A model of a systemic bank run*

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PRELIMINARY
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Abstract

The 2008 financial crisis is reminiscent of a bank run, but not quite. In particular, it is financial institutions withdrawing deposits from some core financial institutions, rather than depositors from their local banks, and the core financial institutions have invested the funds in asset-backed securities rather than committed to long-term projects. These securities can potentially be sold to a large pool of outside investors: the question arises, why these investors require steep discounts to do so. I therefore set out to provide a model of a systemic bank run delivering six stylized key features of this crisis. I consider

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†I am grateful to Douglas Diamond for a two very useful conversation at an early stage of this project. I am grateful to a seminar audience at Wisconsin suffering through a first draft of this paper and its presentation.
two different motives for outside investors and their interaction with banks trading asset-backed securities: loss-aversion versus moral hazard. I shall argue that the version with loss-averse investors is more consistent with the stylized facts than the moral hazard perspective: in the former, the crisis deepens, the larger the market share of distressed core banks, while a run becomes less likely instead as a result in the moral hazard version.

I conclude from that that the variant with loss averse investors is more suitable to analyze policy implications. This paper therefore provides an argument that an outright purchase of troubled assets by the government at prices above current market prices can both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

Keywords:

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1 Introduction

Bryant (1980) and Diamond and Dybvig (1983) have provided us with the classic benchmark model for a bank run. There, an individual bank engages in maturity transformation, using demand deposits to finance long-term loans, which can be liquidated in the short term only at a cost. If too many agents claim short-term liquidity needs and withdraw their demand deposits, the value of the bank assets are thus not sufficient to meet these liquidity demands, in turn justifying even patient depositors to get their money while they can: a bank run ensues. One policy conclusion then is for a central bank to follow the classic Bagehot principle of committing to inject liquidity to illiquid but otherwise solvent bank, in order to stop bank runs.

The financial crisis of 2007 and 2008 is reminiscent of a bank run, but not quite, see Brunnermeier (2008). First, this was (with few exceptions) not a run of depositors on their local house bank, but a run of banks and money funds on some core financial institutions. Second, the health of some core financial institutions (I shall call them “core banks” for the purpose of this paper) was called into question not because of their commitment to costly-to-call long-term loans, but rather because of the questionable value of a variety of “exotic” securities, most notably their guarantees for particular tranches of mortgage-backed security derivatives and credit default swaps. These are assets which could be marked to market at least in principle. So, when a bank cannot repay its depositors because the market value of their assets is below the value of its liabilities, the traditional prescription is to declare the bank to be bankrupt and not to provide it with additional liquidity.

In the current situation, this would mean for the affected banks to mark their assets to market and to sell e.g. their questionable assets, while they can meet withdrawals. There is a widespread perception, however, that current market prices are below fundamental values, and that further sales of these assets are akin to fire sales, leading to further depression of the price of these assets, triggering additional bankruptcies. This conclusion appears unpalatable to many and therefore, the Federal Reserve Bank and the Treasury
have instead expanded interventions where these assets will be bought at above-laissez-faire prices. There is the perception that current events should be understood as some version of a systemic bank run, despite the inapplicability of the original Diamond-Dybvig framework. This creates a gap in our understanding. A new or at least a modified theory is needed.

This paper seeks to contribute to filling that gap, and provide a model (in two variants) of a systemic bank run. It thereby seeks to provide a framework for analyzing or evaluating policy options in a financial crisis similar to the one experienced in 2007 and 2008. For example, who are the gainers and losers of the March-2009 Geithner plan? Or: is the “plan B” proposed by Zingales (2008) a better alternative to the October-2008 Paulson plan? The model proposed here seeks to be provide one vehicle for answering these questions.

It seeks to capture the following stylized features of the 2008 crisis:

1. The withdrawal of funds was done by financial institutions at other financial institutions, rather than depositors at their bank.

2. The troubled financial institutions held their portfolio in asset-backed securities rather than being invested directly in long-term projects.

3. These securities are traded on markets. In the crisis, the prices for these securities appears low compared to some benchmark fundamental value benchmark (“underpricing”).

4. There is a large pool of investors willing to purchase securities, as evidenced e.g. by market purchases of newly issued US government bonds or the volume on stock markets.

5. Nonetheless, these investors are only willing to buy these asset-backed securities at additional discounts compared to some benchmark fundamental value calculation.

6. The larger the market share of troubled financial institutions, the steeper the required discounts.
I describe the model in section 2. A starkly simplified version of the model, that may be useful as a guide to some ideas for the full model is in appendix B. For the full model, I start from an environment inspired by Smith (1991), in which depositors interact with a local bank, which in turn refinances itself via an (uncontingent) deposit account with one of a few core banks, who in turn invest in long-term securities backed by locally run projects (think: mortgage-backed securities). Clearly, the observable world of securities is considerably richer (and harder to describe), but this framework may capture the essence of the interactions. I assume that there are two aggregate states, a “boom” state and a (rare) “bust” state. In the “boom” state, everything follows from the well-known analysis in the benchmark bank run literature, see section 3: essentially, things are fine. More serious problems arise in the bust state. I assume that the long-term securities become heterogeneous in terms of their long-term returns, and that local banks (together with their local depositors) hold heterogeneous beliefs regarding the portfolio of their core bank. Therefore, some local banks may withdraw early, even in the local consumption demands are “late”.

The key idea is now to allow for outside investors with unbounded liquidity to become active in the market for the long-term securities which the core banks seek to unload, but that these investors demand steeper discounts for the long-term securities than one would expect to see under “normal” conditions, described in section 4.1. Generating a rationale for the additional discounts, and the interaction of these discounts with the selling decisions by core banks is at the center of the analysis.

This requires modeling the motives of outside investors. I investigate two variants in particular. The first hypothesizes that only a small subset of outside investors has the expertise to evaluate the asset which the core banks wish to sell, and that the remaining vast majority of investors is highly loss averse: they fear getting “stuck” with the worst asset among a diverse portfolio, and are therefore not willing to bid more than the lowest price, see section 4. The second reason is assuming risk-neutral investors together
with adverse selection, i.e. an Akerlof-style lemons problem: whatever the market price, liquid core banks have an incentive to sell assets that will be a good deal for them and a bad deal to the buyers, leading to a low market price, see section 5. Both models generate a downward sloping demand curve or, more accurately, an upward sloping market discounting for the long-term securities from the perspective of the individual core bank. As more local banks seek to withdraw early, this will increase the discounting a core bank has to endure, in turn triggering further local banks to withdraw from this as well as other core banks. This creates a systemic bank run.

However, the two variants have sharply different implications for certain thought experiments, and I shall argue that a straight moral-hazard explanation appears to be at odds with some stylized view of current events, while the variant with loss-averse investors is not, see section 6. Since the models also have sharply different policy conclusions, I shall therefore argue to rather trust the policy conclusions from the loss-averse model and to discard the policy conclusions emerging from the moral hazard framework.

There obviously is a large literature expanding the Diamond-Dybvig bank run paradigm, and it includes investigations into systemic risk and the occurrence of fire sales. Additionally and due to recent events, a plethora of papers have appeared, seeking to provide explanations and coherent frameworks. A number of these papers share questions and insights with the paper at hand, but differences remain. A complete discussion is beyond the scope of this paper and excellent surveys are available elsewhere. Allen and Gale (2007), for example, have succinctly summarized much of the bank run literature, including in particular their own contributions, in the their Clarendon lectures. Rochet (2008) has collected a number of his contributions with his co-authors which help to understand banking crises and the politics and policy of bank regulation. A number of papers regarding the recent financial crises and avenues towards a solution have been collected in Acharya and Richardson (2009), and that literature keeps evolving quickly. Nonetheless, it may be good to provide at least a sketch on some related ideas and to
describe how this paper relates to them\footnote{I apologize for the very sketchy and possibly inaccurate nature of this sketch!}.

While the Diamond-Dybvig model is originally about multiple equilibria ("bank run" vs "no bank run"), Allen and Gale put considerable emphasis instead on fundamental equilibria, in which it is individually rational for a depositor to "run", even if nobody else does. In this paper I lean towards this fundamental view, but take somewhat of a middle ground. For the "bust" state, I shall argue, that some investors may believe the situation to be sufficiently bad that they withdraw, even if few others or nobody else does, while others are more optimistic. This can generate a partial fundamental run (based on the underlying beliefs), which may tip into a full-fledged bank run, see section 4.1.

Allen and Gale (1994, 2004b) have investigated the scope and consequences of cash-in-the-market pricing to generate fire sale pricing and bank runs. In the context here, the idea is that the additional investors need to bring cash to period 1, in case the core banks need to sell securities in period 1 in the bust state. If the bust state is sufficiently unlikely, the incentives to do so and therefore the additional liquidity is small: asset prices in the bust state are then not determined by the usual asset pricing equations, but rather by the amount of liquidity available. This may suffice as an explanation for current events. However, there clearly are plenty of investors out there who have liquidity available, when, say, the US government seeks to sell additional Treasury bonds. Why, then, should one assume the same investors to forget to bring their wallet, when other securities are auctioned off at firesale prices? The cash-in-the-market pricing surely therefore is a stand-in assumption for an endogenous reluctance of an otherwise deep market to buy the securities which the core banks are desperate to sell. Thinking about this reluctance and its implications is one of the key goals of this paper.

Diamond and Rajan (2009) have argued that banks have become reluctant to sell their securities at present, if they foresee the possibility of insolvency due to firesale prices in the future: the option of waiting allows banks to
redistribute losses of depositors from the insolvency state in the future into private gains in the case of continued solvency. Their paper helps to explain the reluctance of banks to resolve their predicament by trading, but additional reasons are needed to generate the firesale price in the first place: the latter is the focus of this paper.

While the popular press views financial crises and bank runs as undesirable desasters, e.g. Allen and Gale (1998, 2004a) have shown that they instead may be an integral part of business cycles and can serve a socially useful rule by partially substituting for a missing market due to the uncontingent nature of deposit contracts. A number of regulatory and policy issues arise as a result. It follows directly, that a policy avoiding bank runs or financial crises under all circumstances may be welfare decreasing. On a more subtle level, Ennis and Keister (2008) have shown that ex-post efficient policy responses to a bank run of allowing urgent depositors to withdraw may actually increase the incentives to participate in a bank run and the conditions for a self-fulfilling bank run in the first place. Given these and a number of related results, the focus of this paper is on the positive analysis of current policy proposals rather than a normative “second-best” analysis, though this would be a desirable part of further research (or a future draft of this paper). Likewise (and regarding potentially welfare-improving private sector solutions), we assume a particular structure of the contracts, markets and asymmetries of beliefs and information, rather than requiring contracts to be optimal, as in Green and Lin (2003) or Ennis and Keister (2008).

