in the US, see e.g. Blanchard and Diamond (1989).
by \( \frac{b}{n} \) to obtain

\[
s_w = (1 - \epsilon) \left[ \alpha + \psi s_v + (1 + \psi) s_v \frac{p}{\lambda} \right] + \epsilon \frac{1 + \tilde{b}}{1 + \eta} \alpha, \tag{53}
\]

where \( \tilde{b} \equiv \frac{b}{v(h)} \) is fixed over variable work disutility in the steady state. On the other hand, equation (17) in the steady state can be rescaled by \( \frac{\psi'(c)}{\kappa} y \) to obtain

\[
\frac{1 + \psi}{\lambda} s_v \left\{ 1 - \beta [1 - \lambda - (1 - \epsilon)p] \right\} = \beta \epsilon \left[ \eta - \tilde{b} \right] \frac{\eta + \psi s_v}{1 + \eta} \alpha \tag{54}
\]

Assuming a labor share of GDP of \( s_w = 2/3 \), I can use (53) and (54) to solve for \( \alpha \) and \( \tilde{b} \), obtaining values of 0.68 and 0.58, respectively.

Regarding the New Keynesian side of the model, I assume an elasticity of substitution across goods \( \gamma \) of 7.67, which in the absence of the sales subsidy would imply a retailer mark-up of 15%. Klenow and Kryvtsov (2005) find that 26% of prices are changed every month in the US. I therefore set \( \delta_p \) to 0.75, which implies an average duration of price contracts of 4 months. This implies an elasticity of inflation with respect to marginal costs of \( \kappa_p = 0.084 \). The latter implies a weight on inflation stabilization of \( \lambda_p = 91.09 \).

Most wages in the US are renegotiated once a year (see e.g. Gottschalk, 2004). This suggests an average duration of nominal wage contracts of \( \frac{1}{1 - \delta_w} = 12 \) months, which implies \( \delta_w = 0.92 \). The resulting degree of real rigidity in wage bargaining, \( \phi = 0.15 \), turns out to be quite small. The slope of the wage inflation equation is given by \( \kappa_w = 0.0094 \). The elasticity of vacancy rates with respect to relative wages is given by \( \psi = 0.019 \). These values imply a weight on wage inflation stabilization of \( \lambda_w = 387.53 \). This implies relative weights on price and wage inflation of \( \frac{\lambda_p}{\lambda_p + \lambda_w} = 0.19 \) and \( \frac{\lambda_w}{\lambda_p + \lambda_w} = 0.81 \), respectively. See Table 1 for a summary of parameter values.

<table>
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<th>( \lambda )</th>
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<th>( \gamma )</th>
<th>( \tau_z )</th>
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<tr>
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<td>( \tilde{b} )</td>
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<td>0.58</td>
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<td>0.0094</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 1. Parameter values
6.5 Quantitative analysis

6.5.1 Zero inflation policy

I start the quantitative analysis by simulating the behavior of the decentralized economy when the central bank implements a policy of full inflation stabilization, $\pi_t = 0$.\textsuperscript{33} Panels (a) and (b) of Figure 1 plot the economy’s response to a 1% negative productivity shock.\textsuperscript{34} The fall in the marginal product of labor reduces the flexible real wage on impact. Given that the latter is the target in wage negotiations, nominal wages in renegotiating firms fall accordingly. This results in wage deflation and inefficient wage dispersion.

On the other hand, the fall in productivity reduces vacancy posting. As a result, unemploy-

\textsuperscript{33}The log-linear model is solved using Uhlig’s (1999) undetermined coefficients method.
\textsuperscript{34}Price and wage inflation responses are expressed as annualized rates, i.e. $12\pi_t$ and $12\pi_{wt}$.
ment increases. The latter effect is amplified by the fact that real wages are too high relative to their flexible-wage levels. Finally, the response of hours per worker (not shown on the figure) is relatively weak, with a peak increase of 0.15%.

6.5.2 Optimal policy commitment

At time 0, the central bank chooses the state-contingent plan that minimizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \lambda_p \pi_t^2 + \lambda_w \pi_{wt}^2 + L_t^{n,h} \right), \]

where \( L_t^{n,h} \) is given by equation (50). This minimization is subject to the aggregate log-linear equilibrium conditions, given by equations (33) to (42), as well as equations (46) and (47).

Panels (c) and (d) of Figure 1 show the economy’s response to a 1% negative productivity shock. Relative to the situation under zero inflation, the optimal policy improves matters by creating a certain amount of inflation. This accelerates the convergence of real wages towards their flexible-wage levels. This can be seen by comparing panels (b) and (d): whereas such convergence takes 40 months under the zero inflation policy, it takes only 20 months under the optimal policy. This way, the central bank can reduce the two distortions created by nominal wage staggering. First, since real wages are now closer to their flexible-wage targets, renegotiating firms reduce their nominal wages by less, which reduces wage deflation and the extent of wage dispersion. Second, the smaller gap in real wages reduces the negative effect of the shock on vacancy posting. As a result, unemployment increases by less than under the zero inflation policy. Hours per worker (not shown on the figure) also increases by less, with a peak at 0.09%.

6.5.3 Welfare loss from inflation targeting

I now turn to the welfare consequences of different monetary policy regimes. The suboptimality of the zero inflation policy depends on the degree of nominal wage staggering, \( \delta_w \), or equivalently on the average duration of nominal wage contracts, \( \frac{1}{1-\delta_w} \). Figure 2 plots (half) the unconditional expectation of the loss function

\[ \frac{1}{2} E(L_t), \]
as a function of the average duration of nominal wage contracts. Under my assumption of log-utility, this metric represents the average welfare loss relative to the first-best allocation, as a percentage of steady state consumption. The solid and dotted lines represent the period welfare loss under the optimal and the zero inflation policy, respectively. If nominal wages are completely flexible (i.e. if contracts last for 1 month on average), from Proposition 1 the optimal policy is precisely the zero inflation policy and the equilibrium mimics the efficient one. Welfare losses relative to first best are therefore zero. For moderate contract durations, zero inflation is still a very good approximation to the optimal policy. However, as nominal wage contracts become longer, the distortions created by infrequent wage bargaining become larger and larger, and the failure of the central bank to use price inflation to counteract these distortions starts taking its toll on welfare. Welfare losses under the zero inflation policy are 25% higher than under the optimal policy for mean contract durations of 6 months. However, they are 91% higher when contracts last for 9 months, and 294% higher under the 12-month wage contracts assumed in the baseline calibration.

