Abstract

We study the importance of labor market frictions and the evolution of labor market variables for the design of monetary policy, using a micro-founded macroeconometric model with sticky prices, search and matching frictions on the labor market, and staggered nominal wage bargaining, estimated on U.S. data. We demonstrate that our model is very successful in describing the U.S. business cycle since 1960: all contractions dated by the NBER are matched by steep increases in unemployment above the estimated natural rate (or decreases of output below the natural level). Furthermore, model estimates of the output and unemployment gaps are fairly precise. We also find that uncertainty concerning parameters and the natural rates of output and unemployment has little effect on the performance or design of monetary policy rules. Finally, monetary policy rules that respond to the output or unemployment gap are more efficient than rules responding to output or unemployment growth rates, also in the presence of uncertainty about the natural rates.

Keywords: Monetary policy, Labor market search, Unemployment, Parameter uncertainty, Natural rate misperceptions.

JEL Classification: E24, E32, E52, J64.
1 Introduction

In recent years, monetary business cycle models with monopolistic competition and staggered price setting have been widely used to study the implications of alternative specifications of monetary policy. One shortcoming of these models, however, is that they typically do not include a very detailed description of the labor market, and are therefore not suited to discuss the relationship between monetary policy and unemployment. In the labor market literature, on the other hand, search and matching models with equilibrium unemployment have been fairly successful in explaining aggregate labor market fluctuations. Such labor market specifications have recently been extended to monetary business cycle models, originally by Trigari (2004, 2006) and Walsh (2005b), and thus present a natural alternative to the standard monetary framework.

Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) have demonstrated that nominal wage rigidities are a crucial ingredient when explaining U.S. business cycles, using monetary business cycle models without search and matching frictions. Within a similar model, Levin, Onatski, Williams, and Williams (2005) have shown that wage rigidities account for the main welfare cost of business cycle fluctuations, and that a monetary policy rule that responds only to nominal wage inflation performs almost as well as the welfare-optimizing policy. However, these results are very sensitive to the precise form of wage rigidities, suggesting that the specification of the labor market has important consequences for monetary policy.

The aim of this paper is to better understand the importance of labor market frictions and the evolution of labor market variables for the design of monetary policy. We study a micro-founded macroeconometric model with sticky prices, search and matching frictions on the labor market, and staggered nominal wage bargaining, following Gertler and Trigari (2006) and Gertler, Sala, and Trigari (2007). Compared with the models of Christiano et al. (2005), Smets and Wouters (2003), and Levin et al. (2005), our model includes a more realistic description of the labor market, featuring equilibrium unemployment, and wage rigidities are not subject to the Barro (1977) critique. It is therefore a natural laboratory for studying issues related to monetary policy and the labor market. In addition, Gertler et al. (2007) show that this new framework fits U.S. data well.

Using this model we study the behavior of the natural rate of unemployment and the implied unemployment (and output) gap(s), and we quantify the trade-offs facing the monetary authorities. We also analyze the design of monetary policy in the estimated model and the effects of parameter and natural rate uncertainty on optimized monetary policy rules.

Compared with the existing literature on monetary policy in models with search and matching frictions, for instance, Blanchard and Galí (2006), Thomas (2007), and Faia (2006), we use a quantitative framework and we study the implications for monetary policy of uncertainty concerning parameters and the natural rates of unemployment and output. While many authors have studied robust monetary policy with parameter and model uncertainty, for example, Levin, Wieland, and Williams (1999, 2003), Leitemo and Söderström (2005), Levin et al. (2005), Batini, Justiniano, Levine, and Pearlman (2006), and Edge, Laubach,
and Williams (2007), to our knowledge no one has considered uncertainty in a model with equilibrium unemployment.

Our analysis proceeds in the following steps. We first develop our model (in Section 2) and estimate it on U.S. data using Bayesian techniques (in Section 3). This part of the paper follows closely Gertler et al. (2007). We show that the estimated model fits U.S. data very well, also for the rate of unemployment and the degree of labor market tightness, variables that were not used when estimating the model.

We then discuss some properties of the model that are important for the design of monetary policy (see Section 4). In particular, we study the behavior of the estimated natural rates of output and unemployment and the implied output and unemployment gaps, with particular focus on the precision of these estimated gaps. We find that the implied path for the natural rate of unemployment is very similar to other estimates obtained with very different methodologies, for instance, by Staiger et al. (1997, 2002), and that the estimated unemployment and output gaps coincide very closely with the standard view of the U.S. business cycle (for example, contractions dated by the National Bureau of Economic Research). This feature of the model is in stark contrast with other estimated DSGE models, e.g., Levin et al. (2005). We also discuss the trade-offs facing monetary policymakers in terms of inflation and unemployment stability, showing that this trade-off is mainly driven by price markup shocks, and to some extent by technology and wage bargaining shocks. The trade-off is also significantly worsened by the presence of wage rigidities.

Finally, we study the design of monetary policy in our framework, assuming that the central bank aims at minimizing a loss function that is consistent with the mandate of the U.S. Federal Reserve (see Section 5). In particular, we compare the performance of standard Taylor rules that respond to the rate of inflation and the output gap with Taylor rules that respond to the rate of unemployment, the output growth rate, or the change in the unemployment rate. We also study the effects of uncertainty concerning model parameters and the natural rates of output and unemployment on the appropriate conduct of monetary policy. We show that the optimized monetary policy rules are superinertial, that is, the interest rate should respond to the lagged interest rate with a coefficient larger than one. Preliminary results also suggest that parameter and natural rate uncertainty has small effects on the performance of monetary policy rules and the optimized rules themselves. Finally, monetary policy rules that respond to the output or unemployment gaps clearly dominate rules responding to the growth rates of output and unemployment, also when the natural rates (and therefore the gaps) are misperceived by the central bank.

The results are still preliminary, and we end the paper by suggesting some possible extensions to the analysis.

2 The model

The model is based on Gertler, Sala, and Trigari (2007) and is a monetary Dynamic Stochastic General Equilibrium (DSGE) framework with habit formation, investment adjustment costs, variable capital utilization, and nominal price and wage rigidities. The model also includes
growth in the form of a non-stationary productivity shock, implying that all real variables grow at the same rate in steady state, as in Altig, Christiano, Eichenbaum, and Lindé (2005). In contrast to conventional DSGE models, the labor market involves search and matching in the spirit of Mortensen and Pissarides (1994) and others, and nominal wage rigidity in the form of staggered Nash bargaining as in Gertler and Trigari (2006).

There are three types of agents in the model: households, wholesale firms, and retail firms. Following Merz (1995) we assume a representative family in order to introduce complete consumption insurance. Production takes place at competitive wholesale firms that hire workers and negotiate wage contracts with them. Retail firms buy goods from wholesalers and repackage them as final goods. Retailers are monopolistic competitors and set prices on a staggered basis.

2.1 Households

There is a representative household with a continuum of members of measure unity. At each time $t$ a measure $n_t$ of household members are employed and a measure $1-n_t$ are unemployed. Household members are assumed to pool their labor income to insure themselves against income fluctuations. The household consumes final goods, saves in one-period nominal government bonds, and accumulates physical capital through investment. It transforms physical capital to effective capital by choosing the capital utilization rate, and then rents effective capital to firms.

The household thus chooses consumption $c_t$, bond holdings $B_t$, the rate of capital utilization $z_t$, investment $i_t$, and physical capital $k^p_t$ to maximize the utility function

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \varepsilon^b_{t+s} \log (c_{t+s} - h c_{t+s-1}) \right\},$$

where $\beta$ is a discount factor, $h$ measures the degree of habits in consumption preferences, and $\varepsilon^b_t$ is a preference shock with mean unity. This shock follows

$$\log (\varepsilon^b_t) = \rho_b \log (\varepsilon^b_{t-1}) + \zeta^b_t,$$

where all innovations, including $\zeta^b_t$, are zero-mean i.i.d. random variables.$^{1}$

The capital utilization rate $z_t$ transforms physical capital into effective capital according to

$$k_t = z_t k^p_{t-1},$$

which is rented to wholesale firms at the rate $r^k_t$. The cost of capital utilization per unit of physical capital is given by $A(z_t)$, and we assume that $z_t = 1$ in steady state, $A(1) = 0$ and $A'(1)/A''(1) = \eta_z$, as in Christiano et al. (2005) and others.

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$^{1}$As in Gertler et al. (2007), we do not allow for variation in hours on the intensive margin, for two reasons. First, most of the cyclical variation in hours in the U.S. is on the extensive margin. Second, earlier estimates confirmed that the intensive margin was unimportant to the cyclical variation, as the estimated Frisch elasticity was close to zero, in line with the microeconomic evidence.
Physical capital accumulates according to

\[ k_t^p = (1 - \delta)k_{t-1}^p + \varepsilon_t^i \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] i_t \]  

(4)

where \( \delta \) is the rate of depreciation, \( \varepsilon_t^i \) is an investment-specific technology shock with mean unity, assumed to follow

\[ \log (\varepsilon_t^i) = \rho_i \log (\varepsilon_{t-1}^i) + \zeta_i, \]  

(5)

and \( S(\cdot) \) is an adjustment cost function which satisfies \( S(a_{\gamma}) = S'(a_{\gamma}) = 0 \) and \( S''(a_{\gamma}) = \eta_k > 0 \), where \( \gamma_k \) is the steady-state growth rate.

Let \( p_t \) be the nominal price level, \( r_t \) the one-period nominal interest rate, \( w_t \) the real wage, \( b_t \) the flow value of unemployment (including unemployment benefits), \( \Pi_t \) lump-sum profits, and \( T_t \) lump-sum transfers. The household’s budget constraint is then given by

\[ c_t + i_t + B_{t-1} p_t = w_t n_t + (1 - n_t) b_t + r_t^k z_t k_{t-1}^p + \Pi_t + T_t - A(z_{t+1}) k_{t-1}^p + \frac{B_{t-1}}{p_t}, \]  

(6)

and the first-order conditions with respect to \( c_t, B_t, z_t, i_t \), and \( k_t^p \) are

\[ \lambda_t = \frac{\varepsilon_{t-1}^i}{c_t - h c_{t-1}} - \beta h E_t \left\{ \frac{\varepsilon_{t+1}^i}{c_{t+1} - h c_t} \right\}, \]  

(7)

\[ \lambda_t = r_t \beta E_t \left\{ \frac{\lambda_{t+1} p_t}{p_{t+1}} \right\}, \]  

(8)

\[ r_t^k = a'(z_t), \]  

(9)

\[ q_t^k \varepsilon_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] = q_t^k \varepsilon_t S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} - \beta E_t \left\{ q_{t+1} \varepsilon_{t+1}^i \frac{\lambda_{t+1} S'}{\lambda_t} \left[ \frac{i_{t+1}}{i_t} \right]^2 \right\} + 1, \]  

(10)

\[ q_t^k = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta) q_{t+1}^k + r_t^k z_{t+1} - A(z_{t+1}) \right] \right\}, \]  

(11)

where \( \lambda_t \) is the marginal utility of consumption and \( q_t^k \) is the value of installed capital in consumption units (that is, Tobin’s Q).

### 2.2 Wholesale firms

There is a continuum of wholesale firms measured on the unit interval. Each firm \( i \) produces output \( y_t(i) \) using capital \( k_t(i) \) and labor \( n_t(i) \) according to the Cobb-Douglas production function

\[ y_t(i) = k_t(i)^{\alpha} [a_t n_t(i)]^{1-\alpha}, \]  

(12)
where $a_t$ is a common labor-augmenting productivity factor, whose growth rate $\varepsilon_t^a = a_t/a_{t-1}$ follows the exogenous stochastic process

$$\log (\varepsilon_t^a) = (1 - \rho_a) \log (\varepsilon^a) + \rho_a \log (\varepsilon_{t-1}^a) + \zeta_t^a. \tag{13}$$

The steady-state value $\varepsilon^a$ corresponds to the economy’s steady-state growth rate $\gamma_a$. Thus, technology is non-stationary in levels but stationary in growth rates.