There is a large literature on systemic risk and contagion, both for international financial crises (which I shall not even attempt to review here) as well as for banking crises. For example, Cifuentes et al (2005) have studied the interplay between uncontingent capital adequacy requirements and the endogenous collapse of prices and balance sheets, as banks need to unload assets in order to meet these requirements. They assume that demand for these assets is downward sloping: this paper seeks to investigate why. Allen and Gale (2000) have studied the possibility for contagion in a sparse net-
work of banks interlinked by mutual demand deposits, where a collapse of one bank can lead to a domino effect per their large withdrawals on their direct neighbor. Here, a hierarchy is instead assumed, where local banks hold deposit contracts on core banks, who in turn use the market to obtain liquidity, rather than other core banks. Diamond and Rajan (2005) have investigated the contagious nature of bank failures, arguing that bank failures can shrink the common pool of liquidity, thereby possibly leading a meltdown of the entire system. They assume that the returns on long-term projects can only be obtained by banks, and that any securities written on these returns can only be traded by banks. While this paper shares the central idea of a shortage of a common pool of liquidity and the feature, that projects are run by “managing” (local) banks, I allow outside investors to buy the securities written on these projects and collect their returns. In essence, I assume that a mortgage-backed security will pay its return, irrespective of who actually holds that security. If that perspective is appropriate, then one needs to understand why outside deep-pocket investors do not buy these securities, if they are indeed severely undervalued.

2 The model

There are three periods, $t = 0, 1, 2$. There are two fundamental aggregate states: “boom” and “bust”. The aggregate state will be learned by all participants in period 1. There are four types of agents or agencies:

1. Depositors in locations $s \in [0, 1]$.
2. Local banks in locations $s \in [0, 1]$.
3. Core banks, $n = 1, \ldots, N$.
4. Outside investors $i \in [0, \infty)$.

There are two types of assets
1. A heterogeneous pool of long-term securities ("mortgage backed securities"), backed by long-term projects in locations \( s \in [0, 1] \).


Let me describe each in turn. As in Allen and Gale (2007),, I assume that depositors have one unit of resources in period 0, but that they care about consumption either in period 1 ("early consumer") or in period 2 ("late consumer"). As in Smith (1983), I assume that all depositors at one location are of the same type. They learn their type in period 1. I assume that a fraction \( \varphi \) of locations has early consumers and a fraction \( 1 - \varphi \) has late consumers, where \( 0 < \varphi < 1 \), that the realization of the early/late resolution is iid across locations and that depositors are evenly distributed across locations. I assume that depositors learn of their type in period 1. Ex-ante utility is therefore given by

\[
U = \varphi E[u(c_1)] + (1 - \varphi)E[u(c_2)]
\]

where \( c_1 \) and \( c_2 \) denotes consumption at date 1, if the consumer is of the early type and \( c_2 \) denotes consumption at date 2, if the consumer is of the late type and where \( u(\cdot) \) satisfies standard properties. This heterogeneity in consumption preference induces a role for liquidity provision and maturity transformation, as in Diamond and Dybvig (1983) and the related literature. I assume that depositors only bank with the local bank in the same location. This lack of diversification can be thought of as arising from some unspecified cost to diversification, e.g. the impossibility for banks or depositors to travel to other locations. An alternative way to think about this is that \( s \) actually enumerates the deposit banks in existence and each location denotes its customer base, noting that depositors are observed to typically spread their bank accounts across very few banks only.

At date zero, local banks can invest in long-term projects ("mortgages") of location \( s \) or short-term securities, and they can invest in short-term securities in period 1, but they cannot invest in long-term securities. Long-term
projects pay off only in period 2. I assume that long-term projects cannot be terminated ("liquidated") prematurely and that they require nonnegative investments in period 0. I assume that local banks administer the local long-term projects, delivering their payment streams to whoever finances them originally.

I allow local banks to open accounts with the core banks, depositing resources in period 0 and taking withdrawals in period 1 and/or period 2. Again, for some unspecified cost reasons, we assume that local banks operate a deposit account only with one of the core banks.

Core banks invest the period-0 deposits received from local banks in local long-term projects, and turn their period-2 payments into long-term securities. In all periods, core banks can trade in short-term as well as long-term securities.

In the aggregate "boom" state, local long-term projects return $R_{boom} + \epsilon_s$, where $\epsilon_s$ is a random variable with mean zero, distributed independently and identically across locations $s \in [0, 1]$. Long-term securities pool these risks. Thus, in the aggregate "boom" state, the long-term securities all return $R_{boom}$. I assume that

$$\varphi u(0) + (1 - \varphi) u(R_{boom}) < u(1) \quad (2)$$

$$1 < R_{boom} < \frac{u'(1)}{u'(R_{boom})} \quad (3)$$

If $u(c)$ is CRRA with an intertemporal elasticity of substitution below unity and if $R_{boom} > 1$, both equations are satisfied. Further, (2) is generally satisfied, if $(1 - \varphi)R_{boom} < 1$.

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2To provide this with a bit of formal structure, suppose there are $m = 1, \ldots, M$ long-term securities, suppose that $(A_m)_{m=1}^M$ is a partition of $[0, 1]$ with each $A_m$ having equal Lebesque measure, and suppose that the payoff for the long term security with index $m$ is the integral of all long-term projects $s \in A_m$. The law of large numbers in Uhlig (1996) then implies the safe return here. Conversely, knowing the return of the long-term securities, one might directly assume that the long-term projects return this return plus the idiosyncratic noise $\epsilon_s$. This structure can also be used for the "bust" episode. We will not make further use of this formal structure, though.
In the “bust” state, the long term securities offer heterogeneous returns \( R \sim F \), drawn from some distribution \( F \) on some interval \([ R, \bar{R}]\), where \( 0 < R \leq R < \infty \), with unconditional expectation \( R_{\text{bust}} \), satisfying

\[
R_{\text{bust}} \leq R_{\text{boom}}
\]  

(4)

Once the aggregate state is revealed to be a “bust” in period 1, I assume that core banks all know the type of long-term securities in their portfolio, i.e. know the period-2 return of the securities in their portfolio. Depositors and local banks only know the distribution \( F \), but not the portfolio quality of their (or other) core banks. Instead form heterogeneous beliefs about that. I assume that local banks at location \( s \) and its depositors believe their core bank to hold a portfolio with return distribution \( F(\cdot; s) \), where \( F(\cdot; \cdot) \) is measurable and \( F(\cdot; s) \) is a distribution function. I assume that

\[
F(R) = \int F(R; s) ds
\]

Therefore, aggregate beliefs accurately reflect the aggregate distribution, but there is a potential disagreement at the local level. For simplicity, I shall assume that core banks actually all hold exactly the same portfolio, i.e. there is a mismatch between the beliefs of the local banks and the portfolio of their core bank. I assume that core banks do not know the belief \( F(\cdot; s) \) of their local banks at date 0 and contracting time\(^3\) and cannot condition allowed withdrawals on these beliefs at time 1 or time 2.

Finally, there is a large pool of outside investors \( i \in [0, \infty) \). These investors can invest in the long-term securities or the short-term securities in period 1, though not in period 0. Each investor is endowed with one unit of resources. I do not allow them to engage in short-selling.

I shall investigate three variants in particular:

1. **[Benchmark:]** As a benchmark, I assume that outside investors are risk-neutral, discounting resources between period 1 and period 2 at

\(^3\)E.g., suppose that the believes are \( F(R; s) = G(R; s + \xi \mod 1) \), where \( \xi \) is a random variable uniformly distributed on \([0, 1]\) and drawn at date 1 and \( G(\cdot; \cdot) \) is a commonly known function.
rate $\beta$. Furthermore, I assume that core banks sell bundles of their long-term securities, which have the same return distribution as their total portfolio (or, equivalently, sell randomly selected long-term securities, but cannot “adversely select” the long-term security they wish to sell).

2. **[Loss Aversion:]** I assume them to be extremely loss averse\(^4\), drawing on Tversky and Kahnemann (1991) and Barberis, Huang and Santos (2001). Given a security drawn from a pool of securities with $[R_{\text{low}}, \bar{R}]$ as the support of its returns, these investors are willing to pay

$$\beta R$$

per unit invested, i.e. the investor is willing to entertain gains, but not losses, even if the expected gains are far larger than the expected losses. I assume that

$$\beta R_{\text{boom}} < 1$$ \hspace{1cm} (5)

Another way of reading these preferences is that investors are suspicious or perhaps even paranoid. If offered to trade a security from the described set, they will fear that they will always be offered the security with the lowest of these returns, even though this cannot happen to all investors in equilibrium. A third interpretation is that these are traders working on behalf of institutional investors drawn to the profit opportunities in the market, who face lopsided incentives for investing in a bust market: due to the complexity of these securities, they cannot afford to risk losing money ex post, as their managers may not be able to tell whether this was indeed just a case of bad luck or a case of poor research.

One can make this a bit more formal as follows. Suppose that the initial “status quo” consumption plan for an investor is $a_1^*$ and $a_2^*$ in

\(^4\)Alternatively, one may interpret these investors as allowing for a high degree of ambiguity concerning the downside risk, which one could model more formally using the approach in Hansen and Sargent (2008).
periods 1 and 2, before trading in the long-term securities. Suppose the purchase would then result in the (possibly stochastic) plan \( a_1, a_2 \).

The investor evaluates these plans according to

\[
E[V], \text{ where } V = \begin{cases} 
a_1 + \beta a_2 & \text{if } a_1 + \beta a_2 \geq a_1^* + \beta a_2^* \\
-\infty & \text{if } a_1 + \beta a_2 < a_1^* + \beta a_2^* \end{cases}
\] (6)

where the expectation is taken with respect to the information available to the investor at date 1. Suppose that a security was offering a safe return of \( R \). Even with the upper part of the definition of \( V \) alone, the investor would be willing to invest in this security if and only if \( R\beta \geq 1 \).

Suppose now, the return is risky. What these preferences then say is that the investor is willing to invest in that security if and only if the minimal return is at least \( 1/\beta \).

The group \( i \in [0, \omega] \) of these investors is assumed to have the expertise of discerning the quality of the long-term securities, i.e. they know the return of a given long-term security and are therefore willing to buy them when the return exceeds \( 1/\beta \). I call them the expert investors. All other investors \( i > \omega \) only know the distribution \( F \) and the equilibrium, but not the specific return of some offered long-term security.

