Inflation Dynamics with Search Frictions: A Structural Econometric Analysis*

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October 30, 2007

Abstract

The New Keynesian Phillips curve explains inflation dynamics as being mainly driven by current and expected future real marginal costs. In competitive labor markets, these would be proxied by the labor share. We explore in this paper the role of the labor market variables that are implied by search frictions in the labor market. Allowing for search frictions, real marginal cost should also incorporate the cost of generating and maintaining long-term employment relationships, along with conventional measures, such as real unit labor costs. In order to construct a synthetic measure of real marginal costs, we use newly available labor market data on worker finding rates that reflect hiring costs. The measure turns out to be highly correlated with the labor share. Estimates of the New Keynesian Phillips curve reveal that inflation dynamics are mainly driven by forward-looking behavior. Interestingly, for the period 1985 to 2005, we find that the our real marginal cost measures are procyclical, while from the 1960s, they are countercyclical. General equilibrium analysis of our model suggests which shocks must have driven fluctations to generate the observed behavior of real marginal costs.

*Paper prepared for the Carnegie Rochester Conference on Public Policy, November 9-10, 2007. The views expressed in this paper are those of the authors and should not be interpreted as those of the Deutsche Bundesbank, Federal Reserve Bank of Richmond, the Federal Reserve System, or the Board of Governors. We thank seminar participants at the Kiel Conference ´The Phillips Curve and the Natural Rate of Unemployment', the Econometric Society Summer Meetings at Duke University, the SCE conference in Montreal, the Catholique Universite de Louvain, and the Banque de France conference on 'Wage bargaining, employment, monetary and economic policies' for helpful comments on an earlier version of the paper.

1 Introduction

This paper explores the determinants of real marginal cost fluctuations when there are search frictions in the labor market. Without such frictions, or any other type of labor adjustment costs, real marginal costs are identical to unit labor costs, that is, competitively set wages relative to productivity. Not only do search frictions imply that wages are somewhat insulated from competitive pressures. They are also a particular form of labor adjustment costs that are determined by aggregate labor market conditions, rather than being internal to the firm. They give rise to long-term employment relationships because both firm and worker save on future search costs by keeping their match alive. This dual role of search frictions motivates our interest in how they change the nature of real marginal costs. In turn, real marginal cost are the key determinant of inflation dynamics in business cycle models with monopolistic price setting.

We illustrate the linkages between inflation, real marginal cost and the labor market in a standard new Keynesian model with search and matching frictions in the labor market.[1] We first use the model to derive an equation for real marginal cost that can be linked to observable labor market data under an appropriate timing assumption on labor flows. Our strategy to generate a synthetic time series for real marginal costs is to calibrate the parameters multiplying the labor market variables. We then use newly available labor market data for 1960 to 2005. on job finding rates, vacancies, unemployment and wages, to construct the series. This in turn forms the basis of our estimation of the hybrid new Keynesian Phillips curve from the model. In a second step, we simulate the full general equilibrium model, calibrated with the same parameters, to further understand the interaction between labor market variables and inflation.

We find that search frictions do indeed matter for inflation dynamics, in that they tend to reduce the role of backward-looking price setting for generating persistence, and by changing the sensitivity of inflation to real marginal costs. But it turns out that the synthetic measure of real marginal costs is fairly closely related to the labor share. Interestingly, we find that, among the variables that matter for real marginal costs, the real wage has become more volatile since the eightes, even though

[1] Here we follow a literature that has adopted the labor market model by Mortensen and Pissarides (1994) and Pissarides (2000) into dynamic general equilibrium frameworks, such as Merz (1995), Andolfatto (1996), and Den Haan, Ramey and Watson (1999) in real models, and Walsh (2005), Trigari (2006), and Krause and Lubik (2007) for monetary models.

consumption is less volatile.[2] Further, real marginal costs have become procyclical from the eighties.while they are countercyclical for the whole sample.

The general equilibrium perspective allows us to understand how real marginal cost, the labor share, and labor market variables are related. Here we make use of the cross-equation restrictions that arise in general equilibrium, which puts additional constraints on the joint dynamics of the marginal costs components. The impulse responses partly confirm our empirical findings. Depending on the shocks driving fluctuations in the model, such as technology and monetary shocks, labor share and marginal costs are negatively or positively correlated. This suggests which shocks may have been the dominant driving forces of inflationary dynamics. For many shocks, we find that the dynamics of the labor market drive a wedge between the labor share and real marginal costs. The dynamics most plausible from an empirial viewpoint are monetary shocks.

Finally, we explore extensions such as real wage rigidity and internal labor adjustment costs. With rigidity in wages per worker, the labor market becomes somewhat more volatile, but real marginal costs and inflation are barely affected.This mirrors the findings of Krause and Lubik (2007) who show in a similar model that higher rigidity in wages generates more volatile hiring costs in the frictional labor market, which leave the overall volatility of real marginal cost almost unchanged. Labor adjustment costs smooth fluctuations of employment, but do not much affect real marginal cost dynamics.

We proceed as follows. The next section describes the full new Keynesian DSGE model. Firms simultaneously choose hiring and prices given hiring and price adjustment costs. Households supply labor at both the intensive and extensive margins: workers search in order to find employment, and when employed, they supply labor and earn wages determined in bilateral Nash bargaining. Wages are fully flexible. In section 3, we derive an equation for real marginal costs that shows the explicit link with the labor market variables implied by search frictions. Section 4 presents construction of a real marginal cost series using calibrated parameters, and conducts the GMM estimation. In section 5, we take the general equilibrium perspective, analysing the behavior of the key labor market variables. Here we also discuss extensions of the model, such as rigid wages. Section 6 concludes.

[2] A fact know as the 'great moderation'.

2 The model

The economy consists of firms, households, and a government. Households are represented by an aggregate household that chooses its members' consumption to maximize aggregate welfare. Thus there is insurance against consumption risk. Employment is determined by the hiring behavior in the frictional labor market, and hours worked are the result of bargaining between worker and firm. Firms in turn hire workers, separate from them at an exogenous rate, and choose the price of their differentiated product in a monopolistically competitive market. The demand derives from household preferences according to a Dixit-Stiglitz CES specification. The government issues a one period bond, spends income on goods, and levies a lump-sum tax.

2.1 Households

We assume that each household i in the economy seeks to maximize intertemporal utility

$$\mathcal{W}(n_{it}) = \max E_t \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \left[\frac{(c_{it} - \varsigma C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \chi \varepsilon_t^n n_{it} \frac{h_{it}^{1+\mu}}{1+\mu} \right]$$

where $\beta \in (0,1)$ is the discount factor, and we allow that the household utility depends on the average consumption of its members (relative to external – aggregate – habits, C_{t-1}) and the hours supplied by the members of the household, h; where n represents the numbers of members of the household, i.e. the extensive margin.[3] Consumption is a CES aggregate of differentiated products bought from monopolistic competitors. Notice that by introducing time dependence in preferences we aim at improving the match of the model to the hump-shaped and persistent responses of output to nominal and real shocks found empirically.[4] The parameter $\sigma > 0$ governs risk aversion; and ς reflects the importance of habits. Notice that under $\varsigma = 0$ this preferences correspond to the standard CRRA preferences. Leisure is valued ($\chi > 0$), and the paramter $\mu \geq 0$ is the inverse of the Frisch labor supply elasticity. The labor market will be considered in more detail later on. Finally, we allow household welfare to be affected by both an intertemporal preference shock, and an intratemporal labor supply shock.

The household's flow-budget constraint for period t is given by:

[3] Note that in equilibrium, hours supplied will be identical across employed members of the household.

[4] See for instance, Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), and Fuhrer (2000).

$$C_{it} + \frac{B_{it}}{P_t} = R_{t-1}\frac{B_{it-1}}{P_t} + w_t n_{it} h_{it} + (1 - n_{it})b + D_{it} + T_t$$

where households enter period t with bonds B_{t-1}, that pay and gross interest rate R_{t-1}. At the beginning of the period, they receive lump-sum nominal transfers T_t, labor income $w_t n_{it} h_{it}$, where w_t denotes the real wage, and a nominal dividend D_t from the firms. The term $(1-n_{it})b$ denotes income of unemployed household members ($u_t = 1 - n_{it}$) which might also be interpreted as total output of a home production sector where the technology parameter $b > 0$, alternatively this parameter will capture the access to unemployment benefits by some members of the household. P_t is an aggregate price level that will be defined later.

The first-order conditions with respect to bonds and consumption imply:

$$\begin{aligned}\lambda_{it} &= \varepsilon_t^b (c_t - \varsigma C_{t-1})^{-\sigma} \\ 1 &= \beta E_t \left[R_t \frac{\lambda_{it+1}}{\lambda_{it}} \frac{P_t}{P_{t+1}} \right]\end{aligned}$$

where λ_{it} represents the marginal utility of consumption, and hence ε_t^b can also be interpreted as an exogenous shifter to λ_{it} (see, e.g. Hall (2007)). In general equilibrium, this condition governs the stochastic discount factor used in the firms' problem. Due to perfect risk sharing, the sole problem of the household is thus to determine the consumption path of its members. There is no explicit household labor supply choice. It is chosen at the firm level during negotiations. We now turn to the specification of the firm's problem.

2.2 Firms and the labor market

We assume that there is a continuum of firms of measure one. Each firm is a monopolistic competitor and produces a differentiated good. Let P_{jt} and Y_{jt} denote nominal price and output for firm j, and P_t and Y_t be the corresponding aggregate values. A firm's output is sold in a monopolistically competitive market with demand, derived from consumer preferences, given by:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon_t} Y_t, \quad (1)$$

where $Y_t = \left(\int_0^1 Y_{jt}^{(\epsilon_t - 1)/\epsilon_t} dj \right)^{\epsilon_t/(\epsilon_t - 1)}$. The parameter $\epsilon_t > 1$ represents the elasticity of substitution between differentiated products and is subject to stochastic variation.

Accordingly, $P_t = \left(\int_0^1 P_{jt}{}^{1-\epsilon_t}\, dj\right)^{\frac{1}{1-\epsilon_t}}$ is the consumption-based, aggregate price index. Finally, a firm produces its differentiated good using n_{jt} workers according to the following technology:[5]

$$Y_{jt} = \varepsilon_t^A \left(n_{jt} h_{jt}\right)^{\alpha}, \tag{2}$$

where ε_t^A is an aggregate productivity shocks, and $0 < \alpha < 1$.

During period t, the firm sets its nominal price P_{jt}, subject to the requirement that it satisfy demand at that price. Following Rotemberg (1982), the firm faces a quadratic cost of adjusting its nominal price between periods, measured in terms of aggregate output and given by

$$\mathcal{P}_{jt} = \frac{\psi}{2}\left(\frac{1}{\widetilde{\pi}_{t-1}} \frac{P_{jt}}{P_{jt-1}} - 1\right)^2 Y_t, \tag{3}$$

with $\psi > 0$ controlling the importance of the price adjustment costs, and $\widetilde{\pi}_{t-1} = \pi_{t-1}^{\gamma} \pi^{1-\gamma}$; the parameter $0 < \gamma < 1$ governs the degree of backward-lookingness of the price setting, and finally π represents steady-state inflation.[6] This cost function penalizes deviations of the firms price change from an average between past aggregate inflation π_{t-1} and steady-state inflation, π. When, in particular, $\gamma = 0$, then price setting is purely forward-looking, in the sense that it is costless for firms to increase their prices in line with steady-state inflation. When, on the other hand, $\gamma = 1$, price setting is purely backward-looking, in the sense that it is costless for firms to increase their prices in line with the previous period's actual rate of inflation. Interestingly, this formulation yields a Phillips curve analogous to the one deriving from Calvo-price setting with backward-looking firms, as in Galí and Gertler (1999); or with backward indexation, as in Christiano, Eichenbaum and Evans (2005).