We assume that capital is perfectly mobile across firms and that there is a competitive rental market for capital. These assumptions ensure that there are constant returns to scale at the firm level, which simplifies the wage bargaining problem (which is specified below).

To attract new workers wholesale firms need to post vacancies $v_t(i)$. The total number of vacancies and employed workers are then equal to $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. All unemployed workers are assumed to look for a job, and unemployed workers who find a match go to work immediately within the period. Accordingly, the pool of unemployed workers, $u_t$, is given by

$$u_t = 1 - n_{t-1}. \tag{14}$$

The number of new hires is determined by the number of searchers and vacancies according to the matching function

$$m_t = \sigma_m u_t^\sigma v_t^{1-\sigma}. \tag{15}$$

The probability that a firm fills a vacancy is then given by

$$q_t = \frac{m_t}{v_t}, \tag{16}$$

and the probability that a worker finds a job is

$$s_t = \frac{m_t}{u_t}. \tag{17}$$

It is useful to define the hiring rate $x_t(i)$ as the ratio of new hires $q_t v_t(i)$ to the existing workforce $n_{t-1}(i)$:

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)}, \tag{18}$$

where the law of large numbers implies that the firm knows $x_t(i)$ with certainty at time $t$, as it knows the likelihood $q_t$ that each vacancy will be filled. Therefore, we can treat the hiring rate as the firm’s control variable.

Firms exogenously separate from a fraction $1 - \rho$ of their existing workforce $n_{t-1}(i)$ in each period, and workers who lose their jobs are not allowed to search until the next period. Thus, fluctuations in unemployment are only due to variation in hiring, in accordance with the empirical evidence in Hall (2005a, b) and Shimer (2005, 2007). The total workforce is
then the sum of the number of surviving workers and new hires:

\[ n_t(i) = \rho n_{t-1}(i) + x_t(i)n_{t-1}(i). \]  

(19)

Let \( p^w_t \) denote the relative price of intermediate goods and \( \beta E_t \Lambda_{t,t+1} \) be the firm’s discount rate, where \( \Lambda_{t,t+s} = \lambda_{t+s}/\lambda_t \). Then the value of firm \( i \), \( F_t(i) \), is given by

\[ F_t(i) = p^w_t y_t(i) - w_t(i)n_t(i) - \frac{\kappa_t}{2} x_t(i)^2 n_{t-1}(i) - r^k_t k_t(i) + \beta E_t \{ \Lambda_{t,t+1} F_{t+1}(i) \}, \]

(20)

where \( (\kappa_t/2)x_t(i)^2 n_{t-1}(i) \) is a quadratic labor adjustment cost. In order to maintain a balanced steady-state growth path, this adjustment cost is allowed to drift proportionately with productivity, \( \kappa_t = \kappa a_t \).  

(21)

The firm maximizes its value by choosing the hiring rate \( x_t(i) \) (by posting vacancies) and its capital stock \( k_t(i) \), given its existing employment stock \( n_{t-1}(i) \), the probability of filling a vacancy \( q_t \), the rental rate on capital \( r^k_t \), and the current and expected path of wages \( w_t(i) \). The first-order condition for capital is given by

\[ r^k_t = p^w_t \alpha y_t(i) k_t(i) = \alpha p^w_t \frac{y_t}{k_t}, \]

(22)

and is equal across firms, as the Cobb-Douglas production function with perfect capital mobility implies that all firms choose the same capital/output ratio, and therefore the same labor/output and capital/labor ratios.

The optimal hiring decision yields

\[ \kappa_t x_t(i) = p^w_t f_{nt}(i) - w_t(i) + \beta E_t \{ \Lambda_{t,t+1} \frac{\partial F_{t+1}(i)}{\partial n_t(i)} \}, \]

(23)

where

\[ f_{nt}(i) = (1 - \alpha) \frac{y_t(i)}{n_t(i)} = (1 - \alpha) \frac{y_t}{n_t} = f_{nt} \]

(24)

is the marginal product of labor, which is also equal across firms. Applying the envelope theorem to calculate \( \partial F_{t+1}(i)/\partial n_t(i) \) we obtain

\[ \kappa_t x_t(i) = p^w_t f_{nt} - w_t(i) + \beta E_t \{ \Lambda_{t,t+1} \frac{K_{t+1}}{2} x_{t+1}(i)^2 \} + \rho \beta E_t \{ \Lambda_{t,t+1} K_{t+1} x_{t+1}(i) \}. \]

(25)

The hiring rate thus depends on the discounted stream of earnings and the saving on adjustment costs.

Finally, we derive the value to the firm of having another worker at time \( t \) after new workers have joined the firm and adjustment costs are sunk. Differentiating \( F_t(i) \) with respect

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\(^2\) A constant adjustment cost would become relatively less important as the economy grows.
to \( n_t(i) \), taking \( x_t(i) \) as given, yields

\[
J_t(i) = p_t^n f_{nt} - w_t(i) + \beta E_t \left\{ \Lambda_{t,t+1} \frac{\partial F_{t+1}(i)}{\partial n_t(i)} \right\}.
\]

(26)

Using the hiring condition (25) and the expression for the evolution of the workforce in (19), \( J_t(i) \) can be written as

\[
J_t(i) = p_t^n f_{nt} - w_t(i) - \beta E_t \left\{ \Lambda_{t,t+1} \kappa_{t+1} x_t(i) \right\} + \beta E_t \left\{ \frac{n_{t+1}(i)}{n_t(i)} \Lambda_{t,t+1} J_{t+1}(i) \right\},
\]

(27)

that is, expected average profits per worker net of the first period adjustment costs, with the discount factor adjusted for future changes in workforce size.

2.3 Workers

Let \( V_t(i) \) be the value to a worker of employment at firm \( i \), and let \( U_t \) be the value of unemployment. These values are defined after hiring decisions at time \( t \) have been made and are in units of consumption goods. The value of employment is given by

\[
V_t(i) = w_t(i) + \beta E_t \left\{ \Lambda_{t,t+1} \left[ \rho V_{t+1}(i) + (1 - \rho) U_{t+1} \right] \right\}.
\]

(28)

To construct the value of unemployment, denote by \( V_{x,t} \) the average value of employment conditional on being a new worker, given by

\[
V_{x,t} = \int_0^1 \left[ V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} \right] di.
\]

(29)

Then \( U_t \) can be expressed as

\[
U_t = b_t + \beta E_t \left\{ \Lambda_{t,t+1} \left[ s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1} \right] \right\},
\]

(30)

where, as before, \( s_t \) is the probability of finding a job, and

\[
b_t = b_t k_t^p
\]

(31)

is the flow value of unemployment (measured in units of consumption goods). The flow value is assumed to grow proportionately with the physical capital stock in order to maintain balanced growth. The value of unemployment thus depends on the current flow value \( b_t \) and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that is currently unemployed is \( V_{x,t+1} \), the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.

Finally, the worker surplus at firm \( i \), \( H_t(i) \), and the average worker surplus conditional
on being a new hire, $H_{x,t}$, are given by

$$H_t(i) = V_t(i) - U_t$$

(32)

and

$$H_{x,t} = V_{x,t} - U_t.$$  

(33)

It follows that

$$H_t(i) = w_t(i) - b_t + \beta E_t \{ \Lambda_{t,t+1} [\rho H_{t+1}(i) - s_{t+1}H_{x,t+1}] \}.$$  

(34)

### 2.4 Wage dynamics

Firms and workers are not able to negotiate their wage contract in every period, but wage bargaining is assumed to be staggered over time, as in Gertler and Trigari (2006). As in Gertler et al. (2007), firms and workers bargain over nominal wages. In each period, each firm faces a fixed probability $1 - \lambda_w$ of being able to renegotiate the wage. The fraction $\lambda_w$ of firms that cannot renegotiate the wage instead index the nominal wage to past inflation according to

$$w_n^t(i) = \gamma_w \pi^{\gamma_w}_t w_n^{t-1}(i),$$  

(35)

where $\pi_t = p_t/p_{t-1}$ is the gross rate of inflation, $\gamma_w = \gamma_a \pi^{1-\gamma_w}$, and $\gamma_w \in [0,1]$ measures the degree of indexing. In the case of no indexing ($\gamma_w = 0$) the nominal wage grows over time with productivity $\gamma_a$ and the steady-state rate of inflation $\pi$.

Let $w_n^{*t}$ denote the wage of a firm-worker pair that renegotiates at $t$. Given constant returns, all sets of renegotiating firms and workers set the same wage. The firm negotiates with the marginal worker over the surplus from the marginal match. Assuming Nash bargaining, the contract wage $w_n^{*t}$ is chosen to solve

$$\max H_t(i)^{w_n} J_t(i)^{1-\eta_t},$$  

(36)

subject to

$$w_n^{*t}(i) = \begin{cases} 
\gamma_w w_n^{*t-1}(i) \pi_{t+j-1}^{\gamma_w} & \text{with probability } \lambda_w \\
\gamma_w w_n^{*t-1}(i) \pi_{t+j-1}^{\gamma_w} & \text{with probability } 1 - \lambda_w.
\end{cases}$$  

(37)

The variable $\eta_t \in [0,1]$ reflects the worker’s relative bargaining power, and is assumed to evolve according to

$$\eta_t = \eta^{\varepsilon_w}_t,$$  

(38)
where $\varepsilon^w_t$ is a shock with mean unity that follows

$$\log (\varepsilon^w_t) = \rho^w \log (\varepsilon^w_{t-1}) + \zeta^w_t.$$  

(39)

This bargaining power shock implies a disturbance to the wage equation.

The first-order conditions for the Nash bargaining solution are given by

$$\chi_t(i)J_t(i) = [1 - \chi_t(i)]H_t(i),$$  

(40)

where

$$\chi_t(i) = \frac{\eta_t}{\eta_t + (1 - \eta_t)\Sigma_t(i)/\Delta_t}$$  

(41)

is the (horizon-adjusted) effective bargaining power of workers,

$$\Sigma_t(i) = 1 + \beta \lambda w E_t \left\{ \Lambda_{t,t+1} \frac{n_{t+1}}{n_t} \frac{p_t}{p_{t+1}} \pi^w_{t+1}\Sigma_{t+1}(i) \right\}$$  

(42)

is the firm’s cumulative discount factor, and

$$\Delta_t = 1 + \beta \rho \lambda w E_t \left\{ \Lambda_{t,t+1} \frac{p_t}{p_{t+1}} \pi^w_{t+1}\Delta_{t+1} \right\}$$  

(43)

is the worker’s cumulative discount factor.