3. **Moral hazard:** I assume that outside investors are risk-neutral, discounting period-2 payoffs at rate \( \beta \), but cannot distinguish between the qualities of the long-term securities sold to them. I assume that core banks can “adversely select” the long-term security they wish to sell.

The timing of the events is now as follows. In period 0, core banks offer deposit contracts to local banks, offering state-uncontingent withdrawals of \( r \) in period 1 per unit deposited. Local banks offer state-uncontingent withdrawals of \( \tilde{r} \) in period 1 per unit deposited. In period 1 and depending on the aggregate state, local banks may withdraw \( r \) from their core bank. The core banks match these withdrawal demands from payoffs of their portfolio of short-term securities as well as sales of long-term securities. If they cannot meet all withdrawal demands, they declare bankruptcy. In that case, I
assume that all local banks, who have decided to withdraw, obtain an equal pro-rata payment, splitting the entire resources of the bankrupt core bank across local banks in proportion to their withdrawal demands.

I assume that Bertrand competition in these contracts makes local banks pay out everything to their depositors\(^5\) and likewise makes core banks pay out everything to local banks. Therefore, any resources left in period 2 will be paid in proportion to the remaining deposits. Furthermore, local banks will be indifferent which particular core bank to choose. Let

\[ \nu : [0, 1] \to \{1, \ldots, N\} \]

be the core bank selection function, i.e. let \( \nu(s) \) be the core bank selected by the local bank \( s \). I assume \( \nu(\cdot) \) to be measurable\(^6\) In the numerical examples, I will let \( \nu(s) = \max \{n \mid n < Ns + 1\} \), i.e. assume that the local banks distribute themselves uniformly across core banks.

Finally and for simplicity, I assume that the “bust” state is sufficiently unlikely a priori, so that \( r \) and \( \tilde{r} \) are determined entirely from the “boom” state calculus\(^7\).

3 Analysis: Preliminaries

It is useful to first analyze some special cases in order to set the stage of the analysis of the bust state. The analysis of these special cases are the same, no matter which assumption has been made about the type of outside investors.

\(^5\)For that, one may want to assume that there are at least two local banks in each location, though that assumption is immaterial for the rest of the analysis

\(^6\)An alternative is to assume \( \nu(\cdot) \) to be random and use Pettis integration, see Uhlig (1996).

\(^7\)It would not make much difference for the analysis, if instead one were to calculate \( r \) and \( \tilde{r} \) from a full probabilistic analysis.
3.1 No core banks

Consider first the environment above without core banks. The investors then do not matter: they would love to short-sell the short-term securities, but they cannot do so (and that certainly seems reasonable, if one imagines the short-term securities to be Treasury bills). In that case, local banks offer contracts to their local depositors. Note that all their depositors wish to either only consume at date 1 or at date 2. Due to local Bertrand competition, the local banks will choose the deposit contract that maximizes expected utility (1).

Consider first the choice between investing everything in the long-term project versus investing everything in short-term securities. In the first case, depositors only get to consume in case they turn out to be late consumers, and their ex ante utility is

\[ U = \varphi u(0) + (1 - \varphi)E[u(R)] \leq \varphi u(0) + (1 - \varphi)u(R_{\text{boom}}) \]

due to concavity of \( u(\cdot) \) as well as (4). In the second case, depositors can consume in both periods, at ex ante utility equal to \( u(1) \). Since the latter is larger than the former due to (2) and therefore also larger than any convex combination of the latter and the former, local banks will only invest in short-term securities.

One can view this as a version of 100% reserve banking. Note that there cannot be a bank run or financial crisis in this situation, but, as is well known and as we shall see, this solution is inefficient.

3.2 Only “boom” state

To set the stage of the “bust” state analysis as well as an important benchmark, consider the situation with only a boom state. Competition drives banks to maximize the ex-ante welfare of depositors. This amounts to choosing the amount \( x \) to be invested in the long-term securities, \( y \) to be invested in the short-term security and the amount \( z \) of the investment in the long-term...
security to be sold to outside investors at date 1 in order to solve

\[
\max_{x,y,z} \varphi u(c_1) + (1 - \varphi) u(c_2) \\
\text{s.t.} \quad \varphi c_1 = y + \beta R_{\text{boom}} z \\
(1 - \varphi) c_2 = R_{\text{boom}} (x - z) \\
0 \leq x, \ 0 \leq y, \ x + y = 1, \ 0 \leq z \leq x \\
c_2 \geq c_1 \geq 0
\]

where the last constraint prevents local banks in locations with late consumers to withdraw their funds in period 1 and investing in the short security. Note that the optimal solution will have \( z = 0 \) due to (5): it is cheaper to deliver resources for period 1 per investing in the short-term security rather than investing it in the long-term security and selling it at a steep discount to the outside investors. With the interpretation of the sale to outside investors as the liquidation value of long-term projects, this problem is a baseline problem in the literature on banking and has been thoroughly analyzed in the literature, see e.g. Allen and Gale (2007), in particular chapter 3. Due to (3) there will be an interior solution with \( R_{\text{boom}} > c_2 > c_1 > 1 \) with

\[
\frac{u'(c_1)}{u'(c_2)} = R_{\text{boom}}
\]

(7)

As a consequence of this as well as (5),

\[
c_2 < \frac{r}{\beta}
\]

(8)

holds: generally, this is rather far from being a sharp bound.

The period-1 withdrawals offered by the deposit contracts are

\[
r = \tilde{r} = c_1 = \frac{y}{\varphi}
\]

As is well-understood, the solution is more efficient than the solution with 100% reserve banking of subsection 3.2, but potentially subject to bank runs.
For example, if preferences are CRRA with an intertemporal elasticity of substitution below unity,

\[ u(c) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}, \quad \text{where } 0 < \sigma < 1 \]  

and if \( R_{\text{boom}} > 1 \), then (3) is satisfied and

\[ r = \left( \phi + (1 - \phi)R_{\text{boom}}^{\sigma-1} \right)^{-1}, \quad c_1 = r, \quad c_2 = R_{\text{boom}}^\sigma r = R_{\text{boom}} \frac{1 - \phi r}{1 - \phi} \]  

There are perhaps two twists compared to the standard solution. First, core bank runs (i.e. local banks running on the core banks) can occur but they invoke the resale of long-term securities to outside investors at the market discount rate rather than the early termination of projects. This already could be viewed as a solution to the task set forth in the introduction of creating a bank-on-bank run in terms of marketable securities. It is obviously a rather trivial solution, as it simply amounts to one of many possible interpretations of the standard bank run model. That literature is typically silent on what it means to “liquidate” the long-term projects, and selling them at a steep discount certainly is consistent with these models.

Second, aside from liquidity provision, the core banks also offer insurance against the idiosynchratic fluctuations in the returns of long-term projects. Consider a slightly different environment, in which local depositors split into fractions \( \phi \) of early consumers and \( (1 - \phi) \) of late consumers at each location. The local bank may still solve a problem as above, but with the random return \( R_{\text{boom}} + \epsilon_s \) in place of the safe return \( R_{\text{boom}} \). It is obvious, that the solution involving securitization is welfare improving compared to this “local-only” solution, which exposes local depositors to additional local risks. Moreover, it is more likely to trigger “fundamental” bank runs, where long-term depositors run on the local bank, if \( R_{\text{boom}} + \epsilon_s < c_1 \). Indeed, absent intermediation by core banks, these fundamental bank runs are welfare-improving compared to regulating that deposit contracts need to avoid fundamental bank runs at the local level: these bank runs provide
a partial substitute to the missing insurance market, see Allen and Gale (2007). Put differently, securitization improves welfare and makes the system less prone to local bank runs, but exposes it instead to the possibility of “systemic” runs on core banks and thereby to “contagion” across different locations. This interdependence has been analyzed in the literature previously, see e.g. the exposition in chapters 5 and 10 of Allen and Gale (2007), and the literature discussion there.

3.3 The “bust” state and the classic bank run case

To analyze the full model, we assume that the probability of the “bust” state is vanishingly small\(^8\). It therefore remains to analyze the “bust” state, fixing the first-period withdrawal \(r\) of the deposit contracts and the total investments \(r\) in the short-term securities and the long-term securities \(1 - r\) as provided by the solution to the “boom”-only situation above.

Note first, that in the absence of a run, \(c_{2,\text{bust}}(0) = R_{\text{bust}} \frac{(1 - \varphi r)}{1 - \varphi}\)

where I use the argument “(0)” to denote that the fraction zero of late consumers run. Therefore, if \(c_{2,\text{bust}}(0) < r\), there will be a fundamental bank run, even if core banks hold the same “market” portfolio of long-term securities and local banks believe them to do so, as insurance against the “boom-bust” aggregate uncertainty is not available. For CRRA preferences (9) and therefore (10), this will be the case if

\[
R_{\text{bust}} < R_{\text{boom}}^{1 - \sigma}
\]  

(11)

Suppose even further, that all long-term securities offer the return \(R_{\text{bust}}\) and that a fraction \(\theta\) of all local banks serving late consumers opt for early withdrawal. The following algebra is well understood, but will be useful for

\(^8\)Alternatively, assume that the “bust” state was “irrationally” ignored at the time the deposit contracts were signed.
comparison to the more general case. The core banks meet the additional liquidity demands by selling a fraction $\zeta$ of its long-term portfolio or $z = \mu x \zeta$ units of its long-term securities to obtain additional liquidity $L$, where

$$r\theta(1 - \varphi) = L = \beta R_{\text{bust}} \mu x \zeta$$

The securities are effectively discounted at $q = \beta$ and $1/\beta$ is the opportunity cost in terms of period-2 resources for providing one unit of resources of period-1 withdrawals. This leaves the remaining late-consumer local banks with

$$c_2(\theta) = \frac{c_{2, \text{bust}}(0) - \frac{\zeta \theta}{1 - \theta}}{1 - \theta} = \frac{1}{1 - \theta} \left( \frac{1 - \varphi r}{1 - \varphi} R_{\text{bust}} - \frac{r}{\beta} \right)$$

in period 2. Let $\theta^*$ solve $c_2(\theta) = r$,

$$\theta^* = \frac{\left( \frac{1}{r} - \varphi \right) \beta R_{\text{bust}} - \beta (1 - \varphi)}{1 - \beta (1 - \varphi)}$$

If $\theta^* < 0$, there is a fundamental bank run: all local banks will try to withdraw early, because even if no one else did, second-period consumption would be below the promised withdrawal at date 1, $c_2(0) < r$. Fundamental bank runs may actually be welfare-improving, as they partially complete missing, markets, see Allen and Gale (2007). If $0 < \theta^* < 1$, there is scope for a Diamond-Dybvig “sunspot” bank run. If late-consumer local banks believe that the fraction of early withdrawals by late-consumer local banks exceeds $\theta^*$, they will withdraw early too, so that $\theta = 1$ in equilibrium. If late-consumer local banks believe that the fraction of early withdrawals by late-consumer local banks is below $\theta^*$, they will choose to wait until period 2, and $\theta = 0$ in equilibrium.