The labor market is subject to search frictions. To form new employment relationships, workers must search and firms must post vacancies. In line with the literature, we assume that the total number of new matches M_t is produced by the aggregate matching function:

$$M_t = \bar{m} \varepsilon_t^M u_t^{\xi} v_t^{1-\xi}, \tag{4}$$

which gives the number of new employment relationships available at the beginning of period $t+1$. In the previous expression, u_t represents the size of the unemployment

[5] For the purpose of this paper, we abstract from capital accumulation since it does not alter the expression of the marginal costs. See Rotemberg and Woodford (1999).

[6] See Ireland (2006). As Ireland shows, steady state inflation is identical to the monetary authority's inflation target.

pool (the measure of non-employed who search), v_t is the total number of vacancies posted (search activity of firms); the constant $\bar{m} > 0$ is match efficiency, and $0 < \xi < 1$ is the elasticity of the matching function with respect to unemployment. Finally, ε_t^M will capture exogenous changes in the matching function.

This matching function is homogeneous of degree one, increasing in each of its arguments, concave, and continuously differentiable. Homogeneity implies that a vacancy gets filled with probability $q(\theta) = \frac{M(u,v)}{v} = M(1, \frac{1}{\theta}) = \bar{m}\varepsilon_t^M \theta^{-\xi}$, which is decreasing in the degree of labor market *tightness* $\theta \equiv v/u$. Analogously, an unemployed worker finds a job with probability $p(\theta) = \frac{M(u,v)}{u} \equiv \theta q(\theta)$, which is increasing in θ.[7] We assume that the new matches at firm j at the beginning of period $t+1$ going to a firm are proportionate to the ratio of its vacancies to total vacancies, v_{jt}/v_t, so that $v_{jt}M_t/v_t = v_{jt}q(\theta)$ is hiring by firm i.

The evolution of employment at firm j can be written as:

$$n_{jt} = (1-\rho)\left[n_{jt-1} + v_{jt-1}q(\theta_{t-1})\right], \tag{5}$$

where ρ is the exogenous separation rate of existing employment relationships, which includes previous employees and the number of new hires.[8] In order for a firm to post vacancies v_{jt}, it has to pay a concave flow cost $c(v_{jt})$. Allowing for $c'' < 0$ follows Rotemberg (2006) and departs from the standard search and matching model where cost of recruiting are assumed to be linear (Pissarides, 2000). As emphasized by Rotemberg (2006), if this cost is interpreted as the cost of advertising openings in an information source it can easily be subject to economies of scale at the firm level.[9] Thus, labor adjustment costs are given by the following expression:[10]

$$\mathcal{N}_{jt} = c(v_{jt}) \tag{6}$$

Finally, the total hirings of a firm in period t depend on last period's search in the labor market and the probability that the match survives, i.e. $(1-\rho)\theta_{t-1}q(\theta_{t-1}) =$

[7] Instead of (4) we could have used: $M_t = \frac{u_t v_t}{[(u_t)^\eta + (v_t)^\eta]^{\frac{1}{\eta}}}$, as in den Haan et al. (2000). Although, the main advantage of this matching function, relative to the Cobb-Douglas specification (4), is that guarantees matching probabilities between zero and one for all u, and v, both specifications have identical implications for the log-linearized version of the model considered in this paper.

[8] All separated workers are assumed to reenter the unemployment pool, thus we abstract from workers' labor force participation decisions.

[9] Note that in models where firms consist of only one worker, the assumption of returns to scale in vacancy posting would be immaterial.

[10] Others components of the labor adjustment costs are considered in a later section.

$(1-\rho)\bar{m}\varepsilon_t^M\theta_{t-1}^{1-\xi}$. Thus, as in Pissarides (2000) the finding rate depends positively on the ratio of vacancies posted by the firm to unemployment. [11]

Firms produce differentiated goods in a monopolistically competitive product market and they maximize the present value of discounted flow profits:

$$\mathcal{J}_t(n_{jt}) = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\left(\frac{P_{jt}}{P_t} \right)^{1-\epsilon_t} Y_t - w_{jt} n_{jt} h_{jt} - \mathcal{N}_{jt} - \mathcal{P}_{jt} \right], \tag{7}$$

with respect to P_{jt}, n_{jt}, and $v_{jt,}$ subject to the constraint that demand (1) equals production (2), the employment constraint (5) and the labor adjustment and price adjustments costs, i.e. variables $\mathcal{N}_{jt}$ and $\mathcal{P}_{jt}$, respectively. The discount factor $\beta^t \lambda_t$ derives from consumer preferences in the presence of perfect capital market and is taken as exogenous by the firms.

The first-order conditions for prices, employment, and vacancies are given by the next set of equations:

$$\psi \left(\frac{\pi_{jt}}{\widetilde{\pi}_{t-1}} - 1 \right) \frac{\pi_{jt}}{\widetilde{\pi}_{t-1}} = E_t \beta_{t+1} \psi \left(\frac{\pi_{jt+1}}{\widetilde{\pi}_t} - 1 \right) \frac{\pi_{jt+1}}{\widetilde{\pi}_t} \frac{Y_{t+1}}{Y_t} + \\ + \left[(1-\epsilon_t) \frac{P_{jt}}{P_t} + \epsilon_t mc_{jt} \right] \left(\frac{P_{it}}{P_t} \right)^{-\epsilon_t}, \tag{8}$$

$$\mu_{jt} = mc_{jt} \alpha \varepsilon_t^A n_{jt}^{\alpha-1} h_{jt}^{\alpha} - w_{jt} h_{jt} + \frac{c'(v_{jt})}{q(\theta_t)}, \tag{9}$$

$$\frac{c'(v_{jt})}{q(\theta_t)} = (1-\rho) E_t[\beta_{t+1} \mu_{jt+1}], \tag{10}$$

where $\beta_{t+1} = \beta \lambda_{t+1}/\lambda_t$ is a stochastic discount factor, μ_{jt} is the Lagrange multiplier associated to the employment constraint and it represents the current-period value of workers for the firm.[12] Intuitively, firms expand employment up the point where the benefit from employing an additional worker is equal to μ_{jt}, and they hire such such that the cost of posting a vacancy is equal to its expected benefit, shown on the right hand side, which consists of the net flow profit per worker $(mc_{jt} \alpha \varepsilon_t^A n_{jt}^{\alpha-1} h_{jt}^{\alpha} - w_{jt} h_{jt})$ and a measure of the future value of the job $(c'(v_{jt})/q(\theta_t))$. This measure is determined by the job creation condition (10), which relates the cost of posting a vacancy to the expected present value of the job.[13] Finally, the multiplier mc_{jt} on the constraint

[11] In Krause, Lopez-Salido and Lubik (2007) we consider the effects of endogenous separations. In this case, the finding rate also depends on how many new matches actually turn into productive jobs.

[12] This is not to be confused with the Frisch labor supply elasticity μ, which is not subscripted.

[13] Notice that in the model the job creation margin, i.e vacancy posting, is an intertemporal margin of employment adjustment.

that demand equals production is the contribution of an additional unit of output to the firm's revenue and is equal to the firm's real marginal cost.

2.3 Wage determination

Along with labor adjustment costs, the behavior of wages is key for the determination of real marginal cost because it affects the hiring incentives of firms. We assume, as in most of the labor search literature, that worker and firm bargain at the individual level over the joint surplus of their match, according to the Nash bargaining solution. Bargaining takes place both over hours per worker and the wage, to maximize

$$\left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right)^{\eta}\left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}\right)^{1-\eta}, \tag{11}$$

where the two terms are the marginal contribution of the worker to the representative households welfare, and the present value of profits of the firm, respectively. The parameter η reflects the bargaining power of the worker. The two resulting optimality conditions are a wage equation

$$w_t h_t = \eta\left(mc_t \alpha A_t n_t^{\alpha-1} h_t^{\alpha} + \theta_t c'(v_{it})\right) + (1-\eta)\left(b + \chi\frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n \frac{h_t^{1+\mu}}{1+\mu}\right).$$

and a labor supply equation

$$h_t = \left(\frac{mc_t \alpha^2 \varepsilon_t^a n_t^{\alpha-1}}{\chi \varepsilon_t^n \varepsilon_t^b}\lambda_t\right)^{\frac{1}{1+\mu-\alpha}}.$$

The first equation is the familiar wage equation, as can be found in Mortensen and Pissarides (1994), and Pissarides (2000). It expresses the total wage payment to the worker a weighted average between the marginal revenue product of the worker plus the cost of replacing the worker, and the outside option of the worker plus the marginal disutility of labor, at the level of hours worked, h_t. The bargaining weight determines how close the wage is to either the marginal product or to the outside option of the worker.

The second condition determines the wage from a condition that equalizes the marginal product of labor to the worker's marginal rate of substitution between leisure and consumption:

$$\text{mrs}_t = \chi\frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n h_t^{\mu} = mc_t \alpha^2 A_t n_t^{\alpha-1} h_t^{\alpha-1} = \text{mpl}_t.$$

Thus hours are chosen as in a competitive labor market, maximizing the joint welfare of worker and firm. However, the choice of hours is independent of the wage. The condition also helps us understand the driving forces of hours variation in the search model. A higher marginal utility of wealth, λ, and a higher marginal product of labor all increase labor supplied, whereas it falls whenever the disutility of labor or the intertemporal preference increase. In models where firms choose the amount of hours supplied (such as in the right-to-manage setup of Trigari, 2005, and Christoffel and Linzert, 2006), workers supply whichever hours are demand by firms who equate the bargained wage with the marginal product of labor input.

2.4 Closing the model

The government budget constraint is:

$$R_{t-1}\frac{B_{t-1}}{P_t} = \frac{B_t}{P_t} + T_t$$

where T_t is a transfer, and B_t is the aggregate of bonds held by the public. In our baseline model, the central bank is assumed to set the nominal interest rate R_t at every period according to a simple interest rate rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi^*}\right)^{r_\pi} \left(\frac{Y_t}{Y^*}\right)^{r_Y}\right]^{1-\rho_r} \varepsilon_t^R \tag{12}$$

where $0 < \rho_r < 1$ captures interest rate smoothing, and where $r_\pi \geq 0$ and $r_Y \geq 0$ and where the asteriscs represent the steady state value of the corresponding variable. An interest rate rule of the form (12) corresponds to the family of Taylor rules (Taylor, 1993).

We assume a symmetric equilibrium throughout, which entails identical choices for all variables. Defining aggregates as the averages of firm specific variables, we have that $1 - u_t = n_t = n_{jt} = \int_0^1 n_{jt}dj$, and $v_t = v_{jt} = \int_0^1 v_{jt}dj$. Furthermore, as $P_{jt} = P_t$, $Y_{jt} = Y_t$, for all t and j. Thus all firms produce the same amounts of output, employ equal amounts of labor, and, in particular, face the same marginal costs mc_t. Finally, using the household budget contraint, firms profits, and the government constraint, the resulting aggregate income identity is $Y_t = C_t + G + c(v_t)$. Output is used for consumption, government spending, and for posting vacancies. See the Appendix for a summary of the models equilibrium conditions and constraints, its steady state, as well as specification of the stochastic variables.