As in Gertler and Trigari (2006), the bargaining solution gives a difference equation for the real wage $w^*_t = w^{n*}_t/p_t$ as

$$\Delta_t w^*_t = w^*_t(i) + \rho \beta \lambda w E_t \left\{ \Lambda_{t,t+1}\Delta_{t+1}w^*_t(i) \right\},$$  

(44)

where $w^*_t(i)$ can be interpreted as the real target wage, and is given by

$$w^*_t(i) = \chi \left[ p^w_t J_t(i) + \beta E_t \left\{ \Lambda_{t,t+1} \frac{n_{t+1}}{n_t} \frac{p_t}{p_{t+1}} \pi^w_{t+1}(i)^2 \right\} \right]$$

$$+ (1 - \chi) \left[ b_t + \beta E_t \left\{ \Lambda_{t,t+1}s_{t+1}H_{x,t+1} \right\} \right] + \Phi_t(i),$$  

(45)

and where

$$\Phi_t(i) = \phi_t(i) - \rho \beta E_t \left\{ \Lambda_{t,t+1}\phi_{t+1}(i) \right\},$$  

(46)

$$\phi_t(i) = [\chi_t(i) - \chi] [J_t(i) + H_t(i)].$$  

(47)

Due to the staggered wage bargaining, the contract wage $w_t^*$ depends not only on the current target wage, but on the expected sequence of future target wages. The target wage, in turn, is a convex combination of what a worker contributes to the match (the marginal product of labor plus the saving on adjustment costs) and what the worker loses by accepting a job (the

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3Firms and workers have different horizons when contracting. The firm cares about the impact of the contract wage on existing workers as well as on new workers expected to join the firm under the terms of the current contract. Workers, on the other hand, care only about the impact of the contract wage on the expected job tenure.
flow value of unemployment plus the expected discounted gain of moving from unemployment this period to employment next period), where the weights depend on the worker’s average effective bargaining power. The additional effect, captured by $\Phi_t(i)$, reflects the horizon effect as well as exogenous movements in the bargaining power.

Finally, the average nominal wage is given by

$$w^n_t = \int_0^1 \left[ w^n_t(i) n_t(i) \right] di. \quad (48)$$

### 2.5 Retailers

There is a continuum of monopolistically competitive retailers indexed by $j$ on the unit interval. These buy intermediate goods from the wholesale firms, differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, and sell them to households. Retailers set prices on a staggered basis.

Following Smets and Wouters (2007), we assume that each firm’s elasticity depends inversely on its relative market share, as in Kimball (1995), who generalizes the standard Dixit-Stiglitz aggregator. This assumption implies a strategic complementarity in price setting that makes it easier for the model to match the micro evidence on price adjustment. Thus, letting $y_t(j)$ be the quantity of output sold by retailer $j$ and $p_t(j)$ the nominal price, final goods, denoted $y_t$, are a composite of individual retail goods following

$$\int_0^1 G \left( \frac{y_t(j)}{y_t}, \varepsilon_t^p \right) dj = 1, \quad (49)$$

where the function $G(\cdot)$ is increasing and strictly concave with $G(1) = 1$, and $\varepsilon_t^p$ is a non-negative random variable that influences the elasticity of demand. We assume that $\varepsilon_t^p$ follows

$$\log (\varepsilon_t^p) = (1 - \rho_p) \log (\varepsilon^t) + \rho_p \log (\varepsilon_{t-1}^p) + \zeta_t^p. \quad (50)$$

Cost minimization then leads to a demand curve facing each retailer given by

$$y_t(j) = G^{-1} \left( \frac{p_t(j)}{p_t \tau_t} \right) y_t, \quad (51)$$

where $p_t$ is the aggregate price index and

$$\tau_t = \int_0^1 \left[ G' \left( \frac{y_t(j)}{y_t} \right) \frac{y_t(j)}{y_t} \right] dj. \quad (52)$$

We assume that prices are staggered as in Calvo (1983), but with indexing as in Christiano et al. (2005) and Smets and Wouters (2003). Thus, each retailer faces a fixed probability $1 - \lambda_p$ of reoptimizing its price in a given period, in which case it sets its price optimally to $p^*_t$, which is equal across all firms that reoptimize their price. Firms that do not reoptimize its price instead index it to past inflation following

$$p_t(j) = \tau_p \pi_{t-1}^{\rho_p} \pi_{t-1}(j), \quad (53)$$
where $\gamma_p = \pi^{1-\gamma_p}$ is an adjustment for steady-state inflation.

By combining equations (49) and (51) and applying the law of large numbers, the aggregate price index is given by

$$p_t = (1 - \lambda_p) p_t^* G_t^{-1} \left( \frac{p_t^*}{p_t} \right) + \lambda_p \pi_t \gamma_p p_{t-1} \tau_{t-1} G_{t-1}^{-1} \left( \frac{\pi_{t-1}^\gamma p_{t-1}}{p_t} \right),$$

(54)

where $p_t^*$ is the price set by reoptimizing firms at time $t$ and

$$\tau_t = \int_0^1 G'' \left( \frac{y(j)}{y_t} \right) \frac{y(j)}{y_t} dj.$$

(55)

Re-optimizing firms set their price $p_t^*$ maximize the expected discounted stream of future profits,

$$E_t \sum_{s=0}^{\infty} \left\{ (\beta \lambda_p)^s \Lambda_{t,t+s} \left[ \frac{p_t^*}{p_{t+s}} \left( \prod_{k=1}^{s} \pi_t^\gamma p_{t+k-1} \right) - p_{t+s}^w \right] y_{t+s}(j) \right\} = 0,$$

(56)

subject to the demand function (51). This yields the first-order condition

$$E_t \sum_{s=0}^{\infty} \left\{ (\beta \lambda_p)^s \Lambda_{t,t+s} \left[ (1 + \Theta_t) \frac{p_t^*}{p_{t+s}} \left( \prod_{k=1}^{s} \pi_t^\gamma p_{t+k-1} \right) - \Theta_t p_{t+s}^w \right] y_{t+s}(j) \right\} = 0,$$

(57)

where

$$\Theta_t = \left[ G''^{-1} \left( \pi_t p_t^*/p_{t+s}^w \right) \right]^{-1} G' \left( \frac{G''^{-1} \left( \pi_t p_t^*/p_{t+s}^w \right)}{G''^{-1} \left( \tau_1 p_t^*/p_{t+s}^w \right)} \right).$$

(58)

Thus, the optimal price depends on the expected discounted stream of the retailers’ nominal marginal cost given by $p_t p_t^w$. Using the hiring condition (25), real marginal cost is given by

$$p_t^w = \frac{1}{E_{t+1}^{\tau_t^\gamma}} \left[ w_t(i) + \kappa_t x_t(i) - \beta E_t \left\{ \Lambda_{t,t+1} \left( \frac{1}{2} \right) x_{t+1}(i) \right\} \right] - \rho E_t \left\{ \Lambda_{t,t+1} \kappa_{t+1} x_{t+1}(i) \right\},$$

(59)

so real marginal cost depends on unit labor cost adjusted for hiring costs.

### 2.6 The government sector

The government determines government spending $g_t$ (which does not yield any utility to households) and a central bank sets the short-term nominal interest rate $r_t$. Government spending is assumed to be proportional to output and follow

$$g_t = \left[ \frac{1}{\alpha_t^2} \right] y_t,$$

(60)
where
\[
    \log(\varepsilon^g_t) = (1 - \rho_g) \log(\varepsilon^g) + \rho_g \log(\varepsilon^g_{t-1}) + \zeta^g_t.
\] (61)

The interest rate is set according to the Taylor rule
\[
    \frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_s} \left[ \left( \frac{E_t \pi_{t+1}}{\pi} \right)^{r_p} \left( \frac{y_t}{y^*_t} \right)^{r_y} \right]^{1-\rho_s} \varepsilon^r_t,
\] (62)

where \(y^*_t\) is the natural (or flexible-price) level of output and \(\varepsilon^r_t\) is a monetary policy shock that follows
\[
    \log(\varepsilon^r_t) = \rho_r \log(\varepsilon^r_{t-1}) + \zeta^r_t.
\] (63)

Following convention (see, for instance, Woodford (2003), or Galí (2007)), we define the natural level of output as the level of output in the equilibrium where there are no nominal rigidities or inefficient shocks. In our model, the inefficient shocks are to the price markup (which affects the degree of monopolistic competition), the bargaining power of workers (which causes time-variation in the deviations from the efficiency of the matching process), and monetary policy. Associated with the natural level of output, there is also a natural rate of unemployment, denoted \(u^*_t\).

### 2.7 Resource constraint and model summary

Finally, the resource constraint implies that output is equal to the sum of consumption, investment, government spending, and adjustment and utilization costs:

\[
y_t = c_t + i_t + g_t + \frac{\kappa_t}{2} \int_0^1 \left[ x_t(i) y_{t-1}(i) \right] di + A(z_t) k_t^p - 1.
\] (64)

The complete model consists of 28 equations for the 28 endogenous variables, and seven equations for the exogenous shocks. We log-linearize the model around its deterministic steady state with balanced growth, allowing for the fact that the quantity variables (output, investment, consumption, etc.) are non-stationary due to the unit-root technology shock. The derivation of the steady state, as well as the log-linearized system of equations are presented in Appendix A.

### 3 Estimation

We estimate the log-linearized version of the model using quarterly U.S. data from 1960Q1 to 2005Q1. Following Christiano et al. (2005), Smets and Wouters (2007), and Primiceri, Schauburg, and Tambalotti (2006), we consider data for seven variables:

1. output: per capita real GDP;
2. consumption: per capita real personal consumption expenditures.

\(^4\)There are some slight differences in the series used by Smets and Wouters (2007) compared with Christiano et al. (2005) and Primiceri et al. (2006). As in Christiano et al. (2005) and Primiceri et al. (2006), we include consumer durables in investment. As in Smets and Wouters (2007), we use an economy-wide measure of hours based on an adjustment of non-farm business hours.
nondurables; (3) investment: per capita real investment, equal to the sum of per capita real private investment and per capita real personal consumption of durables; (4) employment: hours of all persons in the non-farm business sector divided by population, multiplied by the ratio of total employment to employment in the non-farm business sector; (5) the real wage: compensation per hour in the non-farm business sector; (6) inflation: the quarterly growth rate of the GDP deflator; and (7) the nominal interest rate: the quarterly average of the federal funds rate expressed in quarterly terms. To convert any nominal variable to real terms, we use the GDP deflator.

The model contains 22 structural parameters, not including the parameters that characterize the exogenous shocks. There are seven exogenous shocks in the model, one for each variable.

We calibrate three of the five labor market parameters for which independent evidence is available. We choose the average monthly separation rate \(1 - \rho\) based on the observation that jobs in the U.S. last about two years and a half, implying \(\rho = 1 - 0.105\). We choose the match elasticity with respect to unemployment, \(\sigma\), to be equal to 0.5, a value that is the midpoint of the evidence typically cited in the literature, and within the range of plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the empirical literature on the matching function. We set the hiring cost parameter \(\kappa\) such that the steady-state job finding rate is \(s = 0.95\), as in recent estimates of the U.S. average monthly job finding rate (see Shimer (2005)).

We calibrate five “conventional” parameters using standard values: the discount factor \(\beta\) is set to 0.99, the capital depreciation rate \(\delta\) to 0.025, the capital share \(\alpha\) in the Cobb-Douglas production function is set to 0.33, and the steady-state ratio of government spending to output \(g/y\) to 0.2. Finally, we calibrate the sensitivity of the firm’s elasticity of demand with respect to shifts in its market share, the Kimball aggregator parameter, denoted \(\xi\), to 10.

Table 1 summarizes the calibrated parameters.

We estimate the two labor market parameters \(\tilde{b}\), which determines the relative flow value of unemployment, and \(\eta\), the steady-state bargaining power of workers. These parameters are critical determinants of the effective elasticity of labor supply along the extensive margin in the flexible wage case. We also estimate the elasticity of the utilization rate to the rental rate of capital, \(\eta_z\), the elasticity of the capital adjustment cost function, \(\eta_k\); the habit

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5All data were obtained from the FRED data base of the Federal Reserve Bank of St. Louis.