There are therefore three scenarios, namely a fundamental bank run, a “sun spot” bank run and no bank run. I call these the “classic bank run” scenarios, for comparison with the more general case to be analyzed.
below. The classic bank run scenarios may already offer sufficient insight into systemic bank runs, as they will affect the core banks and thereby the entire banking system.

Moreover, as Allen and Gale (...) have shown, the possibility of a fundamental bank run generates incentives for a group of investors to save cash from period 0 to period 1, in order to buy the securities offered by the core banks in need of liquidity. If no other investors are present in period 1, this creates cash-in-the-market pricing and deep discounts for the long-term securities. Indeed, as the probability of the bust state and therefore a fundamental equilibrium becomes vanishingly small, the cash-in-the-market becomes vanishingly small too, resulting in prices for the long-term securities to converge to zero. Again, this analysis may be sufficient to understand the collapse of prices and firesale prices for long-term securities in a systemic banking crisis. Here, however, we wish to think through the possibility that outside investors are in principle interested in buying the long-term securities, but may have trouble evaluating their value.

4 The “bust” state with loss averse investors.

Suppose generally, that a core bank or a group of core banks needs to satisfy additional withdrawal requirements beyond the withdrawals r it expected in period 1 (and which it covered using short-term securities). The core banks will then need to liquidate its long term securities by selling them to the outside investors. Suppose that a given core bank or a group of core banks needs to obtain period-1 liquidity ℓ, while all core banks together raise liquidity L. I assume that the market delivers the following outcome (appendix A provides a more detailed description of the market game, that appears to give rise to this outcome).

Expert outside investors provide at most ω units of liquidity, discounting returns at rate β. The core banks sell their long-term securities of highest-quality to expert investors. If ω ≥ L, all liquidity is obtained in this way. If
$\omega < L$, then core banks raise only a pro-rata share of their liquidity needs from expert investors. The remainder

$$\tilde{\ell} = \ell \left(1 - \frac{\omega}{L}\right)$$

needs to be raised by selling long-term securities to non-expert investors. Since non-expert investors pay $\beta R$ per unit invested in the long-term security, regardless of their true quality, core banks sell the long-term securities of lowest quality to them.

Suppose that the core bank or group of core banks holds a share $\mu$ of the entire local bank market. For example, if all core banks have the same size, $\mu = 1/N$ for a single core bank. Therefore, a total of $\mu x$ units have been invested in long-term securities (before selling to expert investors). The group of core banks therefore needs to sell a fraction $\zeta = \zeta(\ell, L)$ of its portfolio or $z = \mu x \zeta$ units of its long-term securities to non-expert investors, where

$$\tilde{\ell} = \beta R \mu x \zeta$$

Note that there is no dependence on $L$, if $\omega = 0$ or if $\omega = \infty$, i.e. in the absence of expert investors, or if all investors are experts.

Suppose that a core bank or a group of core banks has (or is believed to have) a return distribution $G$ for its total long-term security portfolio (before selling part of it to expert investors). Let

$$g(\tau) = \sup\{R \mid G(R) < \tau\}, \; \tau \in [0, 1]$$

be the inverse function of $G$. Note that

$$E_G[R \mid R \leq g(\zeta)] = \frac{\int_{0}^{\zeta} g(\tau) d\tau}{\zeta}$$

is the expected return of all returns below the level given by $g(\zeta)$, under the distribution $G$. Therefore, if $\omega < L$ and taking into account the amount sold to expert investors, the payoffs of long-term securities in period 2 are reduced
by (or believed to be reduced by)

\[
\Delta = \ell \frac{\omega}{L} + \mu x \int_{0}^{\xi} g(\tau) d\tau
\]

\[
= \frac{\ell}{\beta} \left( \frac{\omega}{L} + \left(1 - \frac{\omega}{L}\right) E_G[R \mid R \leq g(\zeta)] \right)
\]

If \( \omega \geq L \), note that

\[
\Delta = \frac{\ell}{\beta}
\]

If a fraction \( \theta \) of late-consumer-location local banks decides to withdraw early, this group of core banks needs liquidity

\[
\ell = r \theta (1 - \varphi) \mu
\]

One may wish to view

\[
R(\theta, L; G) = \frac{\Delta}{\ell} = \frac{1}{\beta} \left( \min \left\{1; \frac{\omega}{L}\right\} + \left(1 - \min \left\{1; \frac{\omega}{L}\right\}\right) R^{-1} E_G[R \mid R \leq g(\zeta(\theta, L))] \right)
\]

where

\[
\zeta(\theta, L) = \frac{1 - \varphi}{1 - \varphi r} \frac{1 - \min \left\{1; \frac{\omega}{L}\right\}}{\beta R} \theta
\]

as the opportunity costs in terms of period-2 resources for providing one unit of resources of period-1 withdrawals. Note that \( R(\theta, L; G) \) does not depend on the market share \( \mu \). Likewise,

\[
q(\theta, L; G) = \frac{\ell}{\Delta} = \frac{1}{R(\theta, L; G)}
\]

is the effective liquidation discount rate of period-2 resources. Observe that

\[
R(\theta, L; G) = \frac{1}{\beta} \quad \text{and} \quad q(\theta, L; G) = \beta, \quad \text{if} \quad \omega > L
\]

More generally, the properties for \( q(\theta, L; G) \) follow immediately from the following properties about \( R(\theta, L; G) \).
Proposition 1

1. $R(\theta, L; G)$ is increasing in $\theta$.

2. $R(\theta, L; G)$ is increasing in $L$ and satisfies $\beta R(\theta, L; G) \geq 1$. There is no dependence on $L$, if $\omega = 0$ or if $\omega = \infty$, i.e. in the absence of expert investors, or if all investors are experts.

3. Suppose that $H$ first-order stochastically dominates $G$. Then

$$R(\theta, L; G) \leq R(\theta, L; H)$$

i.e. $R(\theta, L; G)$ is increasing in $G$, when ordering distributions by first-order stochastic dominance.

Proof:

1. Note that $\zeta(\theta, L)$ and therefore $E_G[R \mid R \leq g(\zeta(\theta, L))]$ is increasing in $\theta$.

2. Note that $\zeta(\theta, L)$ and therefore $E_G[R \mid R \leq g(\zeta(\theta, L))]$ is increasing in $L$. The claim now follows with the observation that $R^{-1}E_G[R \mid R \leq g(\zeta(\theta, L))] > 1$.

3. Define $h$ as the inverse of $H$ as in 14. Since $H(R) \leq G(R)$ for all $R$, $h(\tau) \geq g(\tau)$ for all $\tau \in [0, 1]$. Equation (15) shows that $E_G[R \mid R \leq g(\zeta)] \leq E_H[R \mid R \leq g(\zeta)]$ and the claim follows.

If a group of core banks is believed to have long-term securities with distribution $G$, the second-period payoff to local banks is perceived to be

$$c_2(\theta; G) = E_G[R] \frac{1 - \varphi r}{1 - \varphi}$$

if there are no withdrawals of late-consumer local banks in period 1, i.e. if $\theta = 0$. Note that $c_2$ is increasing in $G$, when ordering distributions with
first-order stochastic dominance. With withdrawals of a fraction $\theta$ of late-consumer local banks, the (perceived) remaining resources at period 2 per late consumer for this group of banks is therefore

$$c_2(\theta, L; G) = \frac{c_2(0; G) - r\theta R(\theta, L; G)}{1 - \theta}$$

(18)

which generalizes (12). Once again, $c_2(\theta, L; G)$ does not depend on the market share $\mu$. Local banks in some location $s$ with late consumers will surely opt for period-1 withdrawal, if $c_2(\theta, L; F(\cdot, s)) < r$, provided that a fraction $\theta$ of late-location locations of its core bank will opt for withdrawal as well and that total market liquidity needs are $L$. In case of equality, they are indifferent.

We can now proceed to describe the equilibria.

Definition 1 An equilibrium is collections $(S_n)_{n=1}^N$ of subsets of $[0, 1]$, withdrawal fractions $(\theta_n)_{n=1}^N$ and total additional liquidity $L$, so that

1. $\nu(s) = n$ for all $s \in S_n$, i.e. $S_n$ are locations of banks banking with core bank $n$.

2. $\theta_n = \lambda(S_n)$ is the Lebesgue measure of $S_n$.

3. For all $s \in S_n$: $c_2(\theta_n, L; F(\cdot, s)) \leq r$. For all $s \notin S_n$, $\nu(s) = n$: $c_2(\theta_n, L; F(\cdot, s)) \geq r$.

4. $L = L(\theta_1, \ldots, \theta_n)$ where

$$L(\theta_1, \ldots, \theta_n) = r(1 - \varphi) \sum_n \theta_n \lambda(\nu^{-1}(n))$$

(19)

where $\lambda(\nu^{-1}(n))$ is the Lebesgue measure of $\nu^{-1}(n)$.

What is perhaps remarkable is that the actual portfolio of the core banks is immaterial: only the perception of their portfolio matters.

It may be useful to note that $c_2(\theta, L; G)$ is not monotone in $G$, when ordering $G$ according to first-order stochastic dominance: while the first term
is increasing in $G$, the second term is now decreasing, due to the negative sign. Indeed, it is easy to construct examples for both a decreasing or an increasing behaviour, by keeping one of the terms nearly unchanged while the other moves significantly. Therefore, one may not assume a priori that $S_n$ takes the form of intervals, even if the core bank selection function $\nu$ is a step function.

However, $c_2(\theta, L; G)$ is monotonously decreasing in $L$ and furthermore, it is decreasing in $\theta$ under the mild condition (20), which generalizes (8) and which essentially assures, that no late-withdrawal local bank will be happy about other late consumer local banks withdrawing early.

**Proposition 2**

1. $c_2(\theta, L; G)$ is monotonously decreasing in $L$.