3 Real marginal costs and Search Frictions

3.1 Baseline Model

In a symmetric equilibrium, using the first-order conditions (9) and (10) it follows that:

$$\mu_t = mc_t \alpha \frac{Y_t}{n_t} - W_t + (1-\rho) E_t \beta_{t+1} \left[\mu_{t+1} \right], \tag{13}$$

where $W_t = w_t h_t$ denotes total labor costs. It is useful to The key equation for understanding the role of search frictions in the labor market is obtained by rewriting equation (13):

$$mc_t = \frac{W_t}{\alpha(Y_t/n_t)} + \frac{\mu_t - (1-\rho) E_t \beta_{t+1} \left[\mu_{t+1} \right]}{\alpha(Y_t/n_t)} \tag{14}$$

In the presence of search frictions, a firm's real marginal cost has two components, unit labor costs (the labor cost over the marginal product of labor), plus a correction for the shadow hiring costs of a worker relative to the expected value of the additional worker next period. Real marginal revenue (equal to real marginal cost in equilibrium) need to cover each period the wage plus the average hiring cost per period. Thus, the real marginal costs are equal to the wage plus the average hiring cost of a job relative to the saved hiring cost from not needing to hire that worker tomorrow. This latter term is the hiring cost saved by the fact that the worker is in employment today, and using expression (10) can be related to $\frac{c'(v_t)}{q(\theta_t)}$, i.e. expression (14) can be written as follows:

$$mc_t = \frac{W_t}{\alpha(Y_t/n_t)} + \frac{\mu_t - \frac{c'(v_t)}{q(\theta_t)}}{\alpha(Y_t/n_t)} \tag{15}$$

In a more compact form, the previous expression can be written as

$$mc_t = s_t(1 + x_t), \tag{16}$$

where $s_t = \frac{W_t N_t}{\alpha Y_t}$ represents real unit labor costs (the labor income share divided by α, the elasticity of output to employment), and

$$x_t = \frac{1}{W_t} \left[\mu_t - c'(v_t)/q(\theta_t) \right], \tag{17}$$

captures the effects of labor adjustment costs relative to the real wage. In the absence of labor market frictions $mc_t = s_t$. This is familiar from new Keynesian models with competitive labor markets: real marginal costs are proportional to the labor share, $S_t = \alpha s_t$. In the steady state, expression (8) implies that real marginal cost, mc, is a constant that solely depends on the elasticity of demand, ϵ. That is $mc = (1 - 1/\epsilon)$, which turn is the inverse of the steady-state markup. This is a standard implication of monopolistic competition. In addition, it then follows:

$$(1 - \frac{1}{\epsilon})MPL = W(1 + x), \tag{18}$$

where $MPL = \alpha\frac{Y}{n}$ is the marginal product of labor. This equation shows that the benefit of hiring an additional employee – the marginal revenue product of labor – equals the the marginal cost of adjusting labor that include the hiring and firing decisions. Thus (18) is our analogous representation of the expression (19) in Rotemberg (2006).

From an empirical point of view, a crucial implication of the baseline model is that the current value of the job μ_t, which is unobservable, is a key determinant of the marginal costs, and thus it can no longer be directly linked to observables, but would require a structural model to be determined. Alternatively, under the specifications in recent papers by Blanchard and Galí (2006) and Rotemberg (2006), this problem is circumvented by appropriate timing assumptions. We turn now to this analysis.[14]

3.2 Blanchard-Galí-Rotemberg Approach

Blanchard and Galí (2006) have analyzed a version of the New Keynesian model with search and matching frictions that differs from our baseline specification in that *hiring* is instantaneous. That is, vacancies are assumed to be filled immediately by paying the hiring cost, which is assumed to be a function of labor market tightness. This implies that hiring optimal decisions are not determined by an intertemporal condition. Jobs are destroyed at the fixed rate ρ and employment evolves as:

$$n_{jt} = (1 - \rho)n_{jt-1} + H_{jt}. \tag{19}$$

Current-period employment depends on last period's employment that survives the separation shock, and current period hiring. Hiring, H_{jt}, is given by $v_{jt}q(\theta_t)$, as before. Hiring costs per firm are $H_{jt}G_t$, where $G_t = Bh_t^{\delta}$, with $\mathfrak{h}_t = H_t/U_t$ and $\delta \geq 0$. B is a positive constant satisfying $\rho B < 1$. G_t and $q(\theta_t)$ are taken as given by firms. Thus firm j hiring costs are $\mathcal{N}_{jt} = v_{jt}q(\theta_t)G_t$. The first-order condition for employment is:

$$B\mathfrak{h}_t^{\delta} = mc_t\alpha\frac{y_t}{n_t} - W_t + (1 - \rho)E_t\beta_{t+1}B\mathfrak{h}_{t+1}^{\delta},$$

[14]In Krause, Lopez-Salido and Lubik (2007) we discuss the implications of endogenous separations for the measurement of the real marginal costs.

which can be used to write the marginal costs as in our general expression (16), where now,

$$x_t = \frac{1}{W_t}[B\mathfrak{h}_t^{\delta} - (1-\rho)E_t\beta_{t+1}B\mathfrak{h}_{t+1}^{\delta}] \tag{20}$$

As emphasized by Blanchard and Galí, the first term in x_t captures the cost of hiring a marginal employed worker, while the second relates to the savings in hiring costs resulting from the reduced hiring needs in period $t+1$.[15] In our setup, $\mathfrak{h}_t = H_t/U_t = \theta_t q(\theta_t) = m\theta_t^{1-\xi}$, so that $\mathfrak{h}_t^{\delta} = m\theta_t^{(1-\xi)\delta}$.

As mentioned above, Rotemberg (2006) uses the large firm assumption for the purpose of motivating increasing returns to vacancy posting at the firm level. To see the effects of this assumption, notice that the cost of posting v_{jt} can be specified by the following, in principle, non-linear function:

$$c(v_{jt}) = c_v v_{jt}^{\epsilon_c}$$

where $\epsilon_c \leq 1$, and $\epsilon_c = 1$ corresponds to the linear case discussed by Pissarides (2000). Crucially, Rotemberg assumes that the hiring costs are incurred one period later, and that aggregate conditions in $t+1$ are observed at the end of period t, *before* vacancies are chosen. Essentially, this amounts to hiring taking place contemporaneously, as in Blanchard and Galí above. Therefore, we write from the outset the following evolution of employment[16]

$$n_{jt} = (1-\rho)n_{jt-1} + v_{jt}q(\theta_t)$$

The difference to our baseline setup is that new jobs are not affected by job destruction.

The first-order condition for this setup is therefore

$$\frac{\epsilon_c c_v v_t^{\epsilon_c - 1}}{q(\theta_t)} = mc_t\alpha\frac{y_t}{n_t} - W_t + (1-\rho)E_t\beta_{t+1}\frac{\epsilon_c c_v v_{t+1}^{\epsilon_c - 1}}{q(\theta_{t+1})}$$

From the previous expressions follows that the only difference to Blanchard and Galí (2006) is due to the specification of the returns to scale in vacancy posting. This expression can be used to write the marginal costs as in our general expression (16) with

$$x_t = \frac{1}{W_t}\left[B\theta_t^{\xi}v_t^{\epsilon_c - 1} - (1-\rho)E_t\beta_{t+1}B\theta_{t+1}^{\xi}V_{t+1}^{\epsilon_c - 1}\right] \tag{21}$$

where $B = \frac{\epsilon_c c_v}{m}$. Note that Rotemberg assumes that the households' utility of consumption is linear, so that $\beta_t = \beta$, for all t. For $\epsilon_v = 1$, and $\mathfrak{h}_t = \theta_t^{1-\xi} \Leftrightarrow \theta_t^{\xi} = \mathfrak{h}_t^{\xi/(1-\xi)}$,

[15] To avoid potential confusion, note that our $\mathfrak{h}_t$ corresponds to their $x_t = H_t/U_t$.

[16] In Rotemberg (2006), vacancies and labor market tightness would be timed $t-1$.

with $\delta = \xi/(1-\xi)$, and this expression is equivalent to the formulation above, i.e. Blanchard and Galí (2006) is identical to Rotemberg (2006). The new element is the negative dependence of hiring costs on v_t, arising from returns to scale in vacancy posting when $\epsilon_c < 1$. It is worth mentioning that contrary to the expression (17), expressions (20) and (21) have two distinctive features. First, the extra-term depends positively on the current hiring, vacancies and labor market tightness and negatively upon the expected values. This implies that the cyclical behavior of the marginal cost is modified in different forms depending upon the form and timing of both firing and hiring costs. Second, the last two expressions require the specification of a stochastic discount factor, β_{t+1}. Both issues are discussed next.

3.3 The Cyclical Behavior of Marginal Costs

In this section we show that a first-order approximation to the real marginal cost can be expressed as a function of the real unit labor costs – as in the standard model without search – plus terms that arise due to the presence of search frictions. Thus, a log-linear approximation around the steady state of the marginal costs (16) using expression (20) yields:[17]

$$\widehat{mc}_t = \widehat{s}_t + \frac{1-\phi}{1-\widetilde{\beta}}\left[\frac{\xi}{1-\xi}(\widehat{\mathfrak{h}}_t - \widetilde{\beta}E_t\widehat{\mathfrak{h}}_{t+1}) - \widetilde{\beta}E_t\widehat{\beta}_{t+1} - (1-\widetilde{\beta})\widehat{w}_t\right] \tag{22}$$

where $\phi = \frac{s}{mc} = \frac{1}{1+x}$, $\widetilde{\beta} = \beta(1-\rho)$. It is straightforward to see that in a Walrasian labor market, that is, when $mc = s$, then $\phi = 1$ and hence $\widehat{mc}_t = \widehat{s}_t$. This corresponds to the baseline specification in Rotemberg and Woodford (1999) and Galí and Gertler (1999). Notice also that the marginal costs is affected by the stochastic discount factor, $E_t\widehat{\beta}_{t+1} = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t$, where $\widehat{\lambda}_t$ corresponds to the marginal utility of consumption. From the viewpoint of the unemployed, the index $\mathfrak{h}_t$ has two alternative interpretations. First, it may represent the probability of being hired in period t, or, in other words, the job-finding rate. Second, it could be an index of labor market tightness, i.e. defined as the ratio of aggregate hires to the unemployment rate.

In a similar way, using expression (21) we can easily obtain a log-linear approximation of the marginal costs in the Rotemberg's model, i.e.:

$$\widehat{mc}_t = \widehat{s}_t + \frac{1-\phi}{1-\widetilde{\beta}}\left[\frac{\xi}{1-\xi}(\widehat{\mathfrak{h}}_t - \widetilde{\beta}E_t\widehat{\mathfrak{h}}_{t+1}) + (\epsilon_c - 1)(\widehat{v}_t - \widetilde{\beta}E_t\widehat{v}_{t+1}) - (1-\widetilde{\beta})\widehat{w}_t\right] \tag{23}$$

[17] We use the caret '^' to denote log-deviation form the steady state, i.e. for any variable x_t, $\widehat{x}_t \equiv \ln \frac{x_t}{x}$.

which is identical to expression (22) under $\epsilon_c = 1$. When $\epsilon_c < 1$, then higher labor market tightness arises when the future value of jobs is higher, firms perceive lower marginal costs. When wages are higher, the labor share will be higher. In addition, real marginal costs rise by less than the labor share because higher wages make hiring costs lower relative to labor costs. Since marginal cost are the shadow value of relaxing the output constraint, which in turn depend on the shadow value of relaxing the employment constraint, this latter value is part of marginal costs. Finally, given the linear preferences in consumption assumed by Rotemberg (2006), the marginal costs (23) are not affected by the discount factor.

Calibration We now study the properties of real marginal costs, the main driver of inflation dynamics, based on the derivations above. We calibrate the parameters of the model and use data on labor market variables to generate the implied marginal cost series. We then describe the statistical properties of this series, and, in particular, contrast this with marginal cost series and their proxies that have typically been used in empirical studies.

In the calibration exercise, each period is assumed to correspond to a quarter. Table 1 describes the values of the parameters we use for the construction of alternative measures of marginal cost as well as the corresponding steady state. With regard to preference parameters, the benchmark value of the relative risk aversion parameter, σ, is set equal to 1 (as in Blanchard-Galí (2006)), although we also consider the case of linear preferences as in Shimer (2005) and Rotemberg (2006). We set the discount factor $\beta = 1.03^{-\frac{1}{4}}$ which implies a 3 percent annual real interest rate. We keep the steady state labor income share, S, equal to 0.64 as in Cooley and Prescott (1995), and the (quarterly) steady-state rate of exogenous and endogenous separation $\rho = 0.05$, a value consistent with the summary of the evidence recently presented by Yashiv (2006), slightly lower than 0.034 of Den Haan et al (2000) but lower than 0.086, a value recently used by Merz and Yashiv (2007). We set $\epsilon = 21$ as our benchmark value, which implies a steady state markup 5 percent which is consistent with the evidence presented by Basu and Fernald (1997). That is, the firm's steady state gross markup, $\mu_p = mc^{-1} = \frac{\epsilon}{\epsilon - 1} = 1.05$. Finally, we set the short run elasticity of output to labor $\alpha = 0.68$.