6In contrast to the frictionless labor market model, the term \(1 - \alpha\) does not correspond to the labor share, but will depend on the outcome of the bargaining process. However, because a wide range of values for the bargaining power imply a labor share just below \(1 - \alpha\), we follow convention by setting \(1 - \alpha = 2/3\). In our calculations, \(1 - \alpha\) equals 0.667 and the labor share 0.666.

7The relative flow value of unemployment is defined as

\[
\tilde{b} = \frac{b(k/a)}{p^n(f_n/a) + \beta(k/2)x^2}.
\]

8Following Smets and Wouters (2007), we define \(\psi_z\) such that \(\eta_z = \frac{1-\psi_z}{\psi_z}\) and estimate \(\psi_z\). When \(\psi_z = 1\), it is very costly to change the capital utilization rate and the utilization rate does not vary. When \(\psi_z = 0\), the marginal cost of changing the capital utilization rate is constant and, as a result, the rental rate of capital does not vary.
parameter $h$; the steady-state price markup $\varepsilon^p$; the wage and price rigidity parameters $\lambda_w$ and $\lambda_p$; the wage and price indexing parameters $\gamma_w$ and $\gamma_p$; and the Taylor rule parameters $r_\pi$, $r_y$, and $\rho_s$. In addition, we estimate the autoregressive parameters of all the exogenous disturbances, as well as their respective standard deviations.

We estimate the model with Bayesian methods (see An and Schorfheide (2007) for a review). Letting $\theta$ denote the vector of structural parameters to be estimated and $Y$ the data sample, we combine the likelihood function of the model, $L(\theta, Y)$, with priors for the parameters to be estimated, $p(\theta)$, to obtain the posterior distribution: $L(\theta, Y)p(\theta)$. Draws from the posterior distribution are generated with the Random-Walk Metropolis-Hastings algorithm.

Tables 2 and 3 report the prior distribution of the parameters along with the median and the 5th and 95th percentiles of the posterior distribution. For the conventional parameters, we use priors similar to those of Primiceri et al. (2006) and Smets and Wouters (2007). The parameter estimates are discussed in detail in Gertler et al. (2007).

4 Model properties

We now discuss some properties of the estimated model that are particularly important for monetary policy. First, we discuss the fit of the estimated model, both for the variables used in estimation and for unemployment and labor market tightness, which were not used when estimating the model. We then turn to the estimated behavior of the natural rates of unemployment and output, and the estimated unemployment and output gaps (the percent deviation of unemployment and output from their natural rates) over the sample. Finally, we discuss the trade-offs facing the central bank.

4.1 Empirical fit

To illustrate the fit of the estimated model, Figures 1–3 show autocovariance functions for three blocks of the variables used in the estimation: aggregate demand variables (output, consumption, and investment, all in terms of growth rates) in Figure 1, labor market variables (output growth, employment, and real wage growth) in Figure 2, and monetary policy variables (output growth, inflation, and the federal funds rate) in Figure 3.

The solid lines are autocovariances in U.S. data, while the dashed lines are 5th and 95th percentiles from the posterior distribution. We see that the autocovariance functions from U.S. data fall within the 90% interval of the empirical distribution at most leads and lags. Thus, the estimated model captures well the covariance structure of U.S. data.

Next we study the implied behavior of the unemployment rate and the degree of labor market tightness, two variables that were not used in the estimation. Figures 4–5 show U.S. data and model estimates of the unemployment rate and the degree of labor market

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9To construct these intervals, we draw 500 times from the posterior parameter distribution, and simulate 100 samples of 160 observations (as in the data sample) for each draw. Thus, the intervals are determined by both parameter and sampling uncertainty. These autocovariances are discussed in more detail by Gertler et al. (2007).
tightness over the sample. The fit is surprisingly good for both variables: while the model tends to overestimate the volatility of the two labor market variables, it matches remarkably well the movements over the sample period. Figure 6 shows the autocovariance function of unemployment with the variables used for estimation. Again, the estimated probability intervals include the covariances in U.S. data at almost all leads and lags.

4.2 Natural rates and gaps

We now study the behavior of the estimated natural rate of unemployment, as well as the unemployment and output gaps. These gaps are important measures of the degree of slack in the economy, and therefore important indicators for monetary policy.

Figure 7 shows the estimated path for the rate of unemployment (discussed earlier) and the natural rate of unemployment (with a 90% confidence interval). According to our model, the natural rate of unemployment has moved gradually over the sample period, trending upwards in the 1970s and early 1980s and then falling slowly in the 1990s.

The Figure also shows that the unemployment rate was above the natural rate for most of the 1970s, in particular in 1971–72, 1975–78, and 1981–84. In contrast, the rate of unemployment was below the natural rate for most of the 1990s.


Thus, our model is able to generate business cycle fluctuations that are similar to alternative accounts. In contrast, the model estimated by Levin et al. (2005) has been criticized for not generating sensible business cycles (see Walsh (2005a)). In particular, their model interprets the decrease in economic activity during the Volcker disinflation in the early 1980s as a large fall in the natural level of output and a positive output gap, rather than a drop in actual output below the natural level. Our estimates instead suggest that the natural rate of unemployment increased only marginally in this period, while the actual unemployment rate increased substantially. Thus, our model interprets this period as a large increase in unemployment above the natural rate and a fall in output below the natural level.

From Figures 8 and 9, we can also identify an Okun’s law relationship in terms of a negative correlation between the unemployment gap and the output gap. The unconditional

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10 The unemployment series is civilian unemployment divided by the civilian non-institutional population over 16, obtained from the Bureau of Labor Statistics. The series for vacancies used to calculate labor market tightness is the help-wanted index constructed by the Conference Board. These data are percent deviations from the sample mean, while the model estimates (obtained via the Kalman smoother) are percent deviations from the steady state, which has been normalized to zero in the figures. With an average unemployment rate (and natural rate) of 6 percent (similar to the sample mean), a 10% unemployment gap implies an unemployment rate 0.6 percentage points above the natural rate.

11 These intervals were constructed taking into account both parameter uncertainty and Kalman filter uncertainty, see Hamilton (1994, Ch. 13.7).
correlation between the two gaps is $-0.88$, and estimating the Okun’s law regression

$$x_t^y = a + bx_t^u + e_t,$$

(65)

where $x_t^y$ is the output gap and $x_t^u$ the unemployment gap, would give an unconditional slope coefficient of $-0.0711$. Thus, with a 6 percent natural rate, an unemployment rate of 7 percent (a 17% unemployment gap) would tend to coincide with a negative output gap of $-1.2\%$.

### 4.3 Monetary policy trade-offs

We now illustrate the existence and the nature of the trade-off faced by the central bank in conducting monetary policy. In order to do so, we consider a central bank that aims at stabilizing inflation and the unemployment gap and we allow for varying weights on the two objectives. In particular, we consider two extreme policies: strict inflation targeting, that is, when the central bank only aims at stabilizing inflation, and strict unemployment targeting, that is, the central bank only cares about stabilizing unemployment around its natural rate. We also consider intermediate cases in which the central bank puts some positive weight on both targets. We report the standard deviations of the annualized rate of inflation and the unemployment gap when the central bank implements the optimal policy with commitment as the relative weight on inflation varies from one to zero, that is, the efficient policy frontier.

Our aim with this exercise is threefold: first, to understand whether a policy trade-off exists; second, which shock are responsible for the trade-off; and third, how the structure of the labor market affects the nature of the trade-off.

Figure 10 displays the policy frontier in the model first when all shocks are included (in Panel (a)), then considering one shock at a time (in Panels (b) and (c)). Panel (a) shows that there is a marked trade-off between inflation and unemployment volatility. In order to achieve full price stability, the unemployment gap (the percent deviation from unemployment from the natural rate) must be extremely volatile, with a standard deviation around 500. In the other extreme, full unemployment stability (a zero unemployment gap) would be possible at the cost of a standard deviation of the inflation rate around 3.

The remaining two Panels in Figure 10 reveal that most of this trade-off originates from price markup shocks, while technology and wage bargaining shocks contribute to a smaller extent, and demand shocks (to preferences, investment, and government spending) barely create a trade-off at all.

It is well-known that the presence of wage rigidities are an important determinant of the monetary policy trade-off, as first demonstrated by Erceg, Henderson, and Levin (2000), and more recently stressed by Blanchard and Galí (2007). Figure 11 aims at quantifying the importance of wage rigidities in our framework, by showing the trade-offs when wages are flexible. As shown in Panel (a), wage rigidities have a huge effect on the trade-off: with flexible wages, complete inflation stabilization is achieved with a standard deviation of the unemployment gap of 65, compared with 500 with staggered wages. Conversely, strict unemployment targeting implies a standard deviation of inflation of 2.1 rather than 3. Panels (b)
and (c) reveal that with flexible wages, price markup and wage bargaining shocks have a much smaller effect than with staggered wages, while technology and demand shocks do not create a trade-off at all when wages are flexible.

Figure 12 examines how the policy trade-offs are affected by the structure of the labor market. We first explore the effect of variations in the average relative flow benefit from being unemployed, \( \bar{b} \). Panel (a) reports the policy trade-offs for two values of \( \bar{b} \): the estimated value of 0.722 and a lower value of 0.4, which is typically assumed when \( \bar{b} \) is interpreted as unemployment insurance (see Shimer (2005)). As shown in the figure, a reduction in \( \bar{b} \) improves the policy trade-off. For example, the strict inflation targeting policy implies a standard deviation of unemployment of about 300 with the lower value of \( \bar{b} \), rather than 500 in the benchmark case. The improved policy trade-off is a consequence of the higher elasticity of marginal cost to movements in unemployment, which in turn implies a higher elasticity of inflation to unemployment. In this case, smaller changes in unemployment are necessary to obtain a given change in inflation. There are two reasons why marginal costs are more responsive to changes in unemployment with a lower value for \( \bar{b} \). As equation (59) shows, marginal cost depends on the real wage as well as the employment adjustment costs. First, the recent literature\(^{12}\) has emphasized that a high value of the relative unemployment benefit stabilizes the workers’ outside option in bargaining and, through this channel, the period-by-period target wage. Thus, a reduction in \( \bar{b} \) increases the responsiveness of real wages to labor market fluctuations, in particular to unemployment fluctuations. In addition, a smaller value of \( \bar{b} \) is associated with a larger value of the parameter \( \kappa \), which measures the size of the employment adjustment costs.\(^{13}\) The larger is \( \kappa \), the larger will be the elasticity of employment adjustment costs to movements in hiring rates.

We then explore the effect of a proportional reduction in both the inflows into unemployment and the outflows from unemployment, that is a reduction in the turnover rate. In particular, we reduce in the average job finding rate \( s \) from 0.95 to 0.5 and the job destruction rate \( 1 - \rho \) from 0.105 to 0.505. Panel (b) of Figure 12 shows that an economy characterized by a lower turnover faces an improved policy trade-off. Under the full inflation stabilization policy the standard deviation of unemployment decreases from 500 to a value little above 300. First, as before, the experiment increases the size of the employment adjustment costs \( \kappa \) implied by our calibration, leading to a larger elasticity of marginal costs to the hiring rate. Second, the decrease in the average job finding probability reduces the spillover effect that average wages have on contract wages by influencing the workers’ outside option in bargaining through their effect on the expected value of moving from unemployment to employment (see Gertler and Trigari (2006)). The lower job finding rate, in fact, reduces the sensitivity of the workers’ outside option to the value of moving from unemployment to employment, and thus the spillover effect. This makes aggregate wages more responsive to labor market fluctuations. Both effects work in the direction of increasing the elasticity of marginal cost.

\(^{12}\)See Hagedorn and Manovskii (2006).