6.5.4 Implementing the optimal policy

The targeting rule that implements the optimal policy commitment in this framework (not shown here) is quite complex to be used in practice. For this reason, I now consider simple targeting rules that are reasonably close to the optimal policy in terms of welfare. A natural candidate is the policy that stabilizes a weighted average of price and wage inflation, with the same relative weights as in the loss function,

\[
\frac{\lambda_p}{\lambda_p + \lambda_w} \pi_t + \frac{\lambda_w}{\lambda_p + \lambda_w} \pi_wt = 0.
\]

This rule only features two of the central bank’s goals. However, the driving force of wage inflation (the gap between actual and flexible real wages) is also the source of inefficiency in employment fluctuations; see equations (51) and (46). Therefore, by tackling wage inflation directly, the central bank can also indirectly eliminate most of the distortion in the path of employment. Figure 2 confirms this intuition. The dashed line represents the welfare loss under stabilization of the weighted average of price and wage inflation. As the figure makes clear, this simple rule is nearly optimal for any plausible contract length; e.g. for the baseline of 12-month contracts, the welfare loss under this policy is only 9% higher than under the optimal

\[^{35}\]The figure assumes a standard deviation for the shock to productivity, \(\epsilon^\pi\), of 0.7%. However, since the productivity shock is the only source of aggregate uncertainty in this model, the relative welfare losses under alternative policies do not depend on the volatility of the shock.
Figure 2: Welfare losses for alternative monetary policy regimes
commitment.

6.5.5 The bargaining set

As claimed in the introduction, this framework is free of Barro’s (1977) critique, because preset nominal wages do not distort existing employment relationships. Hours per employee are independent of wages, because they are chosen so as to maximize the joint match surplus. Regarding the continuation of existing jobs, the failure of some nominal wages to adjust must not lead to the break-up of otherwise efficient employment relationships, in the form of either quits or firings. That is, in every firm (renegotiating or not) both employer and employees must always enjoy a non-negative surplus: \( S^f_{it}, S^w_{it} \geq 0, \forall i, t \). From equations (18) and (20), this means that the real wage must always lie inside the bargaining set, i.e. the set of wages above the worker’s reservation wage and below the firm’s reservation wage,

\[
\frac{W_{it}}{P_t} \in [w_t - (1 - \lambda)E_t\beta_{t,t+1}S^w_{it+1}, \bar{w}_{it} + (1 - \lambda)E_t\beta_{t,t+1}S^f_{it+1}].
\]

In the flexible-wage case, the Nash wage is just the weighted average of the two bounds of the bargaining set (the continuation values would cancel out as a result of the bargaining rule, producing expression 23). This implies that wages are always inside the bargaining set. However, when renegotiations are infrequent, it is only in renegotiating firms that the wage equals the weighted average of both reservation wages; in all other firms, it is not guaranteed that wages lie inside the bargaining set. With large enough shocks, and in firms with sufficiently old nominal wages, the latter are bound to fall outside the bargaining set. This section shows that, for shocks of empirically plausible size, the vast majority of wages remain inside the bargaining set.

The fraction of firms that last changed their nominal wage \( t \) periods ago is given by \((1 - \delta_w)\delta_w^t\). Following Gertler and Trigari (2006), I calculate the minimum value of \( T \) such that the fraction of firms with nominal wages older than \( T \) periods, \( 1 - \sum_{t=0}^{T}(1 - \delta_w)\delta_w^t \), is less than 1 per cent. For my baseline of 12-month wage contracts (\( \delta_w = \frac{12-1}{12} \)), \( T \) is equal to 52 months.

Up to a log-linear approximation, a firm’s surplus is proportional to \( 1 + \dot{S}^f_{it} \). From Lemma 3, it follows that the firm’s surplus is positive only if

\[
\log W_{it} < \log W_t + \tau_f^{-1} \left( 1 + \dot{S}^f_i \right) \equiv \log R^f_{it},
\]

where \( R^f_{it} \) is the firm’s reservation wage in nominal terms. Log-linearizing equations (26) (without the * subscript on period-\( t \) variables) and (19), and using Lemmas 1 and 3 to average across
all firms, I can express the average firm surplus as

\[ S^f_y S^f_t = \alpha \left( \hat{\varphi}_t + mpf_t + \hat{n}_t \right) + \psi s_v \left[ (1 + \nu) \hat{c}_t + \sigma^{-1} \hat{c}_t \right] - s_w \hat{\nu}_t + (1 - \lambda) \beta S^f_y E_t (\hat{\beta}_{t+1} + \hat{S^f_t}). \]

Similarly, from Lemma 3 the worker surplus is positive up to a log-linear approximation only if

\[ \log W_{it} > \log W_t - \tau_w^{-1}(1 + \hat{S^w_t}) \equiv \log R^w_t, \]

where \( R^f_t \) is the worker’s reservation wage in nominal terms. Log-linearizing equations (27) (without the * subscript on period-t variables) and (21), and averaging across all firms, I obtain the following expression for the average worker surplus,

\[ S^w_y S^w_t = s_w \hat{\nu}_t - \alpha \left( \hat{h}_t + \frac{1 + \bar{b}}{1 + \eta} \sigma^{-1} \hat{c}_t \right) - \beta s^w_y E_t (\hat{\beta}_{t+1} + \hat{S^w_t}). \]

In firms that last negotiated 52 months ago, the nominal wage equals \( W^*_{t-52} \). I therefore require that \( \log W^*_{t-52} \) is between \( \log R^f_t \) and \( \log R^w_t \) at all times,

\[ \log R^w_t \leq \log W^*_{t-52} \leq \log R^f_t. \]

In order to check that this condition holds for a reasonably large sample size, I simulate 1,000 observations of the model economy, under both the zero inflation and the optimal policy. I assume that the shock to exogenous productivity, \( \epsilon^a_t \), is normally distributed with mean zero and a standard deviation of 0.7% (see Prescott, 1986). Figure 3 shows the reservation wages as well as \( \log W^*_{t-52} \), all of them deflated by \( \log P_t \). For both policies, the real wage never falls outside the bargaining set. I conclude that, at least for 99% of all firms, Barro’s critique is avoided.