Using this calibration, as follows from the steady state expression (18) for a given steady state marginal cost over price –inverse of the steady state markup– and the steady state labor share it follows a steady state value for the marginal recruiting costs over the marginal product, i.e. $\phi = 0.98$, a value in line with the recent calibration considered by Blanchard and Galí (2006) and Rotemberg (2006). Finally, given the calibration of $\beta = 1.03^{-\frac{1}{4}}$, and $\rho = 0.05$, then the discount factor $\widetilde{\beta} = 0.943$, so the contribution of the labor market variables to the fluctuations of the real marginal costs becomes small, i.e. $\frac{1-\phi}{1-\widetilde{\beta}} = 0.012$.

We conduct some sensitivity analysis below that focuses on the calibration of the elasticity of the matching function with respect to the vacancies $(1-\xi)$, the concavity of the hiring costs (ϵ_c). The elasticity of the matching function with respect to the vacancies determine how job-finding rate responds to changes in its driving forces, it is relevant also since it determines the sensitivity of marginal costs to the tightness ratio and the finding rate. Thus, the lower $1-\xi$ the higher is the sensitivity of marginal costs variations to the previous labor market variables. We set the elasticity of the matching function with respect to vacancies, $1-\xi$, equal to 0.5 as our benchmark value. This value is in line with the upper bound of the range reported by Petrongolo and Pissarides (2001) in their review of the literature on the matching function, and it has been recently used by Blanchard and Galí (2006) and Mortensen and Nagypal (2006). Nevertheless we also consider the alternative value 0.3 in line with the estimates by Shimer (2005) and that constitutes a lower bound upon the available estimates. Regarding the elasticity of vacancy creation, ϵ_c, we follow Rotemberg (2006) and consider two values for the elasticity of recruiting costs; our baseline corresponds to the one advocated by the previous author, i.e. 0.2. As an alternative calibration we follow Pissarides (2000) and assuming that the recruiting costs are linear in the vacancies posted, i.e. $\epsilon_c = 1$.

Results Figure 1 presents a brief summary of some basic stylized facts about unemployment, vacancies and the finding rates.[18] As shown in the top panel, the unemployment rate is strongly countercyclical, and sometimes with large fluctuations. Vacancies (measured as the help-wanted index) are even more strongly procyclical, so that the vacancy-unemployment ratio (labor market tightness) is procyclical (see the bottom panel of Figure 1. Finally, as also shown at the bottom panel of Figure 1 the correlation between the labor market tighness and the finding rate is very high (0.9), so that recessions are period where there is a substantial fall in the probability of finding a job or are periods where the vacancy-unemployment ratio is low relative to its average level.

[18] A complete description of the data used in the paper can be found in the Appendix.

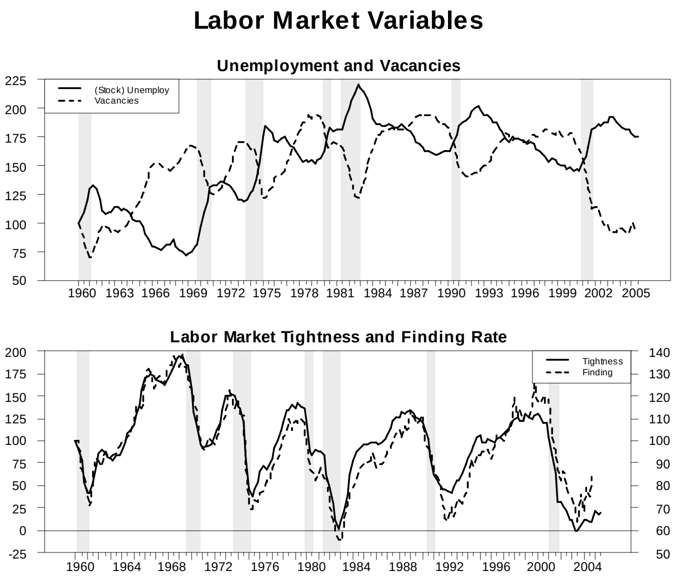


Figure 1: Some Labor Market Variables

Figure 2 depicts our measure of marginal cost, mc_t, and real unit labor cost s_t in the Blanchard-Gali model, we also plot the three main components of the marginal cost associated to the presence of search frictions, i.e. expression (22): the contribution of the expected changed in the finding rates $(\frac{1-\phi}{1-\widetilde{\beta}}\frac{\xi}{1-\xi}(\widehat{\mathfrak{h}}_t - \widetilde{\beta}E_t\widehat{\mathfrak{h}}_{t+1}))$, the contribution of the stochastic discount factor $(\frac{1-\phi}{1-\widetilde{\beta}}\widetilde{\beta}E_t\widehat{\beta}_{t+1} = \frac{1-\phi}{1-\widetilde{\beta}}\widetilde{\beta}[E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t])$, and the contribution of the cyclical component of the real wage $((1-\phi)\widehat{w}_t)$.[19]

The top left compares the typical marginal cost proxy in the New Keynesian Phillips-curve literature, i,e. the real unit labor cost and the marginal costs associated with search frictions in the Blanchard and Gali model. As the figure shows, the two series are very similar. At first glance, it appears that the influence of search and matching frictions on inflation dynamics is not very strong. The two series comove closely, with similar turning points, and exhibit similar persistence and volatility. From the 1980s, though, the new series is somewhat less volatile and smoother. This impression is not substantially altered the elasticity of the matching function with respect to vacancies, $(1-\xi)$ to 0.7 (see Figure 3), although the alternative calibration reduces the volatility of the marginal costs; or when the representation of the real

[19]The trend component was obtained using the HP filter method with a smoothing parameter $\lambda = 100000$.

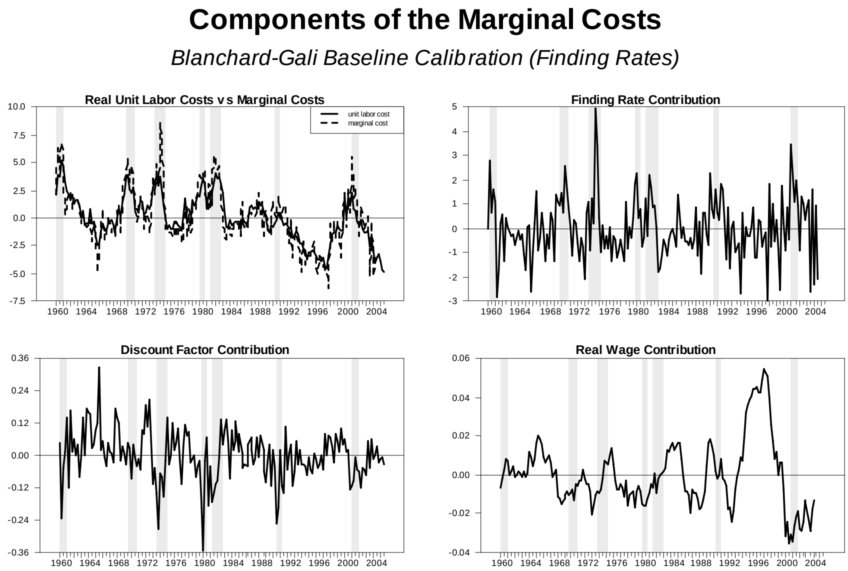


Figure 2: Real Marginal Costs and Search Frictions.

marginal cost equation uses the tightness ratio instead of the job-finding rate – which is the result of the high correlation between the two series show in Figure 1.

The reason for this result is illustrated in the three panels of Figure 2. As can be seen, the contribution of the stochastic discuount factor and the cyclical variation of the real wage are negligible relative to the variability of the unit labor costs. An interesting pattern is worth mention regargind these two components. Consistent with the 'great moderation' period, after mid 80s the reduction in the variability of consumption growth reduces the contribution of the stochastic discount factor to the variation in the marginal costs. Notwithstanding, the variation in the real wage is somewhat higher, but is contribution is still very small.

Table 1 reports some basic statistics underlying the visual evidence in Figures 2 and 3. In particular, the Table reports a set of second moments for the quadractically detrended (log) output – a common indicator of the business cycle, the real unit labor costs and two measures of the marginal costs for the Blanchard-Gali (mc^{BG}) model and the Rotemberg (mc^{R}) model, respectively. Note first that the percent standard deviation of the marginal costs are larger than the one of detrended output and the real unit labor costs.[20] In the Rotemberg model, where the marginal costs

[20]Furthermore, the role of job separations as a means to smooth hiring is eliminated. See also

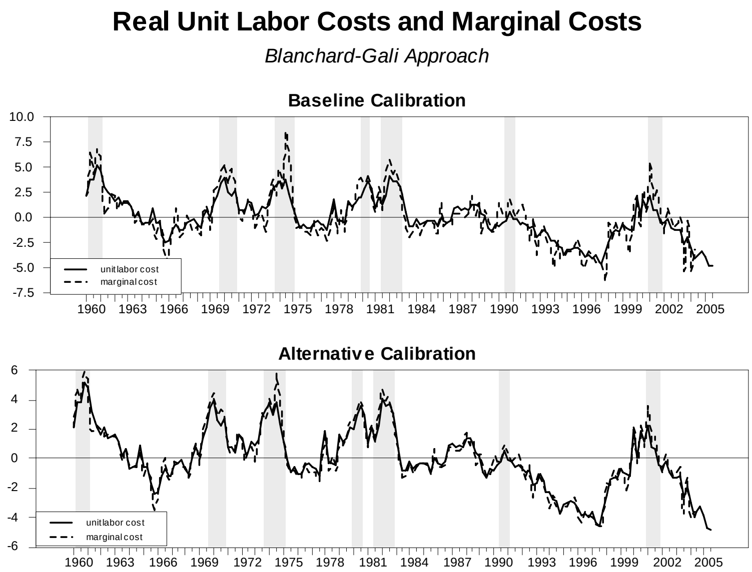


Figure 3: The Real Marginal Costs under an Alternative Calibration.

depend explicitly on the variation in the cost of vacancies, due to a concave cost of adjustment, we accordingly obtain a reduction in the variation of the marginal costs. We will return to this aspect below.

In addition, the departures of marginal costs from steady state are persistent, but less than the persistence of output and the real unit labor costs. In addition, over the first part of the sample the marginal costs are somewhat negatively correlated with the detrended output, while over the second the marginal costs are more procyclical. In this sense, for the second half of the sample period, the presence of search frictions enhances the countercyclical movement in the price markup by making marginal cost more procyclical (e.g. Rotemberg and Woodford (1999).

In Figure 4 we display the robustness of our previous results to the use of alternative specifications of the marginal costs consistent with the Rotemberg'specification. Thus, relative to the Blanchard and Gali measure, the marginal costs inherit the effects of the expected changes in vacancies depending of the value of the the elasticity of vacancy creation, ϵ_c. The bottom panel of Figure 4 presents the time series of this component, i.e. according to expression (23), the contribution of vacancies

Krause, Lopez-Salido and Lubik (2007) for more details on the computation of the marginal costs in models where the separation rate is endogenous.