\(^{13}\)We set the parameter \( \kappa \) to target an average job finding rate of 0.95. For a given job finding rate, the reduction in \( \bar{b} \) increases the surplus from a match and thus the firm’s value of hiring an additional worker. Thus, on average, the marginal cost of hiring an additional worker must also increase, leading to a higher implied value for \( \kappa \).
to unemployment and the elasticity of inflation to unemployment.

5 The design of monetary policy

We now turn to the design of monetary policy in the estimated model. For this purpose, we will consider a central bank with a mandate similar to that of the U.S. Federal Reserve as specified in the Federal Reserve Act, that is, “maximum employment, stable prices, and moderate long-term interest rates.” We formalize this mandate with the intertemporal loss function

\[ L_t = (1 - \hat{\beta}) E_t \sum_{j=0}^{\infty} \hat{\beta}^j \left[ \pi_t^2 + \lambda_u (x^u_{t+j})^2 + \lambda_r \tau_t^2 \right], \]

where \( \pi_t \) is the annualized rate of inflation (four times the quarterly rate), \( x^u_t \) is the unemployment gap, \( \tau_t \) is the annualized federal funds rate, and \( \hat{\beta} \) is the central bank discount factor. Thus the central bank strives at minimizing the volatility of inflation around the steady state (“stable prices”), unemployment around the natural rate (“maximum employment”), and the federal funds rate around steady state (“moderate long-term interest rates”).

We calibrate the weights \( (\lambda_u, \lambda_r) \) in the loss function so that the volatility of the unemployment gap and the federal funds rate relative to that of inflation with the unconstrained optimal policy match those in the estimated model. This gives \( \lambda_u = 0.003 \) and \( \lambda_r = 0.08 \), implying that a one percentage point deviation of the unemployment rate from a natural rate of 6 percent is equivalent in terms of loss to a deviation of inflation from target of 0.91 percent.

We will focus on optimized rules for monetary policy. However, as a benchmark we will use the unconstrained optimal policy with commitment, using the algorithms developed by Dennis (2007), and setting the central bank discount factor to \( \hat{\beta} = 0.99 \). This policy implies standard deviations of inflation, the unemployment gap, and the federal funds rate of 2.19.

14 Much of the recent literature on optimal monetary policy studies the welfare-maximizing policy where the objective is to maximize an approximation to the household’s utility function. This has been done either using numerical methods in larger-scale models (for instance, Schmitt-Grohé and Uribe (2004), Levin et al. (2005), or Faia (2006)), or using smaller models where it is possible to analytically derive an approximation of household utility (recent examples include Blanchard and Gali (2006), Thomas (2007), and Edge et al. (2007)). Unfortunately, our model does not aggregate well in its non-linear form to derive a utility-based welfare criterion or apply the numerical methods to analyze welfare-maximizing policy.

15 We approximate the objective of moderate long-term interest rates with federal funds rate volatility, as such volatility may lead to increased term premia and therefore higher long-term interest rates (see Tinsley (1999)). Woodford (2003) instead shows that the welfare-maximizing policy aims at reducing interest rate volatility when there are money transaction frictions or when the central bank wants to avoid the zero-lower bound of nominal interest rates. Note that as \( \beta \) approaches 1, the loss function (66) approaches the unconditional expectation of \( L_t \), that is

\[ \lim_{\beta \to 1} L_t = E L = \text{Var}(\pi_t) + \lambda_u \text{Var}(x^u_t) + \lambda_r \text{Var}(\tau_t), \]

where \( \text{Var}(\cdot) \) denotes the unconditional variance. We use this specification to optimize and evaluate the monetary policy rules below.

16 With a 6 percent natural rate, an unemployment rate of 7 percent gives an unemployment gap of 16.67%, and therefore a loss of \( \lambda_u \times 16.67^2 \approx 0.834 \). This is in turn equivalent to an inflation rate of \( \sqrt{0.834} \approx 0.91\% \).
25.8, and 2.61, respectively. The estimated rule instead implies standard deviations of 2.37, 27.9, and 2.85, respectively, which are only slightly above those of the optimal policy.\footnote{17}

\section{5.1 Optimized monetary policy rules}

We study the performance of four different rules for monetary policy. The first rule is a standard Taylor rule, where the central bank sets the interest rate as a function of the rate of inflation, the output gap, and the lagged interest rate. In terms of the log-linearized model, this rule is specified as

\[
\hat{r}_t = r_{\pi} \tilde{n}_t + r_y x^y_t + \rho_s \hat{r}_{t-1},
\]

(67)

where hats denote log deviations from steady state, and \( x^y_t \) is the percent deviation of output from its natural level. This rule is similar to our estimated rule above, the only difference being that the estimated rule responds to the one-period ahead expectation of inflation, \( E_t \hat{\pi}_{t+1} \).

The second rule includes the unemployment gap instead of the output gap, so

\[
\hat{r}_t = r_{\pi} \tilde{n}_t + r_y x^u_t + \rho_s \hat{r}_{t-1},
\]

(68)

where \( x^u_t \) is the percent deviation of unemployment from the natural rate. This rule is similar to that used by Orphanides and Williams (2002) in an estimated two-equation model of inflation and unemployment.

These two rules that respond to the output or unemployment gap rely heavily on the central bank’s estimate of the natural rates of output and unemployment. Therefore they may be difficult to implement in practice, and they may be inefficient if the central bank does not have perfect information about the natural rates. We therefore also study two rules that respond to the growth rate of output and unemployment:

\[
\hat{r}_t = r_{\pi} \tilde{n}_t + r_y \Delta \log y_t + \rho_s \hat{r}_{t-1},
\]

(69)

\[
\hat{r}_t = r_{\pi} \tilde{n}_t + r_y \Delta \tilde{u}_t + \rho_s \hat{r}_{t-1},
\]

(70)

which do not rely on the natural rates. Such rules (with \( \rho_s = 1 \)) are shown by Orphanides and Williams (2002) to be robust against natural rate misperceptions.\footnote{18}

We first study the performance of optimized versions of our four rules with the benchmark

\footnote{17}The standard deviations of inflation and the funds rate in U.S. data over our sample period are 2.44 and 3.10, respectively.

\footnote{18}We also experimented with rules that include both the output and unemployment gaps, the level of output and unemployment (that is, the deviation from steady state), the degree of labor market tightness, and the rate of wage inflation. Level rules perform very similarly to rules that respond to the deviation from the natural rates. Rules responding to labor market tightness perform very similarly to those that respond to output or unemployment. Rules that include both output and unemployment and/or the rate of wage inflation give very small improvements compared with rules with only one real variable. This last result is in contrast to Levin et al. (2005) who show that rules that respond to wage inflation are very efficient and a rule with only wage inflation performs almost as well as the welfare-maximizing rule. Our results suggest that wage inflation is important in their framework because it obtains a large weight in the welfare criterion, not because responding to wage inflation is beneficial for macroeconomic stability in general.
parameterization of the model. We will then introduce uncertainty about parameter values and the natural rates, and study the effect of such uncertainty on the performance of the optimized benchmark rules, and the design of rules that take uncertainty into account.

Table 4 shows the optimized coefficients in the four rules. The optimized rules are all “superinertial,” that is, with a coefficient on the lagged interest rate larger than one. As first discussed by Rotemberg and Woodford (1999), this is due to the forward-looking nature of the model: the optimal policy is to offset movements in inflation so that the price level returns towards its initial level. With a monetary policy rule, this can be achieved by threatening to increase (or decrease) the interest rate exponentially if shocks to inflation are not offset in the future. Forward-looking agents foresee this threat and adjust appropriately so that the central bank never needs to carry through its threat.

Table 5 shows how these rules perform in terms of standard deviations of inflation, the output and unemployment gaps, and the interest rate, comparing with the fully optimal policy. Relative to the optimal policy, the rules that respond to the output or unemployment gaps tend to over-stabilize unemployment (and output) and under-stabilize inflation. Thus, these rules suffer from a “stabilization bias” similar to that of the discretionary policy.

To quantify the inefficiency of each optimized rule, we calculate an “unemployment equivalent,” which measures the permanent percent deviation of the unemployment rate from a natural rate of 6 percent which is equivalent in terms of loss to moving from the fully optimal policy to the optimized rule (see also Jensen (2002) and Dennis and Söderström (2006)). In other words, the unemployment equivalent represents the permanent rate of unemployment that the central bank would be willing to accept in order to implement the optimal policy rather than the optimized rule.

As shown in the rightmost column of Table 5, the unemployment equivalent is 0.11 and 0.15 percent for the two gap rules. Interestingly, the rule responding to the output gap is slightly more efficient than the unemployment gap rule. Even more surprisingly, the output gap rule is more efficient in stabilizing the unemployment gap than is the rule that responds directly to the unemployment gap. The differences between the two rules are small, however, and depend to some extent on the parameterization of the loss function: using a larger weight on the unemployment gap eventually reverses the ranking of the two rules, and also implies that unemployment is more stable with the unemployment gap rule.

Table 5 also reveals that the two difference rules perform substantially worse than the gap rules, and the rule that responds to output growth is particularly inefficient, with extreme volatility in the unemployment (and output) gap and the interest rate. As a consequence, the unemployment equivalents for these two rules are 1.02 and 0.56, respectively. Thus, in the case where the central bank has perfect information about the natural rates of output

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19 See Dennis and Söderström (2006) for a discussion and quantification of the stabilization bias of discretionary monetary policy.

20 To calculate the unemployment equivalent, note that the loss function implies that a permanent unemployment gap of \( x \) percent gives a loss of \( (1 - \beta) \sum_{j=0}^{\infty} \beta^j \lambda_u x^2 = \lambda_u x^2 \). Denoting by \( L_R \) the loss under the optimized rule and by \( L^\ast \) the loss under the optimal policy, the permanent unemployment gap that would be equivalent to moving from the fully optimal policy to the optimized rule is given by \( x = \sqrt{L_R / \lambda_u} - \sqrt{L^\ast / \lambda_u} \). The unemployment equivalent is then given by \( (1 + x) u^n \), where \( u^n \) is the steady-state natural rate of unemployment.
and unemployment, responding to the output and unemployment gaps is vastly superior to rules that respond to output or unemployment growth.

5.2 Introducing parameter and natural rate uncertainty

We now introduce uncertainty about the parameters and the natural rates of output and unemployment. We first evaluate the performance of the optimized rules in Table 4 under uncertainty by calculating the expected loss over 5,000 draws from the posterior distribution, keeping the rule coefficients fixed at their optimized values. We then (in the next Subsection) discuss rules that are optimized to take uncertainty into account. As a benchmark for our comparisons, we use the outcome when the central bank implements the unconstrained optimal policy with commitment in each of 5,000 parameter draws.

Table 6 reports the average standard deviations and loss as well as the 5th and 95th percentiles of their distribution for the four rules optimized with the benchmark parameters. In this first case, although parameter uncertainty also implies uncertainty about the natural rates, we assume that the central bank correctly perceives the natural rates of output and unemployment, and thus is able to respond to the correct output and unemployment gaps. We now report two different unemployment equivalents: one relative to the benchmark case without parameter uncertainty, and one relative to the optimal policy with parameter uncertainty, reported at the top of Table 6.

We first see that uncertainty has fairly small effects on the performance of the benchmark policy rules: the effects are on average equivalent to a permanent unemployment rate 0.05 percent above the natural rate. However, the range of outcomes is fairly wide, from −0.3 to 0.5. Uncertainty thus introduces risk for the policymaker: the outcome could be better than in the benchmark model, but also substantially worse. The effects of uncertainty on the gap rules are similar to those on the optimal policy. But the difference rules tend to perform even worse on average than in the benchmark case without uncertainty, with even wider ranges, from −0.4 to 0.9.