2. Assume that

$$c_2(0; G) < \frac{r}{\beta}$$

Then $c_2(\theta, L; G)$ is strictly decreasing in $\theta$.

**Proof:**

1. This follows directly from proposition 1.

2. Write $c_2(\theta, L; G)$ as

$$c_2(\theta, L; G) = c_2(0; G) - \frac{\theta}{1-\theta} \chi(\theta, L; G)$$

where

$$\chi(\theta, L; G) = rR(\theta, L; G) - c_2(0; G) > 0$$

and increasing in $\theta$ per (20) and proposition 1. Let $\theta_a < \theta_b$. Then

$$c_2(\theta_a, L; G) = c_2(0; G) - \frac{\theta_a}{1-\theta_a} \chi(\theta_a, L; G)$$

$$> c_2(0; G) - \frac{\theta_b}{1-\theta_b} \chi(\theta_a, L; G)$$

$$\geq c_2(0; G) - \frac{\theta_b}{1-\theta_b} \chi(\theta_b, L; G)$$

$$= c_2(\theta_b, L; G)$$

24
Assume that (20) is true for all conjectured distributions $G = F(\cdot, s)$. Therefore, if local banks opt for early withdrawals at some level of market liquidity or some fraction of other early withdrawals, they will do also for higher levels of $L$ and $\theta$. Hence, given $L$ and $G$, let $\theta = \theta^*(L; G)$ solve $c_2(\theta, L; G) = r$, i.e. be the smallest fraction of late consumers at a given bank (or pool of banks), so that a local bank will withdraw, if it judges the portfolio of its core bank to have distribution $G$ and if total liquidity demands are $L$. Let

$$S_n(\theta, L) = \{ s \mid \nu(s) = n, c_2(\theta, L; F(\cdot, s) < r) \}$$

(23)

be the set of local banks with deposits at core banks $n$, which will surely withdraw early, if a fraction $\theta$ of depositors at core bank $n$ do, and if there is total liquidity demand $L$.

Define the mapping

$$h : [0, 1]^N \rightarrow [0, 1]^N$$

per

$$(\tilde{\theta}_1, \ldots, \tilde{\theta}_n) = h(\theta_1, \ldots, \theta_n)$$

where

$$\tilde{\theta}_n = \lambda(S_n(\theta_1, L(\theta_1, \ldots, \theta_n)))$$

(24)

and where $\lambda(\cdot)$ denotes the Lebesgue measure. Intuitively, if everyone conjectures the fractions $(\theta_1, \ldots, \theta_n)$ of late consumer local banks to withdraw early, then the fractions $\tilde{\theta}_j$ surely will. Obviously $h$ is an increasing function. Let

$$\Theta = \{ \bar{\theta} = (\theta_1, \ldots, \theta_n) \mid \tilde{\bar{\theta}} \leq h(\bar{\theta}) \}$$

(25)

be the set of of conservative withdrawal conjecture vectors, i.e. actual withdrawals will at least be as high, as the conjecture at each core bank. Let

$$\Theta_{\text{lim}} = \{ \bar{\theta} \in [0, 1]^N \mid \text{There is a sequence } \bar{\theta}_j \in \Theta \text{ with } \bar{\theta}_j \leq \bar{\theta} \text{ and } \bar{\theta}_j \rightarrow \bar{\theta} \}$$

(26)
be a set of upper bounds for Θ and let

$$\Theta_{\text{max}} = \{ \vec{\theta} \mid \text{There is } \epsilon \in R_{++} \text{ so that } [\vec{\theta}, \vec{\theta} + \epsilon] \cap \Theta_{\text{lim}} = \{ \vec{\theta} \} \}$$  \hspace{1cm} (27)

be the set of all local maxima in Θlim. For notation, recall that $[\vec{\theta}, \vec{\theta} + \epsilon]$ is the set of all $\hat{\theta}$ with $\vec{\theta} \leq \hat{\theta} \leq \vec{\theta} + \epsilon$.

**Proposition 3** Assume that (20) is true for all conjectured distributions $G = F(\cdot, s)$.

1. If

$$\vec{\theta} = h(\vec{\theta})$$  \hspace{1cm} (28)

then $\vec{\theta}$ together with $L = L(\vec{\theta})$ and $S_n = S_n(\theta_n, L), n = 1, \ldots, N$ is an equilibrium$^9$.

2. If $\vec{\theta} \in \Theta$, then

$$[\vec{\theta}, h(\vec{\theta})] \subset \Theta$$  \hspace{1cm} (29)

3. $\emptyset \neq \Theta_{\text{max}} \subset \Theta_{\text{lim}} \subset \Theta$.

4. Any $\vec{\theta} \in \Theta_{\text{max}}$ satisfies (28) and therefore has an equilibrium associated with it.

5. Let $\vec{\theta}_0 = (0, 0, \ldots, 0)$. Consider the sequence

$$\vec{\theta}_j = h(\vec{\theta}_{j-1})$$  \hspace{1cm} (30)

Then any $\vec{\theta} \in \Theta_{\text{max}}$ satisfies $\vec{\theta} \geq \vec{\theta}_\infty$.

**Proof:**

1. Check the equilibrium definition.

$^9$Since I have focussed here on finding equilibria per strict preference for withdrawal in period 1, the converse may generally not be true.
2. Let \( \tilde{\theta} \in [\theta, h(\theta)] \). Then
\[
h(\tilde{\theta}) \geq h(\theta) \geq \theta
\]

3. Per (20) or better (21),
\[
h(1, 1, \ldots, 1) = (1, 1, \ldots, 1)
\]
for if everyone else withdraws early, so should you. Thus \((1, 1, \ldots, 1) \in \Theta_{\text{max}}\), which is therefore not empty. The first inclusion is trivial. For the second inclusion, let \( \theta \in \Theta \) and let \( \theta_j \to \theta \) with \( \theta_j \in \Theta \) and \( \theta_j \leq \theta \). Note that
\[
h(\theta) \geq h(\theta_j)
\]
for all \( j \). Therefore,
\[
h(\theta) \geq \theta
\]
and hence \( \theta \in \Theta \).

4. Assume additionally that \( \theta \in \Theta_{\text{max}} \). I need to show that
\[
h(\theta) = \theta
\]
But if instead \( (h(\theta))_n > (\theta)_n \) for some entry \( n \), say, then this together with \([\theta, h(\theta)] \subset \Theta\) would be a contradiction to local maximality.

5. Note that \( \theta_j \) is an increasing sequence. Let \( \theta \in \Theta_{\text{max}} \). Since trivially \( \theta \geq (0, 0, \ldots, 0) \), the conclusion follows per repeated application of \( h(\cdot) \).

If \((0, \ldots, 0) \neq \theta_{\infty} \in \Theta_{\text{max}}\), then\(^{10}\) we call \( \theta_{\infty} \) the fundamental bank run. If \( (h(0, \ldots, 0))_n = 0 \) for \( n \in N \subset \{1, \ldots, N\} \), then the same is true for

\(^{10}\)There probably is a sensible generalization to some minimal element of \( \Theta_{\text{max}} \).
The fundamental bank run therefore only affects the core banks, which experience withdrawals “at the start” of the run, i.e. experience withdrawals of late consumer local banks, even if all local banks assume that nobody else withdraws. Nonetheless, withdrawals at one core bank can spill over to withdrawals at other core banks within this set, due to the dependence of \( R(\theta, L; G) \) on aggregate liquidity, provided that \( \omega \) is nonzero and sufficiently small.

4.1 The “bust” state with risk-neutral investors and no adverse selection.

Suppose instead (and as a benchmark for comparison), that investors are risk-neutral and that there is no adverse selection in selling the long-term securities. This may be a sensible assumption if all long-term securities return the same amount \( R_{\text{bust}} \), despite the heterogenous beliefs of the local banks to the contrary. Or this may be sensible, if one were to assume that core banks can only sell well-defined (or well-audited) portfolios of long-term securities, whose risk-characteristics are known to the market. Finally, this may be sensible if one is to assume that \( \omega = \infty \) in the analysis above. In all these cases, the outside investors discount future payments at rate \( \beta \).

The analysis of the “bust” state is now a corollary to the analysis above by setting \( \omega = \infty \) and using \( R(\theta, L; G) = 1/\beta \) throughout. The details can be skipped, except perhaps for some useful formulas. With (18), second-period consumption will assumed to be

\[
\hat{c}_2(\theta, L; G) = \frac{c_2(0; G) - \frac{r}{\beta} \theta}{1 - \theta}
\]

which is monotone in \( G \), when ordering distributions according to first-order stochastic dominance, and which does not depend on \( L \) (and where I use the \( \hat{\cdot} \) to distinguish it from the scenario above). As in (13), a late consumer local
bank will withdraw early, if \( \theta \geq \hat{\theta}^*(G) \), where

\[
\hat{\theta}^*(G) = \frac{\left(\frac{1}{r} - \varphi\right) \beta E_G[R] - \beta(1 - \varphi)}{1 - \beta(1 - \varphi)}
\]  

(33)

This scenario will serve as a benchmark. While there can also be a fundamental bank run in this case, there is no spillover to other core banks. A fundamental bank run in this scenario and the scenario with loss-averse investors start the same and affect the same core banks. However, a fundamental bank run with loss-averse investors can run considerably deeper.

4.2 A numerical example

To provide a specific, illustrative example, suppose that \( \sigma = 1/2 \), \( R_{\text{boom}} = 1.44 \) and \( \varphi = 1/7 \). Equation (10) then implies

\[ c_1 = r = \frac{7}{6} = 1.1666, \quad c_2 = \frac{7}{5} \]

Assume that \( \beta = 2/3 \), therefore satisfying (5). Assume that 10\% of the returns are uniformly distributed on \([0.6, 1.4]\), whereas 90\% are equal to 1.4 in the bust state: this is the aggregate distribution \( F \), see figure 1. Therefore, \( R_{\text{bust}} = 1.36 \). Note that (11) is satisfied, and that therefore there is no fundamental bank run with complete information in the bust state or if the beliefs \( F(\cdot, s) \) of all local banks coincide with the asset distribution.

Assume that for a fraction \((1 - \mu)\) of core banks, local banks assume the correct aggregate distribution, and will therefore not run in a fundamental bank run equilibrium. However, for the remaining fraction \( \mu \) of the core banks, the local banks believe with certainty that the return is some return \( R \), where \( R \) is randomly drawn from \( F \). I.e., if the local banks of these core banks are enumerated \( \tau \in [0; 1] \), then \( R(\tau) = 0.6 + 8\tau \) for \( 0 \leq \tau \leq 0.1 \) and \( R(\tau) = 1.4 \) for \( \tau \geq 0.1 \). As a result, the local banks are correct in aggregate, but wrong individually, see figure 2.