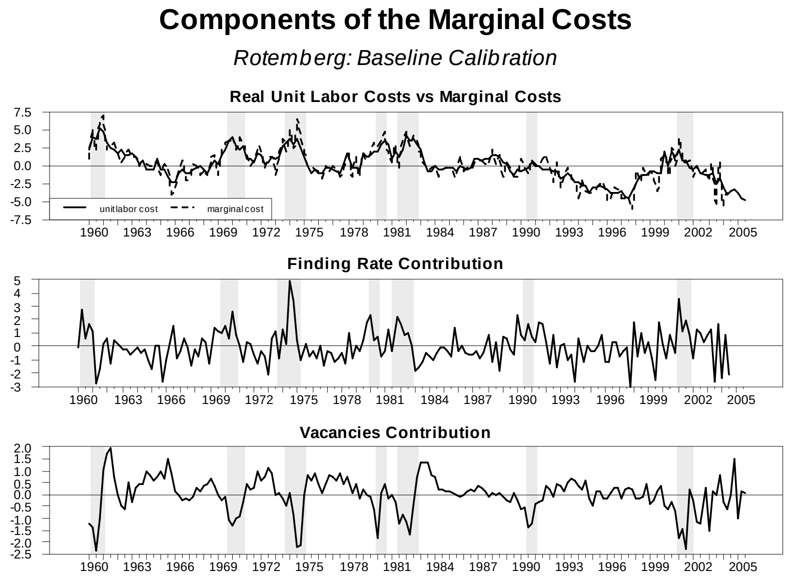


Figure 4: Real Marginal Costs. The Role of Vacancies.

corresponds to $\frac{(1-\phi)(\epsilon_c-1)}{1-\widetilde{\beta}}(\widehat{v}_t - \widetilde{\beta} E_t \widehat{v}_{t+1})$. Overall, the central result from our previous analysis is robust to this alternative. Both the volatility and the persistence of this new measure are slightly lower (see Table 1).

Table 1. Basic Statistics (Baseline Calibration

	Sample : 1960 − 2005					*Sample* : 1985 − 2005				
Variable	s.d.(%)	ρ	Cross Correlation			s.d.(%)	ρ	Cross Correlation		
			s	mc^{BG}	mc^R			s	mc^{BG}	mc^R
y	2.95	.95	-0.12	-0.10	-0.14	1.93	.95	0.30	0.28	0.21
s	2.09	.92	1	0.83	0.84	1.78	.91	1	0.76	0.75
mc^{BG}	2.62	.69		1	0.97	2.27	.53		1	0.97
mc^R	2.39	.63			1	2.08	.45			1

To summarize, we find that adding search and matching frictions in the labor market appears to affect slightly to the cyclical behavior of marginal costs in terms of comovement, persistence and volatility. A typical proxy measure for real marginal costs, such as unit labor costs, behaves similarly. This does not, however, allows us to conclude that these measures have no substantial effects on inflation dynamics. This we investigate now along two dimensions. First, we look at the correlation between inflation and marginal cost using a limited information approach, as in Gali and Gertler (1999). Second, we analyze both theoretically and empirically how the presence of search frictions affect inflation dynamics using a general equilibrium perspective.

4 Inflation and Marginal Costs: A Limited Information Approach

In this section we extend Gali and Gertler (1999) analysis and we pursue a limited information approach aimed at presenting some estimates of the NKPC using a GMM approach when the marginal costs include the effects of labor market search frinctions. Later we use a bayesian full information-approach to estimate the parameters of the general equilibrium model above described.

We begin by noticing that a log-linear approximation of the price setting condition (8) yields the familiar New Keynesian Phillips curve, which describes inflation as driven by lag inflation, expected future inflation, and real marginal cost:

$$\hat{\pi}_t - \gamma\hat{\pi}_{t-1} = \beta E_t[\hat{\pi}_{t+1} - \gamma\hat{\pi}_t] + \kappa\ \widehat{mc}_t - \frac{1}{\psi}\widehat{\epsilon}_t \tag{24}$$

where $\hat{\pi}_t$ is price inflation expressed as a log deviation from steady state, $\widehat{mc}_t$ represents real marginal cost, and $\widehat{\epsilon}_t$ represents exogenous variations in the markup associated to changes in the elasticity of demand. Since we allow for partial indexation to lagged inflation, current inflation is affected by inflation in the previous, where the parameter γ is the indexation parameter. Finally, the pass-through from marginal costs to inflation, κ, is a function of the elasticity of demand, ϵ, and the price adjustment cost parameter ψ. Notice that the slope coefficient $\kappa = \frac{\epsilon-1}{\psi}$ pins down the price adjustment cost parameter, given a value for the elasticity of demand. The previous expression can be rewritten in a more familiar form as a hybrid New Keynesian Phillips curve:

$$\widehat{\pi}_t = \gamma_f E_t\widehat{\pi}_{t+1} + \gamma_b\widehat{\pi}_{t-1} + \frac{\kappa}{1+\beta\gamma}\widehat{mc}_t - \frac{1}{\psi 1+\beta\gamma}\widehat{\epsilon}_t, \tag{25}$$

where $\gamma_f = \beta/(1+\beta\gamma)$, and the parameter on past inflation $\gamma_b = \gamma/(1+\beta\gamma)$. As in the original model of Galí and Gertler (1999), expression (25) corresponds to the hybrid New Keynesian Phillips curve. First, when $\gamma = 0$, the model corresponds to Rotemberg's (1982) original contribution, so that the model reduces to the purely forward looking New Keynesian Phillips curve.

In this paper we deviate from our previous paper (Krause, Lopez-Salido and Lubik (2007)) where we use equation (25) to define the set of orthogonality conditions for all t: $E_t\{(\pi_t - \gamma^b\ \pi_{t-1} - \gamma^f\ E_t\{\pi_{t+1}\} - \frac{\kappa}{1+\beta\gamma}\ \widehat{mc}_t)\ \mathbf{z}_t\} = 0$, such that they define orthogonality conditions that can estimate the model using generalized method of moments (GMM).

Here we rewrite equation (24) as a relationship between inflation and the expected discounted value of the future values of real marginal cost and relative import prices:[21]

$$\widehat{\pi}_t = \gamma\widehat{\pi}_{t-1} + \kappa\sum_{k=0}^{\infty}\beta^k E_t\left[\widehat{mc}_{t+k} + \varphi\widehat{\epsilon}_{t+k},\right]. \quad (26)$$

To estimate our model using (26), we need to forecast real marginal cost. In doing so we use an autoregressive model. Defining $\widehat{mc}_t = \rho_{mc}\ \widehat{mc}_{t-1} + u_t$, where $0 < \rho_{mc} < 1$ and the innovation u_t is iid innovation that is uncorrelated to $\widehat{\epsilon}_{t+k}$.[22] Hence, it is straightforward to compute the forecasts as follows: $E_t\widehat{mc}_{t+k} = \rho_{mc}^k\ \widehat{mc}_t$, and the equation for inflation that we estimate is:

$$\hat{\pi}_t = \gamma\hat{\pi}_{t-1} + \frac{\kappa}{1-\beta\rho_{mc}}\widehat{mc}_t + \epsilon_{\pi t}, \quad (27)$$

Our estimation consists in the join estimation of the inflation equation and the AR(1) process for the marginal costs. Since the exogenous variation in markups may be correlated with our measures of marginal costs, we use lagged variables as instruments. Our benchmark set of instruments includes one lag inflation, one lag of marginal costs, two lags of the output gap – measured as the deviation of NFB sector output from a quadratic trend.[23]

[21] The thodology closely parallels the present-value approach used in the empirical finance literature.

[22] We used the Box-Jenkins methodology to pind down the best AR model for marginal costs.

[23] Since it is possible that our instruments are only weakly correlated with the endogenous variables in our model, we follow Stock, Wright and and Yogo (2002) and Stock and Yogo (2005) and check for the presence of weak instruments based on the g_{min} statistic of Cragg and Donald (1993). We compare this statistic against the critical values compiled by Stock and Yogo (2005), who show how to test for the presence of weak instruments based on this test statistic.

Table 2. GMM Estimates: 1985-2005

Blanchard-Gali Specification

	γ	κ	γ_b	ω	J_T
Finding Rates					
Baseline Calibration	0.635 (0.119)	0.026 (0.013)	0.389 (0.045)	0.854 (0.038)	6.26 (0.044)
Alternative Calibration	0.682 (0.124)	0.013 (0.007)	0.407 (0.045)	0.895 (0.029)	6.55 (0.044)
Using Tightness					
Baseline Calibration	0.692 (0.124)	0.0085 (0.005)	0.410 (0.043)	0.915 (0.025)	6.97 (0.044)
Alternative Calibration	0.698 (0.124)	0.0062 (0.0037)	0.406 (0.044)	0.927 (0.022)	7.17 (0.044)

Note: in all cases the dependent variable is quarterly inflation measured using GDP Deflator. Sample Period: 1985:I-2004:IV. Standard errors are shown in brackets. Instrument set includes two lags of detrended output, and one lag of real marginal costs. The hazard rates are from Shimer (2005). The results remains unchanged under alternative hazard rates.

In Table 2 and 3 we present the results for the hybrid model over the period 1985:I 2005:IV, for the specification of the marginal costs under the Blanchard-Gali model and Rotemberg, respectively. We distinguish the baseline calibration of the marginal costs from the alternative calibration, as specified in the previous section. Finally, we also present the robustness of the results to alternative ways of calculating the marginal costs, i.e. using information on the finding rates or using information based upon the time series of the tightness ratio. The first two columns report the estimates of the two primitive parameters, γ, and the slope coefficient κ. The next two columns present the corresponding parameters associated with these two parameters. First, the fourth column present the backward looking parameter, γ_b, obtained from the value of γ and our calibration of the discount factor β. Second, the following columm presents the index of price rigidity (i.e. the implicit probability of changing price in each quarter) associated to the estimated slope coefficient. Finally, we present the J_T test of overidentifying restrictions and below its corresponding p-value in parenthesis.

The degree of indexation is well estimated across all the specifications and it ranges between 0.6 and 0.7. Hence, even if the forward-looking component is slightly more relevant, the backward-looking component plays a significant role on inflation dynamics with a value for the coefficient γ_b around 0.4. These estimates are fairly stable across specifications, and are in line with the previous results by Gali, Gertler, and Lopez-Salido (2005).

The slope coefficient on the marginal costs is significant but somewhat less precisely estimated, and implies that the estimated probability of changing prices, i.e., the duration of prices being fixed, slightly larger than the one estimated in the literature. In particular, as shown by Sbordone (1998), the Rotemberg model of price rigidity due to firms facing convex adjustment costs of adjusting their price is observationally equivalent to a model based on Calvo, where the price rigidity is determined by a random draw of the firms that are allowed to change prices. Hence, the slope coefficient $\kappa = \frac{\epsilon-1}{\psi}$ under the first interpretation and it is equal to $\kappa = \frac{(1-\beta\omega)(1-\omega)}{\omega}$ under the second one, where ω represents the implicit probability of changing prices. In the Tables we have computed the implicit value of ω, for a value of β and the estimated slope coefficient, κ. For the baseline calibration, the Blanchard-Gali model implies a value or $\omega = 0.85$, which implies a high degree of price inertia. The values are even higher for the specification of the marginal costs using Rotemberg-type of specification.

Table 3. GMM Estimates: 1985-2005

Rotemberg Specification

	γ	γ_b	κ	ω	J_T
Finding Rates					
Baseline Calibration	0.704 (0.129)	0.007 (0.0039)	0.414 (0.044)	0.925 (0.022)	6.87 (0.044)
Alternative Calibration	0.682 (0.124)	0.007 (0.0040)	0.416 (0.045)	0.895 (0.029)	6.72 (0.044)
Using Tightness					
Baseline Calibration	0.692 (0.124)	0.007 (0.0041)	0.417 (0.046)	0.915 (0.025)	6.70 (0.044)
Alternative Calibration	0.698 (0.124)	0.007 (0.0041)	0.416 (0.045)	0.927 (0.022)	7.68 (0.044)

Note: in all cases the dependent variable is quarterly inflation measured using GDP Deflator. Sample Period: 1985:I-2004:IV. Standard errors are shown in brackets. Instrument set includes two lags of detrended output, and one lag of real marginal costs. The hazard rates are from Shimer (2005). The results remains unchanged under alternative hazard rates.