Comparing the optimized rules with the fully optimal policy, we see that the inefficiencies are larger with parameter uncertainty, especially for the difference rules. Thus, not only are difference rules inefficient in the benchmark model, they are also more sensitive to uncertainty, both on average and in terms of risk.

However, the difference rules are by definition insensitive to uncertainty about the natural rates, while such uncertainty may make gap rules inefficient. For instance, Orphanides and Williams (2002) argue that uncertainty about the natural rates of interest and unemployment is so large as to undermine the efficiency of monetary policy rules responding to the natural rates. Instead, they argue that a difference rule, where the central bank sets the change in the interest rate as a function of the rate of inflation and the change in the rate of unemployment, is robust to natural rate uncertainty, as it does not rely on estimates of the natural rates. Furthermore, in their framework such a rule performs well relative to gap rules also when there is no natural rate uncertainty.

To judge the importance of natural rate uncertainty in our model, we follow Edge et al. (2007) and assume that the central bank responds to deviations of output or unemployment.
from misperceived natural rates, which are estimated using the benchmark (median) pa-
parameter values. Thus, although the true parameters are drawn randomly from the posterior
distribution, the gap rules for monetary policy instead respond to the gaps in terms of the
benchmark natural rates. Edge et al. (2007) show that such natural rate misperceptions have
large effects on the performance of output gap rules, which tend to be dominated by rules
that do not include the natural rates.21

Table 7 reports how the gap rules optimized for the benchmark model perform when we
also allow for natural rate misperceptions. Comparing with Table 6, there are almost no
effects at all from natural rate misperceptions: the volatility of all variables are unchanged,
and the unemployment equivalents increase by at most one tenth of a percentage point.

This is in stark contrast to the results of Orphanides and Williams (2002) and Edge et al.
(2007), where the estimated degree of natural rate misperceptions has a strong impact on the
performance of monetary policy. The small differences in our model are largely a consequence
of the high precision with which we estimate the natural rates. Figures 8 and 9 show that the
probability intervals around the estimated unemployment and output gaps are fairly narrow,
and these intervals mainly depend on filtering uncertainty. Thus parameter uncertainty has
an even smaller effect on the precision of the gap estimates.

## 5.3 Optimized monetary policy under uncertainty

Having evaluated how the monetary policy rules estimated for the benchmark model perform
in the presence of uncertainty about parameters and the natural rates, we now re-optimize
the policy rules to take into account such uncertainty, using 1,000 draws from the parameter
distribution.

Table 8 report the optimized rules under parameter uncertainty and natural rate misper-
ceptions. Given that we above showed that the effects of such uncertainty are small, it is
not surprising to see that the optimized rules under uncertainty are very similar to those in
the benchmark model without uncertainty. The optimized gap rules respond slightly more
aggressively to inflation and have slightly more superinertia, but respond slightly less to the
output or unemployment gap. The difference rules show larger effects, but with the opposite
pattern: the inflation and interest rate coefficients are smaller than in the benchmark rules
while the coefficients on output or unemployment are larger.

Thus, our results are only partly consistent with the traditional Brainard (1967) argu-
ment that parameter uncertainty should make optimal policy less responsive. As shown by
Craine (1979) and Söderström (2002), Brainard’s result does not generalize to uncertainty
concerning all parameters in the model; in particular, uncertainty about the persistence of
the economy may instead lead to more responsive policy. Thus, in general the effects of
parameter uncertainty on optimal policy are ambiguous.

Finally, Table 9 shows the performance of the optimized rules with uncertainty. Compar-
ing with Table 6 reveals that the gains from taking uncertainty into account when designing
monetary policy are very small: the rules optimized for the model with uncertainty lead to

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21 Edge et al. (2007) use a fairly small estimated model to study monetary policy rules that are constructed to
maximize household welfare, thus also capturing the effects of parameter uncertainty on the welfare criterion.
improvements in terms of the unemployment equivalent that are typically around one tenth of a percent relative to the benchmark rules. This is also true for the difference rules, where uncertainty had a slightly larger effect on the optimized coefficients. As before, natural rate misperceptions have no perceptible effect on the performance of the gap rules.

6 Concluding remarks

We have demonstrated that an estimated model with sticky prices, search and matching frictions on the labor market, and staggered nominal wage bargaining fits U.S. data very well, also in terms of labor market variables (unemployment and labor market tightness) that were not used when estimating the model. The implied path for the natural rate of unemployment is very similar to other estimates, for instance, by Staiger et al. (1997, 2002), and the estimated unemployment and output gaps fit remarkably well with the standard view of U.S. business cycle (e.g., comparing with contractions dated by the NBER). This feature of the model is in stark contrast with other estimated DSGE models, e.g., Levin et al. (2005).

We also demonstrated that the trade-off in terms of inflation and unemployment volatility facing the central bank are mainly driven by price markup shocks, and to some extent by technology and wage bargaining shocks. This trade-off is worsened significantly by the presence of wage rigidities.

Using a central bank loss function that is consistent with the mandate of the Federal Reserve, we showed that optimized monetary policy rules are superinertial, that is, the interest rate should respond to the lagged interest rate with a coefficient larger than one. Parameter uncertainty was shown to have small effects on the performance of monetary policy rules and the optimized rules themselves, in parallel with the findings of Levin et al. (2005). However, uncertainty about the natural rates of output and unemployment also has small effects, in contrast to Orphanides and Williams (2002) and Edge et al. (2007). Finally, we showed that in general, monetary policy rules that respond to the output or unemployment gaps clearly dominate rules responding to the growth rates of output and unemployment, also when the natural rates (and therefore the gaps) are misperceived by the central bank.

We want to stress that the results are still preliminary. In particular, we plan to extend the analysis in three dimensions. First, by studying the extent to which our results remain in scenarios where parameter uncertainty is more important. Second, by introducing alternative measures of natural rate uncertainty, for instance, by constructing ex-ante estimates of the natural rates (conditional on the model and the full sample parameter estimates) and incorporating a measure of the natural rate uncertainty facing the central bank in real time. Such an extension would be in line with the work of Orphanides and Williams (2002). Finally, we plan to study model specification uncertainty by including an alternative model of labor market frictions in our analysis, for instance a model similar to that estimated by Smets and Wouters (2007). This way we could discuss the design of robust rules across competing specifications of the labor market model and the cost of basing monetary policy on the wrong model.
References


A Model Appendix

A.1 The Steady State

We first obtain

\[ n = \frac{s}{1 - \rho + s}, \quad (A.1) \]
\[ u = 1 - n, \quad (A.2) \]
\[ x = \frac{su}{n}, \quad (A.3) \]
\[ \chi = \frac{\eta}{\eta + (1 - \eta) \Sigma / \Delta}. \quad (A.4) \]

We then get

\[ r^k = \frac{1}{\bar{\beta}} - 1 + \delta, \quad (A.5) \]
\[ \bar{\beta} = \beta / \gamma_a, \quad (A.6) \]
\[ \frac{k}{a} \frac{y}{a} = \frac{\alpha p^w}{r^k}, \quad (A.7) \]
\[ \frac{i}{a} \frac{y}{a} = \left( 1 - \frac{1 - \delta}{\gamma_a} \right) \gamma_a \frac{k}{a} \frac{y}{a}, \quad (A.8) \]
\[ \frac{k}{a} \frac{n}{a} = \left( \frac{k}{a} \frac{y}{a} \right)^{\frac{1 - \alpha}{\gamma_a}}, \quad (A.9) \]
\[ \frac{y}{a} \frac{n}{a} = \left( \frac{k}{a} \frac{n}{a} \right)^{\alpha}, \quad (A.10) \]
\[ f_n / a = (1 - \alpha) \frac{y}{a} \frac{n}{a}. \quad (A.11) \]

We also have

\[ z = q^k = 1. \quad (A.12) \]

Then \( \kappa \) and \( w \) solve the system

\[
\begin{cases}
\kappa x = p^w (f_n / a) - (w / a) + \beta (\kappa / 2) x^2 + \beta p k x \\
(w / a) = \chi \left[ p^w (f_n / a) + \beta (\kappa / 2) x^2 + \beta s k x \right] + (1 - \chi) \left[ \bar{b} \left( p^w (f_n / a) + \beta (\kappa / 2) x^2 \right) \right],
\end{cases}
\]

where

\[
\bar{b} = \frac{b (k / a)}{p^w (f_n / a) + \beta (\kappa / 2) x^2}
\]

Finally, we obtain

\[
\frac{c}{a} \frac{y}{a} = 1 - \frac{g}{a} \frac{i}{a} \frac{y}{a} - \frac{\kappa x^2 n}{2 y / a}
\]
\[
= 1 - \frac{g}{a} \frac{i}{a} \frac{y}{a} - \frac{\kappa x^2}{2 (y / a) / n}
\]

(A.15)
A.2 The log-linearized model

For the stationary variables, let \( \hat{x}_t = \log x_t - \log x \) denote the log deviation from steady state. For the non-stationary variables, we remove the trend by defining the log deviations as \( \hat{z}_t = \log (z_t/a_t) - \log (z/a) \).

Effective capital
\[
\hat{k}_t + \hat{\varepsilon}^a_t = \hat{z}_t + \hat{k}^p_{t-1}
\] (A.16)

Physical capital dynamics
\[
\hat{k}^p_t = \xi \left[ \hat{k}^p_{t-1} - \hat{\varepsilon}^a_t \right] + (1 - \xi) \left[ \hat{z}_t + \hat{\varepsilon}^a_t \right]
\] (A.17)

where \( \xi = (1 - \delta)/\gamma_a \)

Marginal utility of consumption
\[
(1 - h) \left( 1 - \frac{\beta h}{\gamma_a} \right) \hat{\lambda}_t
= \frac{h}{\gamma_a} [\hat{c}_{t-1} - \hat{\varepsilon}^a_t] - \left[ 1 + \beta \left( \frac{h}{\gamma_a} \right)^2 \right] \hat{c}_t + \frac{\beta h}{\gamma_a} E_t [\hat{c}_{t+1} + \hat{\varepsilon}^a_{t+1}]
+ \left( 1 - \frac{h}{\gamma_a} \right) \left[ \hat{\varepsilon}^b_t - \frac{\beta h}{\gamma_a} E_t \hat{\varepsilon}^b_{t+1} \right]
\] (A.18)

Consumption Euler equation
\[
\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + [\hat{r}_t - E_t \hat{\pi}_{t+1}] - E_t \hat{\varepsilon}^a_{t+1}
\] (A.19)

Capital utilization
\[
\hat{z}_t = \eta_z \hat{r}^k_t
\] (A.20)

where
\[
\eta_z = \frac{a'(z)}{a''(z)} = \frac{1 - \psi_z}{\psi_z}
\]

Investment
\[
\hat{i}_t = \frac{1}{1 + \beta} \left[ \hat{r}_{t-1} - \hat{\varepsilon}^a_t \right] + \frac{1}{(1 + \beta)(1 - \delta)\gamma_a} \left[ \hat{q}^k_{it} + \hat{\varepsilon}^a_t \right] + \frac{\beta}{1 + \beta} E_t \left[ \hat{r}_{t+1} + \hat{\varepsilon}^a_{t+1} \right]
\] (A.21)

where \( \eta_k = \frac{s''}{\gamma_a} \)

Tobin’s q
\[
\hat{q}^k_{it} = \beta (1 - \delta) \gamma_a E_t \hat{q}^k_{it+1} + \left[ 1 - \beta (1 - \delta) \gamma_a \right] E_t \hat{r}^k_{it+1} - [\hat{r}_t - E_t \hat{\pi}_{t+1}]
\] (A.22)