Absent a bank run, each late consumer local bank expects a pay out of

\[ c_2(0; F(\cdot; \tau)) = R(\tau) \frac{35}{36} \]
Even for the most optimistic bank, I have
\[ c_2(0; F(\cdot; 1)) = 1.4 \frac{35}{36} < \frac{r}{\beta} = \frac{7}{4} = 1.75 \]

Therefore, the condition (20) is satisfied for all \( G = F(\cdot; s) \).

Suppose first, that there are only risk neutral investors (or only expert investors), as in subsection 4.1. In that case, (33) can be used to calculate the fundamental bank run, if it exists, by calculating the smallest \( \hat{\tau} \) so that
\[ \hat{\theta}^*(F(\cdot; \hat{\tau})) = \hat{\tau} \]

The solution is approximately \( \hat{\tau} = 0.0645 \), i.e. 6.5 percent of late consumer local banks will decide to run, see figure 3.

In the scenario with loss averse investors, note that
\[ L = L(\theta) = r\theta(1 - \varphi)\mu \]
so that
\[ \omega \frac{L}{L} = \frac{\alpha}{\theta} \]
where
\[ \alpha = \frac{\omega}{r(1 - \varphi)\mu} \]
is inverse proportional to \( \mu \). Put differently, the given expertise of outside investors will be diluted, the more core banks are affected by withdrawals, “accelerating” the bank run compared to the experts-only scenario. It is in this sense, that the bank run is systemic.

The fraction of assets that need to be sold to uninformed investors depends on \( \alpha \) and is shown in figure 4, when \( \alpha \in [0, 0.03, 0.06, 1] \). While \( \alpha = 0 \) and \( \alpha = 1 \) are the two possible extreme, the comparison of \( \alpha = 0.06 \) to \( \alpha = 0.03 \) is perhaps most interesting. This reduction in \( \alpha \) would arise, for example, if twice as many core banks are affected by local-bank doubts regarding the qualities of their portfolio, i.e. if \( \mu \) doubles, without any other changes in the parameterization.
“Guessing” that local banks with a low $\tau$ run first, calculate $R(\tau, L(\tau); F(\cdot, \tau))$ as well as $c_2(\tau, L(\tau); F(\cdot, \tau))$. The fundamental bank run then involves the lowest $\theta = \tau$ so that
\[ c_2(\tau, L(\tau); F(\cdot, \tau)) = r \]
or, absent that and depending on the boundary conditions, either $\theta = \tau = 0$ or $\theta = \tau = 1$.

The resulting foregone returns $R(\tau, L(\tau); F(\cdot, \tau))$ are shown in figure 5, whereas the bank run results can be glanced from figure 6, when $\alpha \in [0, 0.03, 0.06, 1]$. That figure essentially shows, that $\alpha = 0.05$ results in a fundamental bank run that is not much different from the bank run in the case of expert-investors only, while for $\alpha = 0.03$, all local banks run. Therefore, a slight increase in the core banks that are subject to withdrawals, and therefore a moderate dilution of the expertise of outside investors can result in a complete collapse of the affected core banks.

5 The “bust” state with moral hazard.

Consider now the variation of the model with adverse selection. More precisely, assume the outside investors to be risk-neutral, discounting the future at rate $\beta$. I assume that all outside investors are non-experts\textsuperscript{11}, and can therefore not distinguish between long-term securities offered to them, while core banks selling them know the returns exactly, and can choose which security to sell. I assume that outside investors know the return distribution $F$. All the long-term securities are therefore sold at the same market price $p$. This creates moral hazard: not only will core banks sell the securities with their worst quality first (and this happens in the analysis above as well, when selling to non-expert investors), but furthermore, some core banks without liquidity needs due to withdrawals may sell long-term securities of low quality, if the price is right. The latter is a key difference between the moral

\textsuperscript{11}It would not be hard but a bit tedious, to generalize this and to include expert investors as well.
hazard variant and the loss-aversion variant presented here: with loss-averse investors and sufficiently high discounting, there never is a reason for “opportunistic” selling by liquid core banks\textsuperscript{12}.

To keep the analysis a bit more tractable, assume that the true portfolio \( F \) is atomless.

Suppose that a core bank or a group of core banks with market share \( \mu \) faces early withdrawals of the same\textsuperscript{13} fraction \( \theta \) of their late-consumer serving local banks. Assume that the other core banks sell long-term securities for purely opportunistic reasons, in case their price exceeds the expected return. I need to determine the market clearing price \( p \) as well as the incentives of local banks to consider withdrawal.

Assume that the distressed core banks have (or are believed to have) a return distribution \( G \) for its long-term securities. They need to sell a share \( \zeta \) of their portfolio or \( z = \mu x \zeta \) of their long-term securities, where

\[
\theta (1 - \varphi) \mu = p \mu x \zeta
\]  

(34)

On average, these securities pay \( E_G[R \mid R \leq g(\zeta)] \) per unit, see equation (15).

Given a market price \( p \) for long-term securities, core banks without early withdrawals will sell all\textsuperscript{14} securities with \( R \leq p \). Assuming their return distribution to be \( H \), they will therefore sell the fraction \( H(p) \). On average, these securities pay \( E_H[R \mid R \leq p] \) per unit.

\textsuperscript{12}Clearly, the distinction here has been sharply drawn, for analytic purposes. It may well be that some mixture of the two variants is a better description than one of these two extreme variants.

\textsuperscript{13}It is straightforward, but tedious to extend this to the case, where \( \theta \) differs from core bank to core bank.

\textsuperscript{14}For equality, there is indifference, and therefore core banks may only sell a fraction of the securities for which there is a equality. The issue does not arise, if \( F(\cdot) \) is atomless, as I have assumed in this section. In the more general case, it will be easy to patch that up at the final step, when calculating market clearing. For reasons of tractability, I shall not pursue this issue further.
The outside investors are risk neutral, discounting the future at $\beta$, but understand this moral hazard problem. They will also (correctly) assume that $G = H = F$. Therefore, the market clearing price $p = p(\theta, \mu)$ and the fraction of the portfolio $\zeta = \zeta(\theta, \mu)$ sold by the distressed core banks solve the two equations

$$p = \beta \mu \frac{\int_{R}^{f(c)} R dF}{\mu \zeta + (1 - \mu) F(p)}$$  \hspace{1cm} (35)

$$\zeta = \frac{r \theta}{p} \left( \frac{1 - \varphi}{1 - r \varphi} \right)$$  \hspace{1cm} (36)

where (per notational convention or per calculation of the integral)

$$0 = \int_{R}^{p(\theta, \mu)} R dF, \text{ if } p(\theta, \mu) < R$$

Note that the right hand side of (35) is simply the average return of the securities sold, discounted at $\beta$. Define

$$\bar{\theta} = \left( \frac{1 - r \varphi}{1 - \varphi} \right) \frac{\beta R}{r} \hspace{1cm} (37)$$

as the maximal $\theta$ compatible with $\zeta \leq 1$, if $p = \beta R$. Note that $\bar{\theta} < 1$.

**Proposition 4**

1. For every $\theta \in [0, \bar{\theta}]$ and $\mu \in (0, 1]$, there is a unique solution $(p, \zeta)$ to (35, 36) with $\beta R \leq p \leq \beta R_{bust}$ and $0 \leq \zeta \leq 1$, so that $p < f(\zeta)$.

2. Given $\theta$, $p(\theta, \mu)$ is an increasing function in $\mu \in (0, 1]$.

**Proof:**

\(^{15}\) Note that the local banks considering withdrawals should be able to learn from the market price, that their beliefs $G$ for their core bank and the market price together are inconsistent with the aggregate return distribution $F$, and should therefore somehow update their belief, learning from the information revealed in market prices. This may be a tough thing to do in practice, and I shall ignore this issue for the purpose of the analysis here.

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1. Recall that the support of $F$ is $[\bar{R}, \bar{R}]$. Define the function $\rho(p)$ per the right hand side of (35), with $\zeta$ replaced with (36). Note that $\rho(p)$ is continuous on $p \in [\beta R, \bar{R}]$ with

$$\rho(\beta R) \geq \beta R, \rho(\bar{R}) \leq \beta E_F[R] = \beta B \leq \bar{R}$$

By the mean value theorem, there is therefore a value $p$ with $p = \rho(p)$. Suppose that $f(\zeta) \leq p$ at this value. Then the right hand side of (35) is not larger than $\beta p$, a contradiction. To show uniqueness, suppose to the contrary that there are two solutions, say $p_a < p_b$, together with $\zeta_a > \zeta_b$. Note generally that

$$\int_{R}^{p_b} RdF - \int_{R}^{p_a} RdF \leq p_b F(p_b) - F(p_a) \leq p_b F(p_b) - p_a F(p_a)$$

Recall that $p_a < p_b \leq B \leq 1$, see (5) and (4). Therefore,

$$\beta \int_{R}^{f(\zeta_a)} RdF > p_j \beta \int_{R}^{f(\zeta_a)} RdF \geq p_j \zeta_j = r\theta \frac{1 - \varphi}{1 - r \varphi}$$

for $j = a, b$, where the second inequality follows from $p_j = \rho(p_j)$ and $\int_{R}^{p} RdF \leq p F(p)$. Define the function

$$\psi(p, \zeta; \theta, \mu) = \beta \mu \int_{R}^{f(\zeta)} RdF + \beta (1 - \mu) \int_{R}^{p} RdF \frac{1 - \varphi}{1 - r \varphi} + (1 - \mu) F(p)p$$

(38)

Note that $p_j = \rho(p_j)$ can be rewritten as

$$1 = \psi(p_j, \zeta_j; \theta, \mu)$$

(39)

for $j = a, b$. Recall generally, that $(y + x)/(z + x)$ is a strictly decreasing function in $x$, if $y > z > 0$. Therefore,

$$1 = \psi(p_a, \zeta_a; \theta, \mu) = \frac{\beta \mu \int_{R}^{f(\zeta_a)} RdF + \beta (1 - \mu) \int_{R}^{p_a} RdF}{\mu r \theta \frac{1 - \varphi}{1 - r \varphi} + (1 - \mu) F(p_a)p_a}$$
2. Given $\theta, \mu$, the unique equilibrium is denoted by $p(\theta, \mu)$ and $\zeta(\theta, \mu)$. Let $\zeta(p)$ denote the expression on the right hand side of (36). The previous calculation shows more generally that

$$\psi(p, \zeta(p); \theta, \mu) > 1 \quad \text{for} \quad p < p(\theta, \mu) \quad (40)$$

$$\psi(p, \zeta(p); \theta, \mu) < 1 \quad \text{for} \quad p > p(\theta, \mu) \quad (41)$$

Note furthermore, that $\psi(p, \zeta; \theta, \mu)$ is increasing in $\mu$ and $\zeta$. Consider thus $\mu_a < \mu_b$. Since

$$1 = \psi(p(\theta, \mu_a), \zeta(p(\theta, \mu_a); \theta, \mu_a))$$

$$< \psi(p(\theta, \mu_b), \zeta(p(\theta, \mu_b)); \theta, \mu_b)$$

it follows from (40) that $p(\theta, \mu_b) \geq p(\theta, \mu_a)$, as claimed.