7 Further discussion

7.1 Output gap targeting

So far I haven’t made any reference to the output gap, i.e. the gap between the actual and the flexible-price-and-wage level of output. This variable features prominently in monetary policy discussions in the New Keynesian literature.\(^{36}\) Under my assumption of an efficient

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\(^{36}\)See e.g. Woodford (2003) and Clarida et al. (1999).
Figure 3: The bargaining set

Optimal policy

Zero inflation policy
steady state, the output gap is equivalently the gap between actual and efficient output. In this economy, output of the intermediate good equals consumption of final goods up to a first order approximation, \( \hat{y}_t = \hat{c}_t \). Combining this with equations (38), (39) and (40), I obtain the following solution for output,

\[
\hat{y}_t = \frac{1 + \omega}{\sigma^{-1} + \omega} a_t + \frac{\eta}{\sigma^{-1} + \omega} \hat{n}_t + \frac{1}{\sigma^{-1} + \omega} \hat{\varphi}_t,
\]

where \( \omega \equiv \frac{\mu}{\alpha} + \frac{1 - \alpha}{\alpha} \). Setting real marginal costs equal to their efficient value, \( \hat{\varphi}_t = 0 \), I obtain the efficient level of output conditional on the employment stock,

\[
\hat{y}_t^e = \frac{1 + \omega}{\sigma^{-1} + \omega} a_t + \frac{\eta}{\sigma^{-1} + \omega} \hat{n}_t.
\]

The latter two equations imply the following,

\[
\hat{\varphi}_t = (\sigma^{-1} + \omega) (\hat{y}_t - \hat{y}_t^e).
\]

That is, real marginal costs are proportional to the output gap, \( \hat{y}_t - \hat{y}_t^e \), where the latter is defined conditional on the employment stock. This in turn allows me to write the Phillips curve, equation (42), as

\[
\pi_t = \kappa (\hat{y}_t - \hat{y}_t^e) + \beta E_t \pi_{t+1},
\]

where \( \kappa \equiv (\sigma^{-1} + \omega) \kappa_p \). Therefore, closing the output gap is equivalent to eliminating inflation up to a first order approximation. This result stems from the following. First, the fact that

\[
\hat{y}_t = a_t + \alpha (\hat{n}_t + \hat{h}_t)
\]

implies that, conditional on the employment stock, stabilizing output is equivalent to stabilizing hours per worker. Second, hours are distorted by price staggering (to the extent that \( \hat{\varphi}_t \neq 0 \)), but not by wage staggering, because they are chosen to maximize the joint surplus of employment relationships. Therefore, the policy that eliminates the distortionary effects of price staggering (i.e. the zero inflation policy) is also the policy that stabilizes output. It follows that closing the output gap is just as suboptimal as the zero inflation policy.

This result is at odds with the standard New Keynesian analysis. As shown by Woodford (2003, ch. 6), in the Erceg et al. (2000) model closing the output gap is equivalent to stabilizing a weighted average of price and wage inflation, with weights depending on the relative stickiness of prices and wages. This result stems from the symmetry between goods markets (where firms
choose prices subject to demand curves for their products) and labor markets (where households choose nominal wages subject to demand curves for their services) in the Erceg et al. (2000) model. Since price and wage inflation receive most of the weight in the loss function in a plausible calibration of that model, it turns out that closing the output gap is nearly optimal for any degree of price and wage stickiness (and exactly optimal in one parametric case). The present analysis shows that, once monopolistic labor markets are replaced with a search and matching labor-market structure where employment relationships are privately efficient, the near-optimality of output-gap targeting disappears.

7.2 Staggered nominal wage bargaining vs. real wage norm

Modelling wage determination as the result of staggered bargaining of nominal wages is motivated by the evidence that nominal wages are indeed staggered (see Taylor, 1999). By contrast, most studies that integrate New Keynesian and search and matching models assume that real wages are set as a weighted average of the Nash real wage and a certain real wage norm. This wage norm can take many forms, but last period’s real wage or a constant real wage are usually considered. A disadvantage of this approach is that the partial adjustment coefficient in this kind of wage equations cannot be calibrated directly using micro data. This is important because the latter parameter determines the extent of the trade-off between unemployment and inflation stabilization in this kind of models (see Blanchard and Gali, 2006). By contrast, the parameter regulating the degree of nominal wage staggering in the Calvo model, $\delta_w$, can be calibrated to match the average duration of nominal wage contracts, which in the model equals $\frac{1}{1-\delta_w}$.

Perhaps more importantly, assuming a real wage norm may affect the model’s policy implications. To show this, I now assume that all real wages follow a partial adjustment equation based on a real wage norm. Equations (29) and (31) are replaced by

$$w_t = \delta w_t^{\text{norm}} + (1 - \delta) w_t^{\text{nash}},$$

where $w_t \equiv \frac{w_t}{P_t}$ is the real wage, $w_t^{\text{norm}}$ is the real wage norm, $w_t^{\text{nash}}$ is the Nash real wage (given by the right hand side of equation 24) and $\delta \in [0, 1]$ measures the degree of wage rigidity. In the absence of wage dispersion, the loss function is given by (49) without the $\pi_t^2$ term. The model remains unchanged otherwise. Figure 4 plots the difference in welfare losses between

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37This is the approach taken in Blanchard and Gali (2006), Krause and Lubik (2007) and Christoffel and Linzert (2005). The wage norm was introduced by Hall (2005).
the optimal and the zero inflation policy, in the cases of a lagged wage norm, $w_{t}^{\text{norm}} = w_{t-1}$, and a constant wage norm, $w_{t}^{\text{norm}} = w$. For comparison, the figure also shows the same welfare difference in the model with staggered nominal wages, under the baseline calibration.

As the figure makes clear, under either type of real wage norm, and for any degree of real wage rigidity, there is virtually no difference between the optimal and the zero inflation policy in terms of welfare. The reason for this result is as follows. Under wage symmetry, wage inflation no longer creates a welfare loss. The central bank is then left with the task of stabilizing inflation and both labor margins. From the previous subsection, stabilizing hours (conditional on the employment stock) is equivalent to stabilizing inflation. This means that the relevant trade-off for the central bank is between inflation and employment stabilization. Employment dynamics are distorted by the gap between actual and Nash real wages. But if real wages follow a weighted average of the Nash real wage and a real wage norm, then manipulating the inflation
rate is relatively ineffective at closing the gap between actual and Nash real wages. That is, the central bank loses most of its leverage over real wages. As a result, it finds it optimal to focus on inflation stabilization.

8 Conclusions

This paper has analyzed optimal monetary policy in a New Keynesian model with search and matching frictions in the labor market. It therefore represents one of the first normative studies in the context of a macroeconomic framework that allows to study the joint dynamics of unemployment and inflation, i.e. the Phillips curve.

I have shown that, when wages are Nash bargained in every period and the economy’s steady state is efficient, the central bank can replicate the first-best equilibrium by keeping the price level constant. I have also analyzed the empirically relevant case in which only a fraction of firms renegotiate nominal wages in each period. Staggered bargaining of nominal wages creates two distortions as the economy is hit by real shocks. First, the failure of some nominal wages to adjust leads to wage dispersion, which in turn creates inefficient dispersion in hiring rates. Second, the resulting rigidity in average real wages distorts job creation and creates inefficient fluctuations in unemployment. It becomes optimal for the central bank to deviate from full price stability. By using price inflation, the central bank can accelerate the convergence of actual real wages towards their flexible-wage levels. This reduces the inefficiency in unemployment fluctuations. It also reduces wage dispersion, by bringing actual wages closer to their targets and thus reducing the size of wage adjustments in renegotiating firms. For a reasonable calibration of the model, the zero inflation policy generates welfare losses (relative to first best) that are about three times as large as under the optimal policy.

The analysis presented here provides both important theoretical advantages and different policy implications relative to earlier monetary policy analysis in the New Keynesian tradition. On the one hand, the adoption of a search and matching labor-market structure allows one to analyze the distortionary effects of nominal wage stickiness in a way that does not impose any arbitrary inefficiencies on employment relationships. On the other hand, and related to the latter, the convenience of output-gap targeting emphasized in earlier research becomes questionable in a search and matching framework in which employment relationships are assumed to be privately efficient.
A. Proofs of Lemmas

Proof of Lemma 1

The firm’s hiring decision, equation (32) in the text, can be written as

\[
1 + \frac{\psi}{\lambda} s_v[(1 - \epsilon)\hat{\theta}_t + \psi\hat{z}_{it}] = \beta E_t \left\{ \begin{array}{l}
\alpha \left( \hat{\phi}_{t+1} + \bar{mp}_{t+1} + \hat{h}_{t+1} \right) \\
\quad + \psi \frac{1 + \psi}{\lambda} s_v \left[ \delta_w \hat{z}^0_{it+1} + (1 - \delta_w)\hat{z}^*_{it+1} \right] - (\alpha - s_w) \sigma^{-1} \hat{c}_{t+1} \\
- s_w [\delta_w \hat{w}^0_{it+1} + (1 - \delta_w)\hat{w}^*_{t+1}] + (1 - \lambda) \frac{1 + \psi}{\lambda} s_v (1 - \epsilon) \hat{\theta}_{t+1}
\end{array} \right\},
\]

where superscripts * and 0 denote the value of a variable conditional on whether the firm can reset its wage or not. I now make the following guess,

\[
\hat{z}_{it} = \hat{z}_t = \tau_z (\log W_{it} - \log W_t).
\]

This implies

\[
\hat{z}^0_{it+1} = \hat{z}^0_{t+1} = \tau_z (\log W_{it} - \log W_t),
\]

where \( \hat{z}^0_{t+1} \equiv E_i \hat{z}^0_{it+1} \) is the average vacancy rate across non-renegotiating firms. I have also used the fact that, since non-renegotiating firms are randomly chosen, this group’s average wage in period \( t+1 \) is equal to economy-wide wage level in period \( t \). I can similarly write

\[
\hat{w}^0_{it+1} = \hat{w}^0_{t+1} + (\log W_{it} - \log W_t).
\]

Inserting the latter two equations into equation (A1), I obtain

\[
1 + \frac{\psi}{\lambda} s_v[(1 - \epsilon)\hat{\theta}_t + \psi\hat{z}_{it}] = \beta E_t \left\{ \begin{array}{l}
\alpha \left( \hat{\phi}_{t+1} + \bar{mp}_{t+1} + \hat{h}_{t+1} \right) \\
\quad + \psi \frac{1 + \psi}{\lambda} s_v \left[ \delta_w \hat{z}^0_{it+1} + (1 - \delta_w)\hat{z}^*_{it+1} \right] - (\alpha - s_w) \sigma^{-1} \hat{c}_{t+1} \\
- s_w [\delta_w \hat{w}^0_{it+1} + (1 - \delta_w)\hat{w}^*_{t+1}] + (1 - \lambda) \frac{1 + \psi}{\lambda} s_v (1 - \epsilon) \hat{\theta}_{t+1}
\end{array} \right\},
\]

where \( \hat{w}_{it} \equiv \log W_{it} - \log W_t \) is the firm’s relative wage and \( \hat{z}_{t+1} = \delta_w \hat{z}^0_{t+1} + (1 - \delta_w)\hat{z}^*_{t+1} \) is the average vacancy rate in period \( t+1 \) (similarly for \( \hat{w}_{t+1} \)). Averaging (A2) across all firms, and using \( E_i \hat{w}_{it} = 0 \), I obtain

\[
1 + \frac{\psi}{\lambda} s_v[(1 - \epsilon)\hat{\theta}_t + \psi\hat{z}_{it}] = \beta E_t \left\{ \begin{array}{l}
\alpha \left( \hat{\phi}_{t+1} + \bar{mp}_{t+1} + \hat{h}_{t+1} \right) \\
\quad + \psi \frac{1 + \psi}{\lambda} s_v \hat{z}_{t+1} - (\alpha - s_w) \sigma^{-1} \hat{c}_{t+1} \\
- (\alpha - s_w) \sigma^{-1} \hat{c}_{t+1} - s_w \hat{w}_{t+1} + (1 - \lambda) \frac{1 + \psi}{\lambda} s_v (1 - \epsilon) \hat{\theta}_{t+1}
\end{array} \right\},
\]

(A3)
which is equation (33) in the text. Substracting (A3) from (A2), and using my guess, I obtain
\[
\beta E_t \{-\psi \frac{1 + \psi}{\lambda} s_v \delta_w \tau_z \bar{w}_{it} - s_w \delta_w \bar{w}_{it}\} = \frac{1 + \psi}{\lambda} s_v \psi (\dot{z}_{it} - \dot{z}_t) \\
= -\frac{1 + \psi}{\lambda} s_v \psi \tau_z \bar{w}_{it}.
\]
This implies
\[
\tau_z = \frac{\beta \delta_w s_w}{(1 - \beta \delta_w) \psi \frac{1 + \psi}{\lambda} s_v}.
\]

Proof of Lemma 2

I define \( \phi \equiv s_w^{-1}(1 - \xi) \psi s_v(1 + \psi) \tau_z \), such that equation (45) in the text can be expressed as
\[
\hat{w}_{it+s|t}^f = \hat{w}_{t+s}^f - \phi (\log W_{it}^* - \log W_{t+s}).
\]
Inserting the latter equation into equation (43) in the text, I obtain
\[
0 = E_t \sum_{s=0}^{\infty} \beta^s (1 - \lambda)^s \delta_w [\log W_{it}^* - \log P_{t+s} - \log w - \hat{w}_{t+s}^f + \phi (\log W_{it}^* - \log W_{t+s})] \\
= E_t \sum_{s=0}^{\infty} \beta^s (1 - \lambda)^s \delta_w [(1 + \phi) \log W_{it}^* + \hat{w}_{t+s} - \hat{w}_{t+s} - (1 + \phi) \log W_{t+s}]].
\]
Notice that all renegotiating firms will agree on the same nominal wage, \( W_{it}^* = W_t^* \). Solving for \( \log W_t^* \) yields
\[
(1 + \phi) \log W_t^* = [1 - (1 - \lambda) \beta \delta_w] E_t \sum_{s=0}^{\infty} \beta^s (1 - \lambda)^s \delta_w [\hat{w}_{t+s}^f - \hat{w}_{t+s}] + (1 + \phi) \log W_{t+s}]. \quad (A5)
\]
Equation (A5) admits the following representation,
\[
(1 + \phi) \log W_t^* = [1 - (1 - \lambda) \beta \delta_w] [\hat{w}_{t}^f - \hat{w}_t] + (1 + \phi) \log W_t] \\
+ (1 - \lambda) \beta \delta_w E_t (1 + \phi) \log W_{t+1},
\]
or alternatively
\[
(1 + \phi)(\log W_t^* - \log W_t) = [1 - (1 - \lambda) \beta \delta_w] (\hat{w}_{t}^f - \hat{w}_t) \\
+ (1 - \lambda) \beta \delta_w E_t (1 + \phi)(\log W_{t+1}^* - \log W_t). \quad (A6)
\]
The law of motion of the wage index, equation (31) in the text, can be approximated by

\[ \log W_t^* - \log W_t = \frac{\delta_w}{1 - \delta_w} (\log W_t - \log W_{t-1}) = \frac{\delta_w}{1 - \delta_w} \pi_{wt}. \]

Using this, equation (56) can be expressed as

\[ (1 + \phi) \frac{\delta_w}{1 - \delta_w} \pi_{wt} = [1 - (1 - \lambda)\beta_0 \delta_w]((\hat{w}_{t}^f - \hat{w}_t) + (1 - \lambda)\beta_0 \delta_w \bar{\pi}_{t+1} \pi_{wt+1}. \]  

(A7)

Dividing (56) by \((1 + \phi)\frac{\delta_w}{1 - \delta_w}\), I finally obtain equation (46) in the text.

**Proof of Lemma 3**

The worker surplus in any firm \(i\), given by equation (27) in the text without the * superscript on period-\(t\) variables, can be approximated by

\[ S_{y}^w \hat{\delta}_{it} = s_w \hat{w}_{it} - s_y \hat{w}_t + (1 - \lambda)\beta_0 S_{y}^{w} E_t[\hat{\beta}_{t,t+1} + \delta_w \hat{\delta}_{it+1} + (1 - \delta_w)\hat{S}_{w}^w], \]  

(A8)

where \(S_{y}^w = \frac{n_{S_{y}^w}}{y}\). I now make the following guess,

\[ \hat{\delta}_{it} = \hat{\delta}_{it} + \tau_w \bar{w}_{it}, \]  

(A9)

For non-renegotiating firms, I can then write

\[ \hat{\delta}_{it}^0 = \hat{\delta}_{it}^0 + \tau_w \bar{w}_{it-1}, \]  

(A10)

where \(\hat{\delta}_{it}^0\) is the average worker surplus across non-renegotiating firms. Using (A10) as well as \(\hat{w}_{it} = \bar{w}_{it} + \hat{w}_t\) in (A8), I can write (A8) as

\[ S_{y}^w \hat{\delta}_{it} = s_w (\bar{w}_{it} + \hat{w}_t) - s_y \hat{w}_t + (1 - \lambda)\beta_0 S_{y}^{w} E_t[\hat{\beta}_{t,t+1} + \hat{\delta}_{it+1} + \delta_w \tau_w \bar{w}_{it}], \]  

(A11)

where \(\hat{\delta}_{it+1} = \delta_w \hat{\delta}_{it+1} + (1 - \delta_w)\hat{S}_{w}^w\) is average worker surplus in period \(t + 1\). I now average (A11) across all firms and subtract the resulting expression from (A11). This, combined with (A9), yields

\[ s_w \bar{w}_{it} + (1 - \lambda)\beta_0 S_{y}^{w} \delta_w \tau_w \bar{w}_{it} = S_{y}^w (\hat{S}_{it} - \hat{\delta}_{it}^w) = S_{y}^w \tau_w \bar{w}_{it}, \]  

41
which implies

$$\tau_w = \frac{s_w}{[1 - (1 - \lambda)\beta\delta_w]S_y}.$$ 

Similarly, any firm's surplus, equation (26) in the text without the * superscript on period-\(t\) variables, can be approximated by

$$S_y^f \hat{S}_it^f = s_w\hat{w}_{it} - s_w\hat{w}_{it} + (1 - \lambda)\beta S_y^f E_t[\hat{\beta}_{t,t+1} + \delta_w\hat{S}_{it+1}^f + (1 - \delta_w)\hat{S}_{i+1}^f],$$

(A12)

where \(S_y^f \equiv \frac{nS_y^f}{y}\). I now guess

$$\hat{S}_it^f = \hat{S}_i^f - \tau_f \hat{w}_{it}.$$  

(A13)

This implies that, for non-renegotiating firms,

$$\hat{S}_it^{f0} = \hat{S}_i^{f0} - \tau_f \hat{w}_{i-1},$$

(A14)

where \(\hat{S}_it^{f0}\) is the average firm surplus across non-renegotiating firms. From the definition of \(\hat{w}_{it}\), equation (19), it follows that, up a log-linear approximation,

$$s_w\hat{w}_{it} = s_w\hat{w}_t + \psi s_v(1 + \psi)\hat{z}_{it}$$

(A15)

$$= s_w\hat{w}_t - \psi s_v(1 + \psi)\tau_z\hat{w}_{it}$$

where in the second equality I have used Lemma 1. Using (A14), (56) and \(\hat{w}_{it} = \hat{w}_{it} + \hat{w}_t\) in (A12), I obtain

$$S_y^f \hat{S}_it^f = s_w\hat{w}_t - \psi s_v(1 + \psi)\tau_z\hat{w}_{it} - s_w(\hat{w}_{it} + \hat{w}_t) + (1 - \lambda)\beta S_y^f E_t[\hat{\beta}_{t,t+1} + \hat{S}_{i+1}^f - \delta_w\tau_f\hat{w}_{it}],$$

(A16)

where \(\hat{S}_{i+1}^f = \delta_w\hat{S}_{i+1}^f + (1 - \delta_w)\hat{S}_{i+1}^f\) is average firm surplus. I average (A16) across all firms and substract the resulting expression from (A16). This, combined with (A13), yields

$$-\psi s_v(1 + \psi)\tau_z\hat{w}_{it} - s_w\hat{w}_{it} - (1 - \lambda)\beta S_y^f \delta_w\tau_f\hat{w}_{it} = S_y^f(\hat{S}_it^f - \hat{S}_i^f) = -S_y^f \tau_f \hat{w}_{it}.$$  

The latter implies

$$\tau_f = \frac{s_w + \psi s_v(1 + \psi)\tau_z}{[1 - (1 - \lambda)\beta\delta_w]S_y^f}.$$  

42
Proof of Lemma 4

Using Lemma 3, I can express the worker surplus in renegotiating firms in period \( t + 1 \) as

\[
\hat{S}_{t+1}^w = \hat{S}_{t+1}^w + \tau_w (\log W_{t+1}^s - \log W_{t+1}),
\]

and similarly for the firm surplus in renegotiating firms,

\[
\hat{S}_{t+1}^f = \hat{S}_{t+1}^f - \tau_f (\log W_{t+1}^s - \log W_{t+1}).
\]

From the bargaining rule, equation (28) in the text, I have \( \hat{S}_{t+1}^w = \hat{S}_{t+1}^f \), which implies

\[
\hat{S}_{t+1}^w = \hat{S}_{t+1}^f - (\tau_f + \tau_w) (\log W_{t+1}^s - \log W_{t+1}). \tag{A17}
\]

Log-linearizing the first order condition with respect to vacancies, equation (4) in text (where \( J_n(n_{it+1}) \) is just \( S_{it}^f \)), and using Lemma 3 to aggregate across all firms yields

\[
E_t(\hat{S}_{t+1}^f - \sigma^{-1} \hat{c}_{t+1}) = (1 - \epsilon) \hat{\theta}_t + \hat{\psi} \hat{z}_t. \tag{A18}
\]

Combining (A17) and (A18), I obtain

\[
E_t(\hat{S}_{t+1}^w - \sigma^{-1} \hat{c}_{t+1}) = E_t(\hat{S}_{t+1}^f - \sigma^{-1} \hat{c}_{t+1}) - (\tau_f + \tau_w) E_t(\log W_{t+1}^s - \log W_{t+1})
\]

\[
= (1 - \epsilon) \hat{\theta}_t + \hat{\psi} \hat{z}_t - (\tau_f + \tau_w) \frac{\delta_w}{1 - \delta_w} E_t \pi_{wt+1}, \tag{A19}
\]

where in the second equality I have used \( \log W_{t+1}^s - \log W_{t+1} = \frac{\delta_w}{1 - \delta_w} \pi_{wt+1} \). Averaging the individual flexible real wage, equation (44) in the text, and using (56) to substitute for \( E_t(\hat{S}_{t+1}^w - \sigma^{-1} \hat{c}_{t+1}) \), I obtain

\[
s_w \hat{u}_{it}^f = s_w \sigma^{-1} \hat{c}_t + (1 - \xi) \left[ \alpha \left( \hat{\varphi}_t + \hat{\eta}_t \right) - \sigma^{-1} \hat{c}_t \right] + \hat{\psi}(1 + \hat{\psi}) s_v \hat{z}_t
\]

\[
\quad + \epsilon \hat{h}_t + \xi \beta S_y^w \left[ \hat{\theta}_t + \hat{\psi} \hat{z}_t - \tau E_t \pi_{wt+1} \right],
\]

where \( \tau \equiv (\tau_f + \tau_w) \frac{\delta_w}{1 - \delta_w} \). Using the fact that, in the steady state, \( \beta S_y^w = \frac{1 - \xi}{\xi} \beta S_y^f \) and \( \beta S_y^f = (1 + \psi) \frac{\delta_w}{\alpha} \), I finally obtain equation (47) in the text.
B. Deriving the loss function

I now obtain a second order approximation of the welfare criterion, \( \sum_{t=0}^{\infty} \beta^t U_t \), where

\[
U_t \equiv \frac{c_t^{1-\sigma}}{1-\sigma} - n_t \left( \frac{h_t^{1+\eta}}{1+\eta} + b \right) - \frac{\chi}{1+\psi} \int_0^1 \left( \frac{v_{it}}{n_{it}} \right)^{1+\psi} n_{it} di
\]

is the period utility flow. Notice that any function \( x_t^a \) can be written as \( e^{a \ln x_t} \). We can then expand every function in the logarithm of its arguments around their steady-state levels, \( \ln x \). Utility from consumption can be approximated by

\[
\frac{c_t^{1-\sigma}}{1-\sigma} = u(c)(1-\sigma^{-1}) \left( \hat{c}_t + \frac{1-\sigma^{-1}}{2} \hat{c}_t^2 \right) + t.i.p. + O^3 \quad \text{(B1)}
\]

where \( O^k \) indicates terms of order \( k \)-th and higher in the size of the shocks and \( t.i.p. \) represents terms independent of policy. In the second equality I have used \( u(c)(1-\sigma^{-1}) = u'(c) c \). Similarly, variable labor disutility can be approximated by

\[
n_t \frac{h_t^{1+\eta}}{1+\eta} = nv(h) \left[ (1+\eta) \left( \hat{h}_t + \frac{1+\eta}{2} \hat{h}_t^2 \right) + \tilde{n}_t + \frac{1}{2} \hat{n}_t^2 + (1+\eta) \hat{h}_t \tilde{n}_t \right] + t.i.p. + O^3 \quad \text{(B2)}
\]

were in the second equality I have used \( v(h) = \frac{v'(h)h}{1+\eta} \), my assumption of efficient steady-state hours, \( v'(h) = u'(c) mpl \), as well as \( mpl = \alpha \frac{y}{nh} \) and \( y = c \). Fixed labor disutility can be approximated by

\[
n_t b = nv(h) \tilde{b} \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + t.i.p. + O^3 \quad \text{(B3)}
\]

where \( \tilde{b} \equiv \frac{b}{v(h)} \). Individual hiring costs can be approximated by

\[
\frac{\chi}{1+\psi} \frac{z_t^{1+\psi}}{1+\psi} n_{it} = \frac{\chi z_t^{1+\psi}}{1+\psi} n \left\{ (1+\psi) \hat{z}_{it} + \tilde{n}_{it} + \frac{1}{2} [(1+\psi)^2 \hat{z}_{it}^2 + \tilde{n}_{it}^2 + 2 (1+\psi) \hat{z}_{it} \tilde{n}_{it}] \right\} + t.i.p. + O^3. \quad \text{(B4)}
\]
Total employment, \( n_t \equiv \int n_{it} di \), and the average hiring rate, \( z_t \equiv \int \frac{wu}{n_t} z_{it} di \), can be approximated, respectively, by

\[
\hat{n}_t = E_t \hat{n}_it + \frac{1}{2} Var_t \hat{n}_it + O^3, \tag{B5}
\]
\[
\hat{z}_t = E_t \hat{z}_it + \frac{1}{2} Var_t \hat{z}_it + E_t \hat{n}_it \hat{z}_it - \hat{n}_t \hat{z}_t + O^3, \tag{B6}
\]

where for any variable \( e_{it} \), \( E_t e_{it} \equiv \int_0^1 e_{it} di \) and \( Var_t e_{it} \equiv E_t (e_{it} - E_t e_{it})^2 \) denote its cross-sectional average and variance, respectively. I have also used the identity \( E_t \hat{n}_it^2 = Var_t \hat{n}_it + (E_t \hat{n}_it)^2 \) and the fact that \( (E_t \hat{n}_it)^2 = \hat{n}_t^2 + O^3 \) (similarly for \( \hat{z}_t \)). Notice that the average hiring rate can also be written as \( z_t = \frac{\hat{u}_t}{\hat{n}_t} \), which implies \( \hat{z}_t = \hat{u}_t - \hat{n}_t \). Combining this with equations (B4), (B5) and (B6), I can write total hiring costs as

\[
\int_0^1 \frac{\chi}{1 + \psi} z_{it}^{1+\psi} n_{it} di = u'(c) s_v \left\{ (1 + \psi) \hat{u}_t - \psi \hat{n}_t + \frac{1}{2} \left[ ((1 + \psi) \hat{u}_t - \psi \hat{n}_t)^2 + \psi(1 + \psi) Var_t \hat{z}_{it} \right] \right\} + t.i.p. + O^3, \tag{B7}
\]

where I have also used \( s_v = \frac{x^2 \chi^{1+\psi}}{1+\psi} \frac{n}{u'(c)\psi} \). The aggregate resource constraint, \( A_t(n_t h_t)^{\alpha} k^{1-\alpha} = c_t \Delta_t \), admits the following representation,

\[
a_t + \alpha(\hat{n}_t + \hat{h}_t) = \hat{c}_t + \Delta_t. \tag{B8}
\]

Combining equations (56) to (56), as well as equations (56) in (B8), I can write the period utility flow as

\[
U_t = u'(c) \left[ \left( \frac{\alpha \eta - \hat{b}}{1 + \eta} + s_v \psi \right) \hat{n}_t - s_v (1 + \psi) \hat{u}_t - \hat{\Delta}_t \right] + \frac{u'(c) c}{2} \left\{ (1 - \sigma^{-1}) \hat{c}_t^2 - \alpha \frac{1 + \hat{b}}{1 + \eta} \hat{n}_t^2 - \alpha (1 + \eta) \hat{h}_t^2 - 2 \alpha \hat{h}_t \hat{n}_t \right\} - s_v \left[ ((1 + \psi) \hat{u}_t - \psi \hat{n}_t)^2 + \psi(1 + \psi) Var_t \hat{z}_{it} \right] + t.i.p. + O^3. \tag{B10}
\]

**The Beveridge curve**

In order to eliminate the linear terms in (56), I perform the following second order approximation of the law of motion of employment, \( n_{t+1} = (1 - \lambda) n_t + v_t u_{t+1}^{1-\epsilon} \),

\[
\hat{n}_{t+1} + \frac{\hat{n}_{t+1}^2}{2} = (1 - \lambda) \left( \hat{n}_t + \frac{\hat{n}_t^2}{2} \right) + \lambda \left\{ \epsilon \hat{v}_t + (1 - \epsilon) \hat{u}_t + \frac{1}{2} \left[ \epsilon \hat{v}_t + (1 - \epsilon) \hat{u}_t \right]^2 \right\} + O^3, \tag{B11}
\]
where I have used the fact that $v^\epsilon u^{1-\epsilon} = \lambda n$. On the other hand, the unemployment rate, $u_t = 1 - n_t$, admits the following approximation

$$\hat{u}_t = -\frac{p}{\lambda} \left( \hat{n}_t + \frac{\hat{n}_t^2}{2} \right) - \frac{\hat{u}_t^2}{2} + O^3,$$

where I have used $\frac{n}{u} = \frac{p}{\lambda}$. Inserting (B12) into (B11), multiplying both sides by $\beta^t$ and integrating across $t$, I obtain

$$\sum_{t=0}^{\infty} \beta^t \{ [\beta^{-1} - (1 - \lambda - (1 - \epsilon)p)]\hat{n}_t - \lambda \epsilon \hat{v}_t \}$$

$$= \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left\{ \lambda [\epsilon \hat{v}_t + (1 - \epsilon)\hat{u}_t]^2 - \lambda (1 - \epsilon)\hat{u}_t^2 - [\beta^{-1} - (1 - \lambda - (1 - \epsilon)p)]\hat{n}_t^2 \right\}$$

$$+ O^3 + t.i.p.,$$

where I have used the fact that $\hat{n}_0$ is independent of policy as of time 0. From the job creation condition in the steady state, equation (54), I have

$$\beta^{-1} - [1 - \lambda - (1 - \epsilon)p] = \frac{1}{1 + \psi} s_v^{-1} \lambda \epsilon \left( \frac{\eta - \bar{b}}{1 + \eta} + \psi s_v \right).$$

This allows me to write (B13) as

$$\sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\alpha}{1 + \eta} + \psi s_v \right) \hat{n}_t - (1 + \psi)s_v\hat{v}_t \right]$$

$$= \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left\{ (1 + \psi)s_v [\epsilon \hat{v}_t^2 - (1 - \epsilon)\hat{u}_t^2 + 2(1 - \epsilon)\hat{v}_t \hat{u}_t] - \left( \frac{\alpha}{1 + \eta} + \psi s_v \right) \hat{n}_t^2 \right\} + O^3 + t.i.p.,$$

I now multiply (56) by $\beta^t$ and integrate across $t$. I can then insert (B14) in the resulting expression to obtain

$$\sum_{t=0}^{\infty} \beta^t U_t = \frac{u'(c)c}{2} \left\{ (1 - \sigma^{-1})\hat{c}_t^2 - \alpha \hat{n}_t^2 - \alpha (1 + \eta)\hat{h}_t^2 - 2\alpha \hat{h}_t \hat{n}_t \right.$$ 

$$- (1 + \psi)s_v \left[ (1 - \epsilon)\hat{\theta}_t^2 + \psi \hat{z}_t^2 + \psi Var_c \hat{z}_t \right] - 2\Delta_t \right\} + t.i.p. + O^3,$$

where I have also used $\hat{\theta}_t = \hat{v}_t - \hat{u}_t$ and $\hat{z}_t = \hat{v}_t - \hat{n}_t$. 

46
Price dispersion and inflation

A second order Taylor expansion of $\Delta_t = \int_0^1 \left( \frac{P_t}{P_t} \right)^{-\gamma} d\gamma$ yields

$$\hat{\Delta}_t + \frac{\hat{\Delta}_t^2}{2} = -\gamma (E_j \bar{p}_{jt} - \frac{\gamma}{2} E_j \bar{p}_{jt}^2) + O^3, \quad (B16)$$

where $\bar{p}_{jt} \equiv \log(P_{jt}/P_t)$ and I have used the fact that $\Delta = 1$. Similarly, a second order approximation of $P_t^{1-\gamma} = \int_0^1 P_{jt}^{1-\gamma} d\gamma$ yields

$$E_j \bar{p}_{jt} = \frac{\gamma - 1}{2} E_j \bar{p}_{jt}^2 + O^3.$$

Therefore, (B16) becomes

$$\hat{\Delta}_t = \frac{\gamma}{2} E_j \bar{p}_{jt}^2 + O^3,$$

where I have used the fact that $\hat{\Delta}_t^2$ is $O^4$. Since $E_j \bar{p}_{jt}$ is $O^2$, it follows that $E_j \bar{p}_{jt}^2 = Var_j \log P_{jt} + O^4$. This implies

$$\hat{\Delta}_t = \frac{\gamma}{2} Var_j \log P_{jt} + O^3, \quad (B17)$$

As shown by Woodford (2003) in his Proposition 6.3, the cross-sectional variance of log prices can be approximated by

$$Var_j \log P_{jt} = \delta_p Var_j \log P_{jt-1} + \frac{\delta_p}{1 - \delta_p} \pi_t^2 + O^3. \quad (B18)$$

Multiplying both sides of (B16) by $\frac{\gamma}{2}$ and using (B17), I find the following law of motion for the price dispersion term,

$$\hat{\Delta}_t = \delta_p \hat{\Delta}_{t-1} + \frac{\gamma}{2} \frac{\delta_p}{1 - \delta_p} \pi_t^2 + O^3. \quad (B19)$$

Multiplying (B19) by $\beta^t$, integrating forward and using the fact that $\hat{\Delta}_{-1}$ is independent of policy as of time 0, I obtain

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\gamma}{2} \frac{\delta_p}{(1 - \delta_p)(1 - \beta \delta_p)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p. + O^3 \quad (B20)$$
Dispersion in hiring rates and wage inflation

Analogously, the cross-sectional variance of nominal wages can be approximated by

\[ \text{Var}_i \log W_{it} = \delta_w \text{Var}_i \log W_{it-1} + \frac{\delta_w}{1 - \delta_w} \pi_w^2 t + O^3. \quad (B21) \]

Multiplying (B21) by \( \beta^t \), integrating forward and using the fact that \( \text{Var}_i \log W_{i,-1} \) is independent of policy as of time 0, I obtain

\[ \sum_{t=0}^{\infty} \beta^t \text{Var}_i \log W_{it} = \frac{\delta_w}{(1 - \delta_w)(1 - \beta \delta_w)} \sum_{t=0}^{\infty} \beta^t \pi_w^2 t + \text{i.p.} + O^3. \]

From Lemma 1, it follows that \( \text{Var}_i \hat{z}_{it} = \tau_z^2 \text{Var}_i \log W_{it} \). This allows me to write

\[ \sum_{t=0}^{\infty} \beta^t \text{Var}_i \hat{z}_{it} = \frac{\tau_z^2 \delta_w}{(1 - \delta_w)(1 - \beta \delta_w)} \sum_{t=0}^{\infty} \beta^t \pi_w^2 t + \text{i.p.} + O^3. \quad (B22) \]

Finally, inserting (B20) and (B22) into (B15) yields

\[ \sum_{t=0}^{\infty} \beta^t U_t = -\frac{u'(c) c}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \alpha \left[ \eta \hat{h}_{t}^2 + 2 \left( \hat{h}_t + \hat{n}_t \right)^2 \right] + (1 + \psi) s_v \left[ (1 - \epsilon) \hat{\theta}_t^2 + \psi \hat{z}_t^2 \right] \right\} + \text{i.p.} + O^3, \]

where \( \lambda_p = \frac{\gamma}{\kappa_p}, \kappa_p = \frac{(1 - \delta_p)(1 - \beta \delta_p)}{\delta_p} \) and \( \lambda_w = \frac{(1 + \psi) s_v \psi}{(1 - \delta_w)(1 - \beta \delta_w)}. \)
References


[34] Pissarides, Christopher A., 2000, Equilibrium Unemployment Theory. MIT Press.