Production function
\[
\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t
\] (A.23)
Unemployment
\[ \hat{u}_t = -\frac{n}{u} \hat{n}_{t-1} \]  (A.24)

Matching
\[ \hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t \]  (A.25)

Transition probabilities
\[ \hat{q}_t = \hat{m}_t - \hat{v}_t \]  (A.26)
\[ \hat{s}_t = \hat{m}_t - \hat{u}_t \]  (A.27)

Aggregate vacancies
\[ \hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_{t-1} \]  (A.28)

Labor market tightness
\[ \hat{\theta}_t = \hat{v}_t - \hat{u}_t \]  (A.29)

Employment dynamics
\[ \hat{n}_t = \rho \hat{n}_{t-1} + (1 - \rho) \hat{m}_t \]  (A.30)

Capital renting
\[ \hat{p}_w^r + \hat{y}_t - \hat{k}_t = r^k_t \]  (A.31)

Aggregate hiring rate
\[ \hat{x}_t = \frac{\epsilon p^w f_n}{a} \left[ \hat{p}_w + \hat{f}_n \right] - \frac{\epsilon w}{a} \hat{w}_t + \beta E_t \hat{x}_{t+1} + \frac{\beta (2 \rho + x)}{2} \left[ E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t \right] \]  (A.32)

where
\[ \epsilon = (\kappa x)^{-1} \]

Marginal product of labor
\[ \hat{f}_{nt} = \hat{y}_t - \hat{n}_t \]  (A.33)

Unemployment benefits
\[ \hat{b}_t = \hat{k}_t^p \]  (A.34)

Nash bargaining weight
\[ \hat{\chi}_t = -(1 - \chi) \left[ \hat{\Sigma}_t - \hat{\Delta}_t \right] \]  (A.35)
Aggregate wage

\[ \hat{\tilde{w}}_t = \gamma_b [\hat{\tilde{w}}_{t-1} - \hat{\tilde{\lambda}}_t + \gamma_w \hat{\tilde{\sigma}}_{t-1} + \hat{\tilde{\sigma}}_t] + \gamma_o \hat{\tilde{\sigma}}_t + \gamma_f [\hat{\tilde{w}}_{t+1} + \hat{\tilde{\lambda}}_{t+1} - \hat{\tilde{\sigma}}_{t+1}] \]  

(A.38)

where

\[ \gamma_b = (1 + \tau_2) \phi^{-1}, \quad \gamma_o = \zeta \phi^{-1}, \quad \gamma_f = (\rho \beta - \tau_1) \phi^{-1}, \]
\[ \phi = 1 + \tau_2 + \zeta + \rho \beta - \tau_1, \quad \zeta = (1 - \lambda_w)(1 - \rho \lambda_w \beta) \lambda_w^{-1}, \]
\[ \tau_1 = \varphi_\Sigma (1 - \rho \lambda_w \beta), \quad \tau_2 = [\varphi_\Sigma \lambda_w - \varphi_\chi (1 - \chi)(1 - \rho)\Psi \Delta^{-1}] \epsilon \Sigma \bar{w}, \]
\[ \Gamma = [1 - \eta(1 - \rho)\Psi] \eta^{-1} \epsilon \Sigma \bar{w}, \quad \Psi = \beta \lambda_w^2 / (1 - \beta \lambda_w), \quad \Sigma = (1 - \beta \lambda_w)^{-1} \]

Spillover-free target wage

\[ \hat{\tilde{w}}_t^o = \varphi_{fn} [\hat{\tilde{w}}_t^o + \hat{\tilde{\lambda}}_t + \varphi_{x} \tilde{E}_t \hat{\tilde{\lambda}}_{t+1} + \varphi_{\Sigma} \tilde{E}_t \hat{\tilde{\sigma}}_{t+1}] \]
\[ \quad + \left( \varphi_{\Sigma} + \frac{\varphi_x}{2} \right) \tilde{E}_t \hat{\tilde{\lambda}}_{t+1} - \hat{\tilde{\lambda}}_t + \varphi_\chi \left[ \hat{\tilde{\lambda}}_t - \beta (\rho - s) \tilde{E}_t \hat{\tilde{\lambda}}_{t+1} \right] + \tilde{\epsilon}_t \]  

(A.39)

where

\[ \bar{w} = w/a, \quad \varphi_{fn} = \chi \rho^w f_n / (a \bar{w}), \quad \varphi_x = \chi \beta (1 - \rho) / (\epsilon \bar{w}), \]
\[ \varphi_{\Sigma} = (1 - \chi) b k / (a \bar{w}), \quad \varphi_{\Sigma} = \chi s / (\epsilon \bar{w}), \]
\[ \varphi_\chi = \chi / [(1 - \chi) \epsilon \bar{w}] \]

Phillips curve

\[ \hat{\tilde{\sigma}}_t = \tau_b \hat{\tilde{\sigma}}_{t-1} + \tau_o \left[ \hat{\tilde{w}}^o_t + \hat{\tilde{\sigma}}^p_t \right] + \tau_f \tilde{E}_t \hat{\tilde{\sigma}}_{t+1} \]  

(A.40)

where

\[ \tau_b = \gamma_p / \phi_p, \quad \tau_o = \zeta_p / (\tau_p \phi_p), \quad \tau_f = \beta / \phi_p, \]
\[ \phi_p = 1 + \beta \gamma_p, \quad \zeta_p = (1 - \lambda_p)(1 - \lambda_p \beta) / \lambda_p, \quad \tau_p = 1 + (\epsilon_p - 1) \xi \]

Government spending

\[ \hat{g}_t = \hat{y}_t + \frac{1 - g_r s_g}{g_r} \tilde{\gamma}_t \]  

(A.41)

Monetary policy rule

\[ \hat{\tilde{r}}_t = \rho_s \hat{\tilde{r}}_{t-1} + (1 - \rho_s) \left\{ r_s \tilde{E}_t \hat{\tilde{r}}_{t+1} + r_y (\hat{y}_t - \hat{y}_{int}) \right\} + \tilde{\varsigma}_t \]  

(A.42)
Resource constraint

\[ \hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + g_y \hat{g}_t + z_y \hat{z}_t + x_y \left[ 2 \hat{x}_t + \hat{n}_{t-1} \right] \] (A.43)

where

\[ c_y = \frac{c/a}{y/a}, \quad i_y = \frac{i/a}{y/a}, \quad g_y = \frac{g/a}{y/a}, \]
\[ z_y = \frac{r_k k/a}{y/a}, \quad x_y = \frac{\kappa x^2 n}{2 y/a}. \]
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital depreciation rate $\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Capital share in production $\alpha$</td>
<td>0.33</td>
</tr>
<tr>
<td>Demand elasticity to market share $\xi$</td>
<td>10</td>
</tr>
<tr>
<td>Government spending ratio $g/y$</td>
<td>0.2</td>
</tr>
<tr>
<td>Matching elasticity to unemployment $\sigma$</td>
<td>0.5</td>
</tr>
<tr>
<td>Survival rate of matches $\rho$</td>
<td>0.895</td>
</tr>
<tr>
<td>Job finding rate $s$</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 2: Prior and posterior distribution of structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Median</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilization rate elasticity $\psi_z$</td>
<td>Beta (0.5,0.1)</td>
<td></td>
<td>0.686</td>
<td>0.603</td>
<td>0.761</td>
</tr>
<tr>
<td>Capital adjustment cost elasticity $\eta_k$</td>
<td>Normal (4,1.5)</td>
<td></td>
<td>2.423</td>
<td>1.639</td>
<td>3.457</td>
</tr>
<tr>
<td>Habit parameter $h$</td>
<td>Beta (0.5,0.1)</td>
<td></td>
<td>0.724</td>
<td>0.672</td>
<td>0.773</td>
</tr>
<tr>
<td>Bargaining power parameter $\eta$</td>
<td>Beta (0.5,0.1)</td>
<td></td>
<td>0.913</td>
<td>0.868</td>
<td>0.946</td>
</tr>
<tr>
<td>Relative flow value of unemployment $\bar{\delta}$</td>
<td>Beta (0.5,0.1)</td>
<td></td>
<td>0.722</td>
<td>0.656</td>
<td>0.79</td>
</tr>
<tr>
<td>Calvo wage parameter $\lambda_w$</td>
<td>Beta (0.75,0.1)</td>
<td></td>
<td>0.718</td>
<td>0.656</td>
<td>0.782</td>
</tr>
<tr>
<td>Calvo price parameter $\lambda_p$</td>
<td>Beta (0.66,0.1)</td>
<td></td>
<td>0.851</td>
<td>0.804</td>
<td>0.887</td>
</tr>
<tr>
<td>Wage indexing parameter $\gamma_w$</td>
<td>Uniform (0,1)</td>
<td></td>
<td>0.806</td>
<td>0.689</td>
<td>0.915</td>
</tr>
<tr>
<td>Price indexing parameter $\gamma_p$</td>
<td>Uniform (0,1)</td>
<td></td>
<td>0.013</td>
<td>0.001</td>
<td>0.055</td>
</tr>
<tr>
<td>Steady-state price markup $\varepsilon^p$</td>
<td>Normal (1.15,0.05)</td>
<td></td>
<td>1.407</td>
<td>1.360</td>
<td>1.455</td>
</tr>
<tr>
<td>Taylor rule response to inflation $r_n$</td>
<td>Normal (1.7,0.3)</td>
<td></td>
<td>2.036</td>
<td>1.916</td>
<td>2.157</td>
</tr>
<tr>
<td>Taylor rule response to output gap $r_y$</td>
<td>Gamma (0.125,0.1)</td>
<td></td>
<td>0.342</td>
<td>0.272</td>
<td>0.421</td>
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<tr>
<td>Taylor rule inertia $\rho_s$</td>
<td>Beta (0.75,0.1)</td>
<td></td>
<td>0.773</td>
<td>0.728</td>
<td>0.810</td>
</tr>
<tr>
<td>Steady-state growth rate $\gamma_a$</td>
<td>Uniform (1,1.5)</td>
<td></td>
<td>1.004</td>
<td>1.003</td>
<td>1.005</td>
</tr>
</tbody>
</table>

For the uniform distribution, the two numbers in parentheses are the lower and upper bounds. Otherwise, the two numbers are the mean and the standard deviation of the distribution.
Table 3: Prior and posterior distribution of shock parameters

<table>
<thead>
<tr>
<th>(a) Autoregressive parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>5%</td>
</tr>
<tr>
<td>Productivity growth rate</td>
<td>$\rho_a$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Preferences</td>
<td>$\rho_b$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Investment-specific technology</td>
<td>$\rho_i$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Price markup</td>
<td>$\rho_p$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\rho_w$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\rho_g$</td>
<td>Beta (0.5,2)</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\rho_r$</td>
<td>Beta (0.5,2)</td>
</tr>
</tbody>
</table>

(b) Standard deviations

<table>
<thead>
<tr>
<th></th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>5%</td>
</tr>
<tr>
<td>Productivity growth rate</td>
<td>$\sigma_a$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Preferences</td>
<td>$\sigma_b$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Investment-specific technology</td>
<td>$\sigma_i$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Price markup</td>
<td>$\sigma_p$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\sigma_w$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\sigma_g$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\sigma_r$</td>
<td>IGamma (0.15,0.15)</td>
</tr>
</tbody>
</table>

The two numbers in parentheses are the mean and the standard deviation of the distribution.