At the distressed core banks, local banks with beliefs $G$ regarding their portfolio will therefore believe the opportunity costs for providing period-1 resources in terms of period-2 resources to be

$$R(\theta, \mu; G) = \frac{E_G[R \mid R \leq g(\zeta(\theta, \mu))]}{p(\theta, \mu)} \quad (42)$$
It is instructive to compare this to (16) for the case $\omega = 0$: the two expressions coincide iff $p(\theta, \mu) = \beta R$. Generally, the returns are quite different. In fact,

$$R(\theta, 1; G) = \frac{E_G[R \mid R \leq g(\zeta(\theta, \mu))]}{\beta E_F[R \mid R \leq g(\zeta)]}$$

(43)
as can be seen by direct calculation. In particular, for $G = F$, I obtain

$$R(\theta, 1; F) = \frac{1}{\beta}$$

(44)

More generally,

**Proposition 5** $R(\theta, \mu; G)$ is decreasing in $\mu$.

**Proof:** This is a direct consequence of (42) together with the fact that $p(\theta, \mu)$ is increasing in $\mu$, implying that $\zeta(\theta, \mu)$ and thus $E_G[R \mid R \leq g(\zeta)]$ are decreasing in $\mu$. •

I obtain the key insight that an increasing market share of distressed banks lessens rather than deepens the crisis. Furthermore, with homogeneous beliefs, $F(\cdot, s) \equiv F$, and with the market share of distressed banks approaching unity, the moral-hazard scenario turns into the standard bank run scenario considered in section 3.3.

The remaining late-consumer local banks will obtain

$$c_2(\theta, \mu; G) = \frac{1}{1 - \theta} \left( \frac{x}{1 - \varphi} E_G[R] - r \theta \frac{E_G[R \mid R \leq g(\zeta(\theta, \mu))] p(\theta, \mu)}{p(\theta, \mu)} \right)$$

(45)

Therefore, a late-consumer-serving local bank in location $s$, banking with a distressed core bank and believing that a fraction $\theta$ of local late-consumer banks will withdraw in period 1 will choose to do so itself, if

$$c_2(\theta, \mu; G) \leq r$$

(46)

The analysis of the resulting equilibrium appears to be similar to the analysis in section 4 and shall be omitted in the interest of space.
5.1 A numerical example

I use the same parameterization as in subsection 4.2. For low values of $\theta \leq \theta$, the market price will be below $R = 0.6$ and the required market discount $R(\theta, \mu; F)$ at the true distribution will equal $1/\beta$. For these low values of $\theta$ and due to the uniform distribution, the market price equals

$$p(\theta; \mu) = \frac{\beta f(\zeta) + 0.6}{2}$$

(47)

Therefore, $\theta$ is low enough, iff $p(\theta; \mu) \leq 0.6$ or, equivalently, $f(\zeta) \leq 1.2$. By the parameterization in (4.2),

$$f(\zeta) = \min\{0.6 + 8\zeta, 1.4\}$$

Therefore, $f(\zeta) \leq 1.2$ corresponds to $\zeta \leq 0.075$. To find $p$ and $\zeta$ when $f(\zeta) \leq 1.2$, I therefore need to solve

$$\zeta = \frac{r\theta}{\beta(0.6 + 4\zeta)} \left(\frac{r - \varphi}{1 - r\varphi}\right)$$

(48)

Let

$$\kappa = \frac{r}{4\beta} \left(\frac{r - \varphi}{1 - r\varphi}\right) \approx 0.538$$

The solutions to (48) are therefore given by

$$\zeta = -0.075 + \sqrt{0.075^2 + \kappa\theta}$$

(where the negative root has been excluded as not sensible). Therefore, $\zeta \leq 0.075$, if

$$\theta \leq \theta = 3 \times 0.075^2 / \kappa \approx 0.0314$$

For $\theta > \theta$, the behavior of the price depends on market share of the distressed core banks. Two extreme scenarios can provide some general insights. If $\mu \to zero$, then the price will remain “stuck” at $p = R = 0.6$, as all remaining banks would sell arbitrarily large chunks of their worst assets otherwise. If $\mu = 1$, then discounting of future returns will remain to be done at the discount rate $\beta$. 
For these as well as the in-between range of values of $\mu$, equation (46) can then be used to determine the threshold value for $\tau$, up to which local banks will decide to withdraw. Proposition (5) generally shows, that a bank run is the less likely, the larger the market share of distressed core banks.

6 Some policy implications

Given the length of this paper, a full discussion of the policy implications is beyond its scope.

A key difference between the model with loss averse investors and the moral hazard are the implications, as the “suspicion” of bad portfolios affect not only a fraction of the core bank but all banks. In the case of loss averse investors, a given core bank now has even less access to expert investors, worsening the situation. In the case of moral hazard, and since all core banks need to obtain equal amounts of liquidity, they will all receive “fair value” for their assets, i.e., the situation essentially turns into a classic bank run. Therefore, the moral hazard scenario violates item six of the stylized description list in the introduction, while the scenario with loss-averse investors does not. For these reasons, I argue that it is more plausible to look at the 2008 financial crisis through the lense of the loss averse investor scenario rather than the moral hazard scenario.

Consider, for example, a government guarantee of payoffs of the securities sold by the core banks, e.g. guaranteeing a return of at least $R_{\text{gov}}$. In that case, the loss averse investors will pay $\beta R_{\text{gov}}$ instead of $\beta R$. In particular, if $R_{\text{gov}} = 1$, i.e. if the government guarantees that investments will not make losses, the “deep” bank run results in discounting with $\beta$ throughout and turns the scenario with loss averse investors into the “classic” bankrun situation of subsection 4.1 for the distressed core banks. The government will loose money on all securities with returns $R < R_{\text{gov}}$. Additionally, if $\beta R_{\text{gov}} > R$, the government now creates an additional moral hazard problem at the core banks which are not distressed, and which now find it at their
advantage to sell all assets with $R \leq R \leq \beta R_{gov}$.

From the tax payers perspective, a more advantageous procedure in the case of loss averse investors seems to be the purchase of the troubled assets outright: if large parts of the portfolios of banks are bought, the tax payer will receive the average payoff and not the bottom payoff as feared by the outside investors. Consider a fixed government purchase price $p$ at which the government stands ready to purchase assets from the core banks. The incentives of the participating core banks then become similar to the analysis in the moral hazard framework: while the distressed core banks will sell for sure, the core banks without distress will only do so if it is in their interest to sell the worst assets. Rather than imposing the equilibrium condition (35), one can then calculate the losses or gains to the tax payers at the mandated government purchase price. If that price is below the moral hazard scenario equilibrium price, the government will earn a return above $1/\beta$, and this scenario is possible and plausible, if investors are loss averse. If the situation is as described in the moral hazard scenario, then the government would only find takers for its offers, if the government price is above the current market clearing price, in which case the government will make losses compared to the benchmark return of $1/\beta$.

Since I have argued that the loss averse scenario is more plausible than the moral hazard scenario, this paper provides an argument that an outright purchase of troubled assets by the government at prices above current market prices can both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

A number of private sector solutions may likewise provide reasonable avenues for resolving the crisis situation, e.g. the complete purchase of portfolios of a distressed core bank or the sale of a distressed core bank and a guarantee of its deposits through the buyer. It may be, however, that the same caution that drives loss-averse investors to demand steep discounts on asset backed securities might also prevent the sale of distressed financial institutions to the same investors at a price that can resolve the situation
sufficiently well. Solutions that mix private sector involvement with government intervention - an idea at the core of the Geithner plan - may likewise offer specific advantages or fallacies, that can be analyzed in this context.

Follow-up work, providing a deeper analysis of the various options and policy scenarios, is surely called for.

7 Conclusions

I have set out to provide a model of a systemic bank run delivering the following features

1. The withdrawal of funds was done by financial institutions at other financial institutions, rather than depositors at their bank.

2. The troubled financial institutions held their portfolio in asset-backed securities rather than being invested directly in long-term projects.

3. These securities are traded on markets. In the crisis, the prices for these securities appears low compared to some benchmark fundamental value benchmark (“underpricing”).

4. There is a large pool of investors willing to purchase securities, as evidenced e.g. by market purchases of newly issued US government bonds or the volume on stock markets.

5. Nonetheless, these investors are only willing to buy these asset-backed securities at additional discounts compared to some benchmark fundamental value calculation.

6. The larger the market share of troubled financial institutions, the steeper the required discounts.

To that end, I have hypothesized two different motives for outside investors and their interaction with banks trading asset-backed securities: loss-aversion...
versus moral hazard. Both variants of the model are capable of delivering on the first five points of the list above. While the variant with loss-averse investors also delivers on the sixth point, this is not the case for the moral hazard scenario. Indeed there, as a larger share of financial institutions are distressed, the discounts lessen rather than rise.

I conclude from that that the variant with loss averse investors rather than the moral hazard scenario is more suitable to analyze policy implications. Therefore, this paper provides an argument that an outright purchase of troubled assets by the government at prices above current market prices can both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

A number of private sector solutions may likewise provide reasonable avenues for resolving the crisis situation. Solutions that mix private sector involvement with government intervention - an idea at the core of the Geithner plan - may likewise offer specific advantages or fallacies, that can be analyzed in this context. Follow-up work, providing a deeper analysis of the various options and policy scenarios, is surely called for.
Appendix

A The market for long-term securities

I assume that selling of the long term securities proceeds as follows. All core banks simultaneously decide what portion of their long-term securities they wish to sell. Outside investors simultaneously decide whether they wish to participate in the market for these securities. A market maker sells these long-term securities sequentially in second-price sealed bid auctions to the participating outside investors, who keep participating until they either won the auction or there are no more securities to buy. The market maker can “break up” the securities into pieces with identical returns, i.e. can decide to auction fractional shares of a given security. The market maker is provided a selling sequence by the core banks, i.e. is given instructions in which sequence its securities are supposed to be sold. The market maker then obtains an overall sequence of the securities by “stretching” the individual core sequences in inverse proportion to the market share of each core bank.

To analyze this market, consider a finite approximation, i.e. where the long-term securities are backed by finitely many long term projects, and where finitely many outside investors are bidding. Note that there is a “unit problem”, because investors can only bid at most one unit of resources. This can be finessed by the market maker predicting the prevailing price per unit in any given auction and selling exactly that fraction of the security, which could be purchased at that price for a one unit of the resource. Suppose, for example, that two core banks are participating, with the first selling securities worth eight units and the second selling securities worth four units. In each round of selling for three units, the market maker sells two units of the first core bank and one unit of the second core bank. To obtain the ordering within each round, the market maker draws one random permutation for the first round, and sticks to that in all future rounds.

Investors know whether expert investors are bidding or not. I assume that
investors myopically bid their valuation. If at least two expert investors are present, the long-term security will be sold at its specific return, discounted with $\beta$. If no or just one expert investors are present, the long-term security will be sold at $\beta R$, due to the loss aversion of the non-expert investors. If there is a finite number of securities, expert investors have a weak incentive to participate, since they have a nonzero probability of being the single expert bidder in one auction, provided the auction receipts for the long-term securities exceeds $\omega$, the combined resources of the expert investors. If this condition is not satisfied, one can still make the expert strictly prefer participation by assuming that experts are slightly heterogeneous, with each expert being slightly more certain about the return of a particular security.

The myopic bidding strategy may not be an appealing assumption, so it is useful to carefully examine the bidding strategies. If there was a single second-price sealed bid auction with at least two bidders, then it is well-known that it is rational to bid ones valuation. For the sequence, it is clear that the non-expert investors will rationally always bid $\beta R$ anyhow, provided there are at least two bidders at each auction. But the expert bidders should recognize the “option value” of having the other expert bidders walk away with having won some auction early, in order to be the single expert bidder at some auction, thereby acquiring the security at $\beta R$. This “option value” should therefore lower the bids in the earlier rounds. The probability of being the single expert investor and walking away with the big price instead gets replaced by an equal “share” to be received by the expert bidders at each auction, making them indifferent as to which expert auction to win. Though I have not shown it formally, I conjecture that the total resources paid by the expert investors end up the same as in the myopic case.

Core banks are indifferent which security to sell to expert investors but strictly prefer to sell the securities of lowest quality to non-expert investors, and will therefore not sell them to expert investors. Without loss of generality, assume therefore that core banks sell the highest-quality assets to expert investors.
If one takes the limit as the number of local projects goes to infinity and become atomless with respect to the aggregate, i.e. with locations \([s, s + δ]\) using δ aggregate units in their long-term projects and with investors \([i, i + ε]\) providing ε units of liquidity, this market appears to deliver the outcome described in the text.

B An overly simple version

B.1 Description

Here is a simple version of the model. There are two periods. Let there be a continuum of banks, \(j \in [0; 1]\), which initially each receive one unit of deposits and hold some extra \(\mu - 1 > 0\) units of capital. Therefore, they have a total of \(\mu\) resources to invest. Banks can invest these resources either in government bonds or in some other asset (think: mortgage-backed security): assume that they invest \(0 \leq ψ_j \leq \mu\) in the latter. Assume that banks differ in their risk management by assuming that \(ψ_j \sim F\). For the calculations below, assume that \(F\) is the uniform distribution on \([0; 1]\). While the government bond has a liquid market in period 1, always trading at price 1 in terms of some consumption good, the other asset may not and will trade at price \(p\) in period 1. Assume that the government bond pays one unit in period 2, but that the other asset will repay the price \(p\) in period 2, at which it is trading in period 1. This is a simple stand-in for the analysis in the main model of this paper. Alternatively, it may be possible to provide a sound underpinning by an extension to multiple periods and valuing the asset by “repetition” of the equilibrium\(^\text{16}\).

Assume that the demand for this security outside the banking system is given by \(D(p)\) and of finite elasticity: for the calculations, assume that \(D(p) = 1 - p\). Note that the total initial demand for the other asset at price \(p = 1\) is given by \(E[ψ] = 0.5\), which we assume to be a constant, aggregate

\(^{16}\)In fact, given this simple assumption, there is no need for a period 2 here!
supply.

Depositors do not know which “type” $\psi$ their bank is, and information is dispersed. For the simple version here and as a rather extreme scenario, I shall assume that depositors are deeply convinced they know the type of their bank, i.e. at each bank $j$, there is a continuum of depositors $i \in [0, 1]$, believing bank $j$ to be of type $\psi_i \sim F$, and each having deposited the same amount. Thus, the aggregate distribution of depositor beliefs is the correct aggregate distribution of bank types, but there is an obvious (and extreme) mismatch at the individual bank level. Depositors can decide to withdraw their deposits in period 1. I assume that depositors do, if they believe that the deposit liabilities exceed the bank asset values. If they do, banks will need to meet their demands, until they run out of funds, in which case they declare bankruptcy.

Note that a bank is indifferent between selling the government bond or the other asset: by construction, their return between period 1 and 2 is unity. Consider, therefore, two extreme equilibria. In the first, the bank always first sells its other assets in order to meet liquidity demands before selling the government bond. In the second, the bank always first sells its bonds in order to meet liquidity demands. We shall analyze both. Of particular interest is the question, which one of these two equilibria may be better suited to avoid a systemic bank run.

B.2 Analysis

I will condition the analysis on assuming some price $p$ for the other asset in period 1. A depositor believing his bank to be of type $\psi$ will withdraw funds, iff

$$p\psi + \mu - \psi \leq 1$$

i.e., if

$$\psi \geq \psi_e = \frac{\mu - 1}{1 - p}$$
The withdrawal at each bank is therefore given by

\[ x(p) = 1 - F(\psi_c) = \max\{0; \frac{2 - \mu - p}{1 - p}\} \]

due to my uniformity assumption about \( F \). Figure 7 shows a plot of the withdrawals. As a result, a bank with insufficient assets will need to declare bankruptcy. This is the case if \( \psi \geq \psi^*(p) \), where

\[ p\psi + \mu - \psi \leq x(p) \]

or

\[ \psi \geq \psi^*(p) = \frac{\mu - x(p)}{1 - p} \]

We now need to analyze the two equilibria for meeting these withdrawals.

1. If banks sell the other asset first, then they sell all of it, if

\[ \psi \leq \psi_A(p) = \frac{x(p)}{p} \]

For larger values of \( \psi \), banks will sell enough so as to meet the liquidity demands. Thus, the total remaining demand by banks for the asset is

\[
D_{A,\text{banks}}(p) = 0.5 - \int_0^{\psi_A(p)} \psi F(d\psi) - \int_{\psi_A}^{1} \frac{x(p)}{p} F(d\psi)
\]

\[
= 0.5 - \frac{\psi_A(p)^2}{2} - (1 - \psi_A(p)) \frac{x(p)}{p}
\]

Thus, aggregate demand is

\[ D_A(p) = D(p) + D_{A,\text{banks}}(p) \]

(which can be simplified further).

2. If banks sell the bond first, then they sell only bonds for

\[ \psi \leq \psi_B(p) = \mu - x(p) \]
and declare bankruptcy for $\psi \geq \psi^*(p)$, selling all of the other asset then. For values in between, $\psi_B(p) \leq \psi \leq \psi^*(p)$, banks sell $(\psi - \psi_B(p))/p$ units of the other asset and all bonds in order to meet the withdrawal amount $x(p)$, as one can see from

$$\frac{\psi - \psi_B(p)}{p} + \mu - \psi = x(p)$$

Therefore, total remaining bank demand for the other asset is

$$D_{B,\text{banks}}(p) = 0.5 - \int_{\psi_B(p)}^{\psi^*(p)} \frac{\psi - \psi_B(p)}{p} F(d\psi) - \int_{\psi^*(p)}^1 \psi F(d\psi)$$

and aggregate demand is

$$D_B(p) = D(p) + D_{B,\text{banks}}(p)$$

### B.3 Results

Figure 8 shows the thresholds for declaring bankruptcy $\psi \geq \psi^*(p)$, for selling all of the other asset in equilibrium A, $\psi \leq \psi_A(p)$ and for selling some of the other asset in equilibrium B, $\psi_B(p) \leq \psi \leq \psi^*(p)$.

Finally, figure 9 shows aggregate demand and - per intersection of aggregate demand with fixed aggregate supply - the equilibria. For the numerical example here and as is evident from that figure, there are multiple equilibria in the case of equilibrium A, because of the strong upward sloping nature of bank demand for the asset. There is the “no bank run” equilibrium at $p = 1$, but there are also two equilibria at levels $p$ below unity. These additional equilibria are systemic bank runs. Essentially, because the asset has a liquidation cost in equilibrium, the conjecture of depositors that the asset has low value causes banks to sell so much of that asset that it will indeed have low value, thereby justifying depositors to line up in period 1.

In contrast, there is only the no-bank-run equilibrium in equilibrium B, when banks sell the bond first. This reduces the downward pressure on the price of the other asset in case of liquidity withdrawals by depositors.
While the demand curve of the banking system for this asset is still upward sloping, it is now not upward sloping enough to cause the existence of bank run equilibria.

Therefore, these preliminary results suggest that it matters substantially, how banks react to liquidity withdrawals by their depositors, even if they individually are indifferent. A pecuniary externality can cause a systemic bank run in this model.

References


[21] Schleifer,


Figure 1: Return distribution in the bust state.

Figure 2: Beliefs.
Figure 3: Bankrun calculus when only expert investors are present.

Figure 4: Fraction of assets that need to be sold to uninformed investors, as a function of late-customer withdrawals.
Figure 5: *Foregone returns due to early withdrawals.*

Figure 6: *Consumption of local banks that wait until the second period, assuming that all banks with a lower \( \tau \) run, and banks with a higher \( \tau \) do not. Comparison to \( c_1 = r \).*

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Figure 7: First-period withdrawals, as a function of the asset price $p$.

Figure 8: Thresholds for trading the other asset and for declaring bankruptcy.
Figure 9: Equilibrium.
local depositors

Demand by outside investors

Price (discount, expected return)

Core banks

Supply of assets

assets
The set $\Theta$