5 Inflation and Marginal Costs in General Equilibrium

This section takes the perspective of dynamic general equilibrium, to see how real marginal cost and the labor share comove when we take aggregate constraints into account. Note that now the labor share and labor market variables are linked by the assumptions we make on wage determination, which we could conveniently ignore in the empirical analysis above. Conditional on the Nash bargaining assumption, labor market tightness directly affects the real wage per worker. As is well-known from the results of Shimer (2005) and Hall (2005), the implied flexibility of wages under this assumption is too high for the standard search and matching framework to generate realistic volatilities of labor market variables. Thus, this basic mechanism will carry over in our model. In the extensions below, we will explore real wage rigidity.

We first present the simulations of our baseline model in terms of the impulse responses, and implied aggregate statistics. Or focus is on the response to technology and monetary shocks. We discuss how the predicted correlations between labor share and real marginal costs differ depending on the shocks driving fluctuations. Finally, we extend the model to include real wage rigidity, and/or additional labor adjustment costs. While the former help us move towards generating more realistic labor market dynamics, the latter are a natural complement to search frictions for generating additional real marginal cost dynamics. We follow the formulation of Rotemberg and Woodford (1999), which has recently also been used by Sveen and Weinke (2007).

5.1 Baseline model

To simulate the baseline model, we need to make additional assumptions on the paramters not specified for the empirical analysis. For habits ς, we assume a value of 0.7, as used in other studies and found in empirical analysis.[24] The costs of vacancy creation c_v are set to 0.5. There is no empirical counterpart for this, but it is a value implied by the requirement that the steady state of the model implies realistic outcomes, such as the observed unemployment and match probabilities. The matching function parameter is set to $\bar{m} = 0.5$ for the same reasons, and the outside option $b = 0.2$, which implies a replacement ratio of the aggregate real wage of about 30 percent. The Frisch elasticity of labor supply is set at $\mu = 1.5$, and the weight on the disutility of labor is $\chi = 0.5$. Finally, the backward looking component of inflation is

[24] See, for example, Smets and Wouters (2003).

set at $\gamma = 0.4$, in line with our estimates. We consider two main shocks, technology and monetary, for the impulse responses.[25] The coefficients of the Taylor rule are set to $\rho_r = 0.7$, $r_\pi = 1.5$, and $r_Y = 0.25$. The model is solved based on the methods intoduced by Sims (1999).

5.1.1 Impulse responses

We illustrate the general equilibrium dynamics for technology, monetary shocks, and preference shocks. The other three shocks (markup and matching function) exhibit very weak propagation. Further, the labor supply shock implies almost identical dynamics as the technology shock.

First consider the technology shock in Figure 5. The graph shows in the first row the impulse responses of the main macro-economic aggregates, inflation, output, and unemployment. While inflation falls – due to the immediate decline in unit labor costs – output rises, and unemployment falls. The second row shows the driving forces of inflation based on the decomposition introduced in this paper. The drop in real marginal costs brings about the drop in inflation. Due to the backward looking dynamics of inflation, inflation stays below steady state for longer than the real marginal cost. Next comes the labor share, which moves in the opposite direction. This is driven by the strong increase in wages following the technology shock, shown in the first panel in the third row.

The reason that real marginal costs do not rise as well is reflected in the behavior of our variable x. Remember that x is largely driven by $\mu_t - (1-\rho)\beta E_t\mu_{t+1}$, the quasi-change in the value of the job μ. One can see in the last panel how this change behaves. After an initial upward jump, and a small additional increase, μ is falling. This is mirrored by the initital drop in x, then further decline, and then subsequent rise. As long as μ is falling back to steady state, the gap $\mu_t - (1-\rho)\beta E_t\mu_{t+1}$ is smaller than in steady state, implying correspondingly an x below steady state. This behavior of x generates the small change in marginal cost even though the labor share is rising. Thus, if technology shocks were the main driving force of fluctuations, the labor share would be an unsuitable measure of inflationary pressures.

Figure 6 shows the response to a contractionary monetary shock, that is, a non-systematic rise in the nominal interest rate. Again, we see a drop in inflation and output, while unemployment rises for a short time. While the real effects of the shock

[25] The choices of volatilities of the additional shocks are somewhat arbitrary, and require estimation of the model, so we concentrate mainly on the monetary and real shocks.

Figure 5: Impulse response to a technology shock

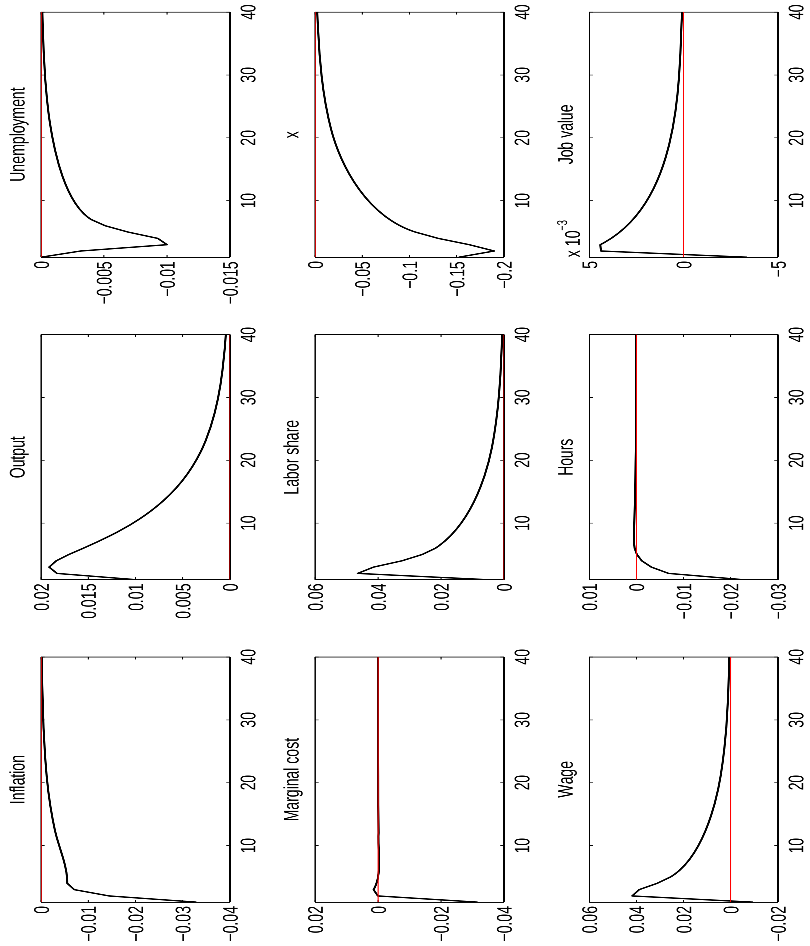
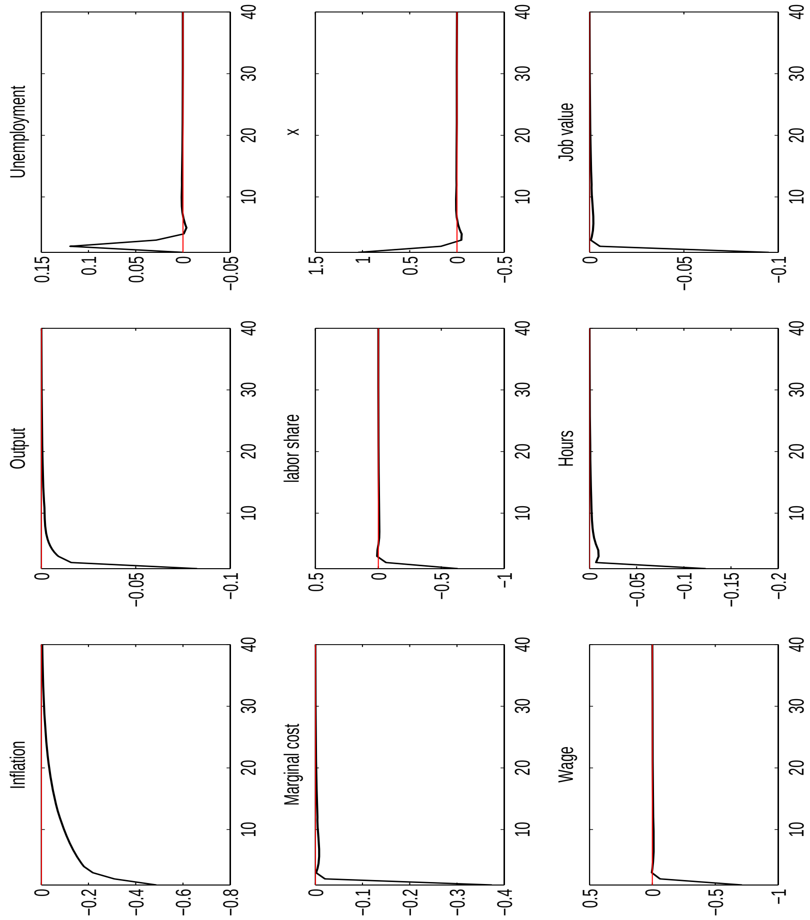


Figure 6: Impulse response to a montary shock



take place within the first 6 quarters, the reponse on inflation is long-lasting. After the monetary shock, real marginal cost and the labor share move down together, and the movement in x is very short-lived. This consistent with the marginal value of jobs μ, that barely changes after the forth quarter. Both the wage, and the value of a job drop in response to the shock. Thus, if monetary shocks were the main driving force of business cycles, the behavior of the labor share would in fact be a good proxy for real marginal costs.

Finally, in Figure 7, the intertemporal preference shocks lets households move consumption to the present. This leads output to rise initially, only to fall later on. The initial boom raises inflation, due to the rise in marginal costs. These in turn rise mainly because of the inital rise in weekly wages. The drop in the labor share would mandate a much stronger decline in inflation. However, after the initial shock, real marginal cost stay rather constant. This is due to a strong rise in x. Since the value of a job drops initially, the subsequent increase, from below, to the steady state value implies a fall in x. Again, the labor share would not be a valid proxy of real marginal costs, and thus inflationary pressures.

Overall, the simulated impulse responses show patterns that are partly consistent with the labor share being a good proxy for real marginal costs. A remark on the behavior of hours worked and employment is at hand. The propagation of the search and matching model on employment is very weak, as Shimer (2005) and Hall (2005) have shown. This carries over to our model. This effect is further mitigated by the fact that output can also adjust by increases in the number of hours worked. A potential remedy is the introduction of real wage rigidity, which we consider next.

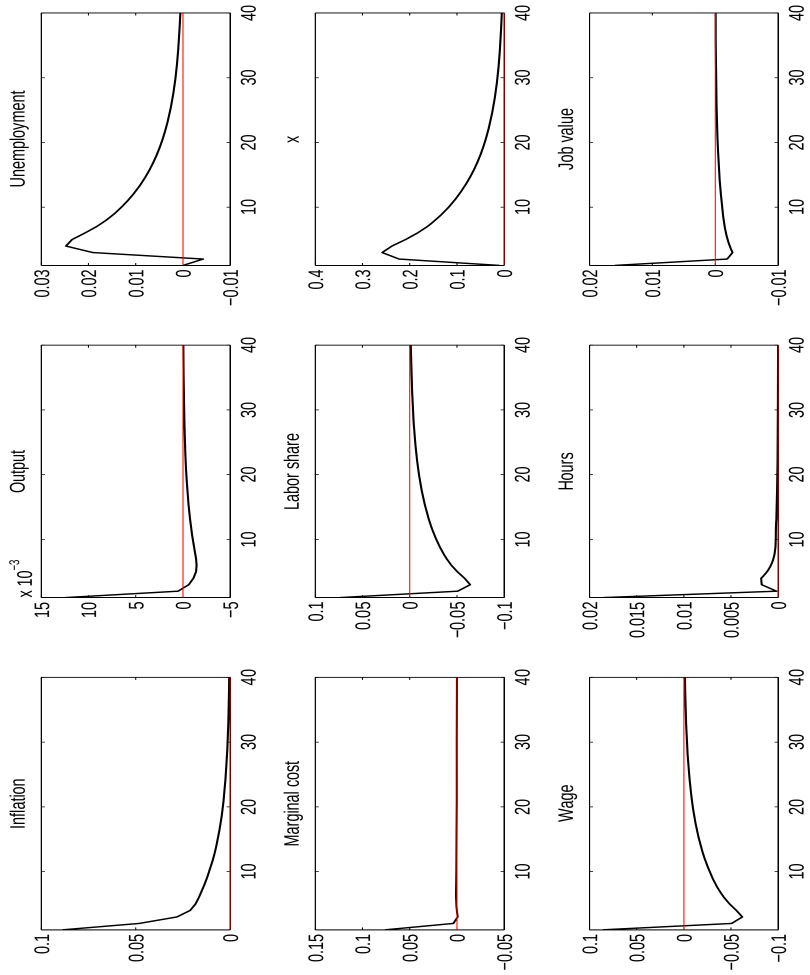
6 Extensions

6.1 Real wage rigidity

A well-known shortcoming of the search and matching framework is its difficulty to explain the cylical dynamics of vacancies and unemployment.[26] Intuitively, the incentives to posting vacancies after a productivity increase are too low to generate the observed magnitude of the rise vacancies and fall in unemployment. Shimer (2005) and Hall (2005) propose real wage rigidity as a possible solution to this puzzle. In this case, the incentives to post vacancies are not eaten up by wages increases, and firms earn most of the returns. An alternative solution has been proposed by Hagedorn

[26] See Hall (2005) and Shimer (2005).

Figure 7: Impulse response to an intertemporal preference shock



and Manovskii (2005), who focus on the details of the calibration: assuming much lower steady state profits of firms than in Shimer's model (due to high outide options of workers), the percentage increase in profits is much larger for a given shock.

We introduce real wage rigidity by alluding to the idea of a wage norm, that constrains real wage adjustments. A wage norm is a social constraint on wage setting. In particular it allows to justify that new hires are paid the same wages as existing workers, due, for example, to internal equity considerations. A possible formulation is to have the wage evolve according to[27]

$$W_t = (1-\gamma_w)W_t^n + \gamma_w \overline{W}_t$$

where W_t depends on the current, notional wage, W_t^n that would obtain if wages were flexible, and the wage norm $\overline{W}_t$. The notional wage would be the Nash bargained wage derived earlier. The wage norm can be the steady state wage, so that $\bar{W}_t = W$, for all t. Alternatively, there can be a lagged wage norm, so that $\bar{W}_t = W_{t-1}$. Here, we concentrate on the steady state wage norm. There is an issue which wage is actually sticky: the hourly or the monthly (or quarterly wage). We consider both cases. For rigid hourly wages $w_t = (1-\gamma_w)w_t^n + \gamma_w \overline{w}_t$, and for rigid monthly wages per worker $W_t = w_t h_t$.[28]

In our simulation, we focus on the steady state wage norm on weekly wages. The result in Figure 8 confirms the finding of Krause and Lubik (2007), that real wage rigidity has barely any effect on inflation dynamics. The intuition is that even though wages and the labor share respond by less to a shock, the other labor market variables move to offset the effect on real marginal costs. This is due to the dynamics of the value of a job brought about by movements in vacancies and unemployment, that increase labor market tightness θ and thus hiring costs. Thus real wage rigidity does not change substantially the behavior of real marginal costs.

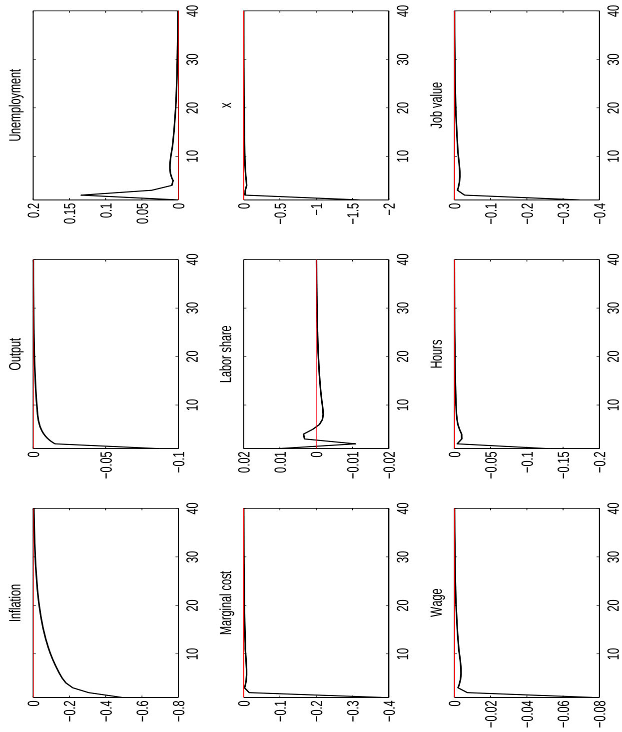
6.2 Additional labor adjustment costs

So far, we have restricted our attention to labor adjustment costs arising from search frictions alone. It is plausible, however, that firms face additional costs of labor adjustment. These may arise, for example, from forgone output when changing the scale of operations, changing production lines and the organization of workplaces.

[27] See Hall (2005) for more motivation, and, for example, Krause and Lubik (2007) who introduced this to analyse the New Keynesian Model with search frictions.

[28] See Sveen and Weinke for the case of rigid hourly wages.

Figure 8: Impulse response to a monetary shock when wages are rigid: $\gamma_w = 0.9$



To account for such considerations, we follow Rotemberg and Woodford (1999) and assume adjustment costs proportional to employment, and depending on the gross change of employment. In contrast to the search cost that also occur in steady state to correct for worker turnover, the costs assumed here are zero in steady state. In particular, assume that total labor costs are now[29]

$$\mathcal{N}_{jt} = c(v_{jt}) + n_{jt}\varphi\left(\frac{n_{jt}}{n_{jt-1}}\right)$$

where $\varphi(1) = \varphi'(1) = 0$, and $\varphi''(1) > 0$. Given this additional constraint, the optimality condition with respect to employment becomes: subject to the corresponding cosntraints. The first-order condition with respect to employment becomes

$$\begin{aligned}\mu_t &= mc_t\alpha\frac{Y_t}{n_t} - w_t h_t + (1-\rho)E_t\beta_{t+1}\mu_{t+1} \\ &\quad - \left[\varphi\left(\frac{n_t}{n_{t-1}}\right) + \varphi'\left(\frac{n_t}{n_{t-1}}\right)\frac{n_t}{n_{t-1}}\right] + E_t\beta_{t+1}\varphi'\left(\frac{n_{t+1}}{n_t}\right)\left(\frac{n_{t+1}}{n_t}\right)^2\end{aligned}$$

where we have ignored the firm indices. The marginal value of a worker is reduced by current labor adjustment cost, but increased by future adjustment costs, because the more workers are added today, the lower are future adjustment costs. Rewriting the equation in terms of marginal costs and linearizing:

$$\widehat{mc}_t = \widehat{s}_t + \frac{1-\phi}{1-\widetilde{\beta}}\left(\begin{array}{c}\hat{\mu}_t - (1-\rho)\beta(\hat{w}_t + \hat{h}_t) + (1-\rho)\beta(\epsilon_v - 1)\hat{v}_t - (1-\rho)\beta\xi\hat{\theta}_t \\ +\mu\varphi''(1)\left[\Delta n_t - \beta E_t\Delta n_{t+1}\right]\end{array}\right)$$

with $\frac{1-\phi}{1-\widetilde{\beta}} = \frac{1-s/mc}{1-(1-\rho)\beta}$. Labor adjustment costs add to real marginal costs if employment is currently rising, while real marginal costs fall when employment is expected to rise in the future. The intuition is that the expectation of growing employment, and thus adjustment costs, makes firms want to smooth adjustment costs. The incentive to do that is provided by falling real marginal costs. This makes firms perceive employing additional workers today cheaper, since this saves on future adjustment costs.

Blanchard-Gali-Rotemberg model In the model with instantaneous hiring, the setup changes along similar lines, resulting the same first-order condition as above, with the only difference being that μ_t can be replaced by $B\mathfrak{h}_t^{\delta}$, the contemporaneous

[29] Sveen and Weinke (2007) follow a similar approach in a search and matching model.

hiring cost faced by the firm.[30] Rewriting in terms of marginal costs and linearizing gives us the modified equation for the Blanchard-Gali framework:

$$\begin{aligned}\widehat{mc}_t &= \widehat{s}_t + \frac{1-\phi}{1-\widetilde{\beta}}\left[\delta(\widehat{\mathfrak{h}}_t - \widetilde{\beta}E_t\widehat{\mathfrak{h}}_{t+1}) - \widetilde{\beta}E_t\widehat{\beta}_{t+1} - (1-\widetilde{\beta})\widehat{w}_t\right] \\ &+ \frac{\phi}{w}\varphi''(1)\left[\Delta n_t - \beta E_t \Delta n_{t+1}\right]\end{aligned}$$

Similarly, in the Rotemberg variant with variable hiring costs, we obtain:

$$\begin{aligned}\widehat{mc}_t &= \widehat{s}_t + \frac{1-\phi}{1-\widetilde{\beta}}\left[\frac{\xi}{1-\xi}(\widehat{\mathfrak{h}}_t - \widetilde{\beta}E_t\widehat{\mathfrak{h}}_{t+1}) + (\epsilon_c - 1)(\widehat{v}_t - \widetilde{\beta}E_t\widehat{v}_{t+1}) - (1-\widetilde{\beta})\widehat{w}_t\right] \\ &+ \frac{\phi}{w}\varphi''(1)\left[\Delta n_t - \beta E_t \Delta n_{t+1}\right]\end{aligned}$$

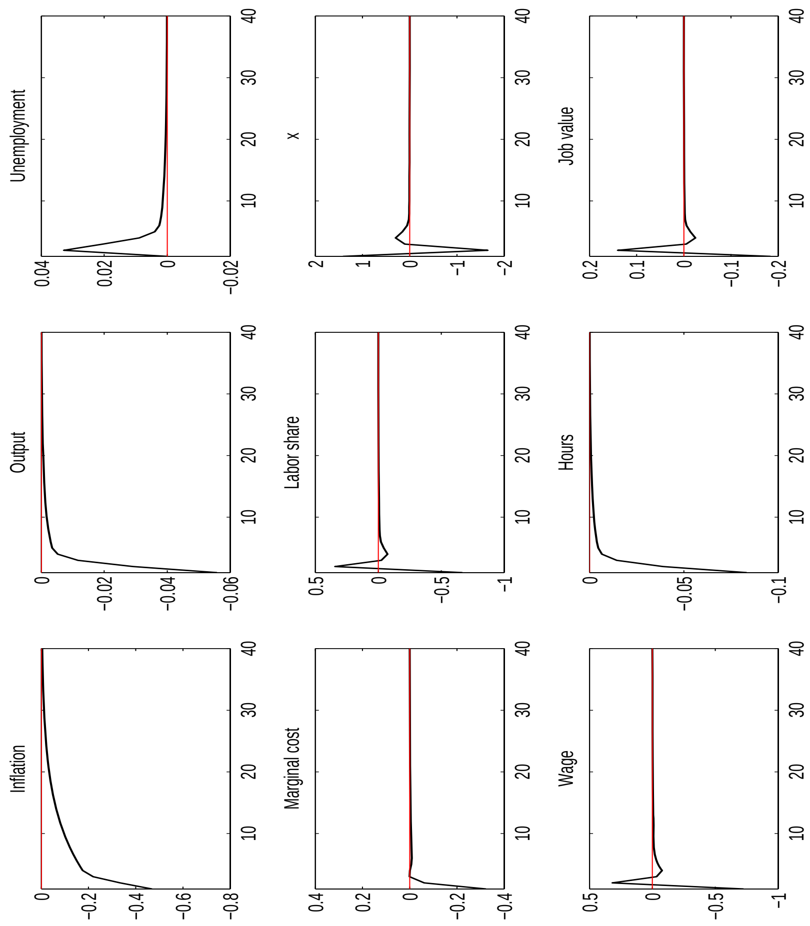
where $\phi = mc/s$.

Impulse responses Interestingly, in the simulations, labor adjustment costs have barely any effect on the dynamics of inflation and real marginal cost as seen in Figure 9. This holds true even for very high values of $\varphi''(1)$. Furthermore, the magnitude of the responses of output, employment, and vacancies are reduced. We illustrate this for the case of a monetary shock. The labor share and wages follow some intricate dynamics. Inititally, wages fall, because workers share in the labor adjustment costs that need to be incurred to bring about a decline in employment. The rise in unemployment then leads to an increase in the value of jobs, because hiring costs have fallen. Thus vacancies even experience a slight increase. The subsequent strong decline in unemployment (and return of employment towards steady state), implies labor adjustment cost, which, again, workers share in terms of lower wages. Even though the labor share moves strongly, inflation is persistently below steady state.

[30] The first order condition is:

$$\begin{aligned}B\mathfrak{h}_t^\delta &= mc_t\alpha\frac{y_t}{n_t} - W_t + (1-\rho)E_t\beta_{t+1}B\mathfrak{h}_{t+1}^\delta \\ &-\phi\left(\frac{n_t}{n_{t-1}}\right) + \phi'\left(\frac{n_t}{n_{t-1}}\right)\frac{n_t}{n_{t-1}} + E_t\beta_{t+1}\phi'\left(\frac{n_{t+1}}{n_t}\right)\left(\frac{n_{t+1}}{n_t}\right)^2\end{aligned}$$

Figure 9: Impulse response to a monetary shock with labor adjustment costs: $\varphi = 100$



7 Inflation and Marginal Costs: A Bayesian Full Information Estimation

7.1 Estimation Method and Choice of the Prior

As shown in the Appendix we can write a log-linear approximation to the non-linear DSGE model. We collect the linearized equilibrium conditions and we write the system in the following state space form:

$$\mathbf{A}(\Theta) E_t \mathcal{X}_{t+1} = \mathbf{B}(\Theta) \mathcal{X}_t + \mathbf{C}(\Theta) \mathcal{X}_{t-1} + \mathbf{D}(\Theta) \mathcal{S}_t,$$
$$\mathcal{S}_t = \mathbf{N}(\Theta) \mathcal{S}_{t-1} + \varepsilon_t, \quad E(\varepsilon_t \varepsilon_t') = \Sigma(\Theta).$$

where Θ denote the vector of parameters that describe preferences, technology, the monetary and fiscal policy rules and the shocks of the model, $\mathcal{X}_t$ be the vector of all endogenous variables, $\mathcal{S}_t$ be the vector of state variables, and ε_t be the vector of innovations. We use standard solution methods for linear models with rational expectations to write the law of motion in state-space form and the Kalman filter to evaluate the likelihood of the five observables (output, inflation, interest rates, unemployment, and vacancies. Formally, we denote the vector $\mathsf{d}_t = (y_t, \pi_t, r_t, u_t, v_t)'$ of variables and $\mathfrak{L}\left(\{\mathsf{d}_t\}_{t=1}^T |\Theta\right)$ is the likelihood function of $\{\mathsf{d}_t\}_{t=1}^T$. In the estimation we include five shocks: production technology, intertemporal preferences, matching technology, exogenous markups, and monetary policy. Let $\Pi(\Theta)$ denote the prior distribution of the model's parameters, then from Bayes rule, we obtain the posterior distribution of the parameters as follows:

$$p(\Theta| \{\mathsf{d}_t\}_{t=1}^T) \propto \mathfrak{L}(\{\mathsf{d}_t\}_{t=1}^T |\Theta)\Pi(\Theta)$$

The posterior density function is proportional to the product of the likelihood function and the prior joint density function of Θ. Given our priors and the likelihood function implied by the state-space solution to the model, we are not able to obtain a closed-form solution for the posterior distributions. However, we are able to evaluate both expressions numerically. We use the random walk Metropolis-Hastings algorithm, to obtain a draw of size 100,000 from $p\left(\Theta| \{\mathsf{d}_t\}_{t=1}^T, \mathfrak{m}\right)$.

We present the list of the structural parameters and its associated prior distribution in Table 4. These priors are assumed to be independent across parameters, and are based upon existing research. Regarding the preferences parameters we set the measure of relative risk aversion $\sigma = 2$, the inverse of the labor supply elasticity

$\mu = 1$, and the habits parameter $\varsigma = 0.5$. These values are in line with conventional wisdom of the business cycle literature are in line with recent micro evidence (see e.g. Hall (2006)). The elasticity of output with respect to hours $\alpha = 0.67$. As discussed above, we follow Shimer (2005) and we set the elasticity of matching to unemployment, $\xi = 0.7$, and as in Pissarides (2000) thge elasticity of vacancy creation, $\epsilon_c = 1$. The prior mean of the firms's price adjustment costs, ψ, is set to 20, and the indexation parameter γ is set to 0.5. These two values are in the lower range of the estimates obtained from the New Keynesian Phillips curve literature. Overall, we also set prior standard deviations that are large enough to incorporate the uncertainty about those parameters in the existing literature.

The policy parameters are chosen as follows. We set the prior mean of the response of the monetary authority to inflation, γ_π, to 1.5, and to output $\gamma_Y = 0.25$. Finally, the prior mean of the smoothing interest rates parameter, ρ_r equals 0.7. These values commonly used in empirical Taylor rules (see e.g. Woodford, 2001). Finally, as described in the last rows of Table 4, we consider a beta distribution for the autocorrelation of the shocks and Gamma Inverse density for the standard deviation of the model's shocks.

Table 4. Prior Distributions of the Model's Parameters

Definition	Parameter	Density	Mean	Param.
Relative Risk Aversion	σ	*Gamma*	2	0.1
Habits	ς	*Beta*	0.5	0.2
Inverse of Labor Supply Elasticity	μ	*Gamma*	1	0.5
Elasticity of output to labor input	α	*Beta*	0.67	0.02
Elasticity of Matching to Unemployment	ξ	*Beta*	0.7	0.05
Scaling Factor Matching Function	$\overline{m}$	*Gamma*	0.7	0.1
Elasticity of Vacancy Creation	ϵ_c	*Gamma*	1	0.5
Scaling Factor on Vacancy Creation	c_v	*Gamma*	0.05	0.02
Bargaining Power of the Worker	η	$Uniform$	0	1
Workere Outside Option	b	*Beta*	0.4	0.1
Separation Rate	ρ	*Beta*	0.1	0.02
Indexation	γ	*Beta*	0.5	0.2
Price Adjustment Costs	ψ	*Gamma*	20	5
Elasticity of Demand	ϵ	*Gamma*	10	1
Interest Rate Smoothing	ρ_r	*Beta*	0.7	0.02
Interest Rate Response to Inflation	γ_π	*Gamma*	1.5	0.1
Interest Rate Response to Output	γ_Y	*Gamma*	0.25	0.05
AR Coefficients of Shocks	$\rho' s$	*Beta*	0.9	0.05
Std. Deviation of Shocks	$\sigma' s$	$Gamma^{-1}$	0.01	1

7.2 Benchmark Estimation Results

8 Conclusions and Further Research

[TBC]

Appendix

1. Derivation of the Wage Equation under Nash bargaining and Endogenous Hours

Here we present the details on how to find the Nash bargaining solution. Firms and workers bargain each period over how the joint surplus of their match is divided. The solution they find is such that it maximizes the Nash product:

$$\left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right)^{\eta}\left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}\right)^{1-\eta}. \tag{28}$$

As is well-known, this solution is pareto-optimal for the bargaining parties. The values in brackets are what each party would lose, if the match broke up. The surplus of the workers is equal to the marginal value of the job to household i:

$$\frac{1}{\lambda_{it}}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_{it}} = w_{it}h_{it} - b - \frac{1}{\lambda_{it}}\varepsilon_t^b\varepsilon_t^n\frac{h_{it}^{1+\mu}}{1+\mu} + E_t\beta_{t+1}\left[(1-\rho)(1-\theta_t q(\theta_t))\frac{1}{\lambda_{it+1}}\frac{\partial \mathcal{W}_{t+1}^i(n_{it+1})}{\partial n_{it+1}}\right].$$

This equation is the derivative of the household's value function with respect to employment n_{it}. Division by λ_{it} translates the utility units of $\mathcal{W}$ in terms of goods.

The firm's surplus is the marginal value of a worker is:

$$\frac{\partial \mathcal{J}_t^j(n_{jt})}{\partial n_{jt}} = mc_t\varepsilon_t^A\alpha n_{jt}^{\alpha-1}h_{jt}^{\alpha} - w_{jt}h_{jt} + E_t\beta_{t+1}\left[\frac{\partial \mathcal{J}_{t+1}^j(n_{jt+1})}{\partial n_{jt+1}}(1-\rho)\right].$$

The first-order condition w.r.t. the wage (dropping the indices j and i)

$$(1-\eta)\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \eta\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}.$$

This equation can be interpreted as a sharing rule according to which each party obtains a fraction of the joint surplus. That is: $\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \eta\left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t} + \frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right)$ and for the firm accordingly.

To find the wage, insert the value functions into this sharing rule:

$$\begin{aligned}&(1-\eta)\left(w_t - b - \frac{1}{\lambda_t}\varepsilon_t^b\varepsilon_t^n\frac{h_t^{1+\mu}}{1+\mu} + E_t\beta_{t+1}\left[(1-\rho)(1-\theta_t q(\theta_t))\frac{1}{\lambda_{t+1}}\frac{\partial \mathcal{W}_{t+1}(n_{t+1})}{\partial n_{t+1}}\right]\right)\\ =\ &\eta\left(mc_t\varepsilon_t^A\alpha n_t^{\alpha-1}h_t^{\alpha} - w_t h_t + E_t\beta_{t+1}\left[(1-\rho)\frac{\partial \mathcal{J}_{t+1}(n_{t+1})}{\partial n_{t+1}}\right]\right)\end{aligned}$$

and rearrange:

$$\begin{aligned} w_t h_t &= \eta mc_t \varepsilon_t^A \alpha n_t^{\alpha-1} h_t^\alpha + (1-\eta)\left(b + \frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n \frac{h_t^{1+\mu}}{1+\mu}\right) \\ &\quad - (1-\eta) E_t \beta_{t+1}\left[(1-\rho)(1-\theta_t q(\theta_t))\frac{1}{\lambda_{t+1}}\frac{\partial \mathcal{W}_{t+1}(n_{t+1})}{\partial n_{t+1}}\right] \\ &\quad + \eta E_t \beta_{t+1}\left[(1-\rho)\frac{\partial \mathcal{J}_{t+1}(n_{t+1})}{\partial n_{t+1}}\right] \end{aligned}$$

Using the fact that because of continuous renegotiation, the sharing rule must also hold in the future $(1-\eta)\frac{1}{\lambda_{t+1}}\frac{\partial \mathcal{W}_{t+1}(n_{t+1})}{\partial n_{t+1}} = \eta \frac{\partial \mathcal{J}_{t+1}(n_{t+1})}{\partial n_{t+1}}$, and simplifying, yields:

$$\begin{aligned} w_t h_t &= \eta mc_t \varepsilon_t^A \alpha n_t^{\alpha-1} h_t^\alpha + (1-\eta)\left(b + \frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n \frac{h_t^{1+\mu}}{1+\mu}\right) \\ &\quad + \theta_t \eta E_t \beta_{t+1}\left[(1-\rho) q(\theta_t)\frac{\partial \mathcal{J}_{t+1}(n_{t+1})}{\partial n_{t+1}}\right] \