Table 4: Optimized monetary policy rules with benchmark parameters

<table>
<thead>
<tr>
<th>Rule</th>
<th>Coefficient</th>
<th>$r_{\pi}$</th>
<th>$r_{\eta}$</th>
<th>$g_s$</th>
<th>$\rho_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap rules</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.206</td>
<td>0.117</td>
<td></td>
<td></td>
<td>1.054</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.201</td>
<td></td>
<td>−0.00813</td>
<td></td>
<td>1.084</td>
</tr>
<tr>
<td>Difference rules</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.721</td>
<td>1.230</td>
<td></td>
<td></td>
<td>1.132</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.136</td>
<td></td>
<td>−0.0848</td>
<td></td>
<td>1.107</td>
</tr>
</tbody>
</table>

This table shows the optimized coefficients in the monetary policy rules $[67] [71]$ in the benchmark estimated model. The objective function is given by equation $[66]$, with $\lambda_u = 0.003, \lambda_r = 0.08$. 

33
Table 5: Performance of optimized monetary policy rules in the benchmark model

<table>
<thead>
<tr>
<th>Rule</th>
<th>Standard deviation</th>
<th>Loss</th>
<th>Unemployment equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>$x_f$</td>
<td>$x_f^*$</td>
</tr>
<tr>
<td>Optimal policy</td>
<td>2.19</td>
<td>2.20</td>
<td>25.82</td>
</tr>
<tr>
<td>Gap rules</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>2.41</td>
<td>1.71</td>
<td>22.38</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2.40</td>
<td>2.01</td>
<td>24.49</td>
</tr>
<tr>
<td>Difference rules</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>2.32</td>
<td>4.60</td>
<td>47.64</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2.28</td>
<td>3.59</td>
<td>36.84</td>
</tr>
</tbody>
</table>

This table shows the unconditional standard deviations for key variables and the value of the loss function in the benchmark estimated model under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 4. Bars denote annualized values (four times quarterly values); the “unemployment equivalent” is the permanent deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the optimal policy to the optimized rule.
Table 6: Evaluating optimized monetary policy rules in the model with parameter uncertainty

<table>
<thead>
<tr>
<th>Rule</th>
<th>Standard deviation</th>
<th>Loss</th>
<th>Unemployment equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>$x^r_t$</td>
<td>$x_t$</td>
</tr>
<tr>
<td>Optimal policy</td>
<td>Mean</td>
<td>2.24</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>1.91</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.59</td>
<td>2.83</td>
</tr>
<tr>
<td>Gap rules</td>
<td>Mean</td>
<td>2.47</td>
<td>1.74</td>
</tr>
<tr>
<td>Output</td>
<td>5%</td>
<td>2.13</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.85</td>
<td>1.92</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Mean</td>
<td>2.46</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.14</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.84</td>
<td>2.28</td>
</tr>
<tr>
<td>Difference rules</td>
<td>Mean</td>
<td>2.35</td>
<td>4.92</td>
</tr>
<tr>
<td>Output</td>
<td>5%</td>
<td>2.12</td>
<td>3.70</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.59</td>
<td>6.44</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Mean</td>
<td>2.33</td>
<td>3.91</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.05</td>
<td>2.79</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.64</td>
<td>5.36</td>
</tr>
</tbody>
</table>

This table shows the unconditional standard deviations for key variables and the value of the loss function as averages and 5th and 95th percentiles over 5,000 draws from the posterior parameter distribution under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 4. Bars denote annualized values (four times quarterly values); the “unemployment equivalents” are the permanent deviation of the unemployment rate from a natural rate of 6 percent that are equivalent in terms of loss to moving either from the benchmark model to the model with parameter uncertainty or from the optimal policy to the optimized rule.
Table 7: Evaluating optimized monetary policy rules in the model with parameter uncertainty and natural rate misperceptions

<table>
<thead>
<tr>
<th>Rule</th>
<th>Standard deviation</th>
<th>Loss</th>
<th>Unemployment equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>$x_t^y$</td>
<td>$x_t^u$</td>
</tr>
<tr>
<td>Optimal policy</td>
<td>Mean</td>
<td>2.24</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>1.91</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.59</td>
<td>2.83</td>
</tr>
</tbody>
</table>

**Gap rules**

| Output                | Mean               | 2.47  | 1.74   | 22.89 | 2.73      | 8.26    | 0.076   |
|                       | 5%                 | 2.13  | 1.58   | 20.30 | 2.38      | 6.40    | -0.30   |
|                       | 95%                | 2.87  | 1.91   | 26.26 | 3.12      | 10.89   | 0.54    |

| Unemployment          | Mean               | 2.46  | 2.04   | 24.87 | 2.60      | 8.45    | 0.069   |
|                       | 5%                 | 2.14  | 1.83   | 22.52 | 2.26      | 6.56    | -0.31   |
|                       | 95%                | 2.85  | 2.27   | 27.73 | 2.98      | 11.10   | 0.53    |

This table shows the unconditional standard deviations for key variables and the value of the loss function as averages and 5th and 95th percentiles over 5,000 draws from the posterior parameter distribution under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 4 when the natural rate of output or unemployment are misperceived by the central bank. Bars denote annualized values (four times quarterly values); the “unemployment equivalents” are the permanent deviations of the unemployment rate from a natural rate of 6 percent that are equivalent in terms of loss to moving either from the benchmark model to the model with parameter uncertainty and natural rate misperceptions or from the optimal policy to the optimized rule.

Table 8: Optimized monetary policy rules with parameter uncertainty and natural rate misperceptions

<table>
<thead>
<tr>
<th>Rule</th>
<th>Coefficient</th>
<th>$r_u$</th>
<th>$r_y$</th>
<th>$g_u$</th>
<th>$\rho_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gap rules, parameter uncertainty only</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td>0.213</td>
<td>0.114</td>
<td></td>
<td>1.056</td>
</tr>
<tr>
<td>Unemployment</td>
<td></td>
<td>0.203</td>
<td></td>
<td>-0.00790</td>
<td>1.085</td>
</tr>
<tr>
<td><strong>Gap rules, parameter uncertainty and natural rate misperceptions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td>0.213</td>
<td>0.113</td>
<td></td>
<td>1.058</td>
</tr>
<tr>
<td>Unemployment</td>
<td></td>
<td>0.203</td>
<td></td>
<td>-0.00789</td>
<td>1.085</td>
</tr>
<tr>
<td><strong>Difference rules</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td>0.695</td>
<td>1.277</td>
<td></td>
<td>1.127</td>
</tr>
<tr>
<td>Unemployment</td>
<td></td>
<td>0.124</td>
<td></td>
<td>-0.0910</td>
<td>1.100</td>
</tr>
</tbody>
</table>

This table shows the optimized coefficients in the monetary policy rules in the model with parameter uncertainty and natural rate misperceptions, using 1,000 draws from the estimated posterior parameter distribution. The objective function is given by equation 66, with $\lambda_u = 0.003$, $\lambda_r = 0.08$. 36
Table 9: Performance of optimized monetary policy rules with parameter uncertainty and natural rate misperceptions

<table>
<thead>
<tr>
<th>Rule</th>
<th>Standard deviation</th>
<th>Loss</th>
<th>Unemployment equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\pi_t)</td>
<td>(x^\gamma_t)</td>
<td>(x^\delta_t)</td>
</tr>
<tr>
<td>Optimal policy</td>
<td>Mean</td>
<td>2.24</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>1.91</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.59</td>
<td>2.83</td>
</tr>
<tr>
<td>Gap rules, parameter uncertainty only</td>
<td>Output</td>
<td>Mean</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.11</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.83</td>
<td>1.98</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Mean</td>
<td>2.44</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.12</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.81</td>
<td>2.33</td>
</tr>
<tr>
<td>Gap rules, parameter uncertainty and natural rate misperceptions</td>
<td>Output</td>
<td>Mean</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.11</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.82</td>
<td>1.98</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Mean</td>
<td>2.44</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.11</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.82</td>
<td>2.33</td>
</tr>
<tr>
<td>Difference rules</td>
<td>Output</td>
<td>Mean</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.16</td>
<td>3.63</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.65</td>
<td>6.37</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Mean</td>
<td>2.38</td>
<td>3.76</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>2.09</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>2.70</td>
<td>5.22</td>
</tr>
</tbody>
</table>

This table shows the unconditional standard deviations for key variables and the value of the loss function as averages and 5th and 95th percentiles over 1,000 draws from the posterior parameter distribution under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 4. Bars denote annualized values (four times quarterly values); the “unemployment equivalents” are the permanent deviations of the unemployment rate from a natural rate of 6 percent that are equivalent in terms of loss to moving either from the benchmark model to the model with parameter uncertainty and/or natural rate misperceptions or from the optimal policy to the optimized rule.
Figure 1: Autocovariance functions of aggregate demand variables in U.S. data and in the estimated model

This figure shows the autocovariance functions of the growth rates of output, investment, and consumption in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.
This figure shows the autocovariance functions of the growth rates of output and the real wage, and the level of employment (total hours per capita) in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.
Figure 3: Autocovariance functions of monetary policy variables in U.S. data and in the estimated model

This figure shows the autocovariance functions of output growth, inflation, and the federal funds rate in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.
This figure shows the rate of unemployment in U.S. data (total unemployment/population) and in the estimated model over the period from 1960 to 2004. The U.S. data is measured as the percent deviation from the sample mean, while the model data is measured as the percent deviation from steady state (normalized to zero).

This figure shows the degree of labor market tightness (unemployment/vacancies) in U.S. data and in the estimated model over the period from 1960 to 2004. The U.S. data is measured as the percent deviation from the sample mean, while the model data is measured as the percent deviation from steady state (normalized to zero).
Figure 6: Autocovariance functions of unemployment with other variables in U.S. data and in the estimated model

This figure shows the autocovariance functions of the rate of unemployment with the growth rates of output, investment, consumption and the real wage, and the rate of employment (total hours per capita), inflation, the federal funds rate, and with itself at different leads and lags. Solid lines represent U.S. data, dashed lines represent the estimated model (5th and 95th percentiles over 500 draws from the posterior parameter distribution with 100 simulated samples of 160 observations for each draw).
This figure shows the estimated path for the actual and natural rates of unemployment over the period from 1960 to 2004. The data is measured as the percent deviation from steady state (normalized to zero). For the natural rate, the solid line is the median and the dashed lines are the 5th and 95th percentiles, taking into account parameter uncertainty and Kalman filter uncertainty.
This figure shows the estimated unemployment gap, that is, the percent deviation of the actual rate of unemployment from the natural rate of unemployment, over the period from 1960 to 2004. The solid line is the median and the dashed lines are the 5th and 95th percentiles, taking into account parameter uncertainty and Kalman filter uncertainty.
This figure shows the estimated output gap, that is, the percent deviation of the actual level of output from the natural level of output, over the period from 1960 to 2004. The solid line is the median and the dashed lines are the 5th and 95th percentiles, taking into account parameter uncertainty and Kalman filter uncertainty.
Figure 10: Efficient policy frontiers in benchmark model

This figure shows the standard deviations of annualized inflation and the unemployment gap (the percent deviation of unemployment from the natural rate) under the fully optimal policy in the benchmark model as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner). Panel (a) shows the frontier with all shocks, Panels (b) and (c) show the frontiers for each shock in the model.
This figure shows the standard deviations of annualized inflation and the unemployment gap (the percent deviation of unemployment from the natural rate) under the fully optimal policy in the model with flexible wages as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner). Panel (a) shows the frontier with all shocks (including also the benchmark model with sticky wages), Panels (b) and (c) show the frontiers for each shock in the model.
Figure 12: Sensitivity of efficient policy frontiers to labor market parameters

(a) Relative flow value of unemployment

(b) Job finding and survival rates

This figure shows the standard deviations of annualized inflation and the unemployment gap (the percent deviation of unemployment from the natural rate) under the fully optimal policy for alternative parameterizations of the labor market as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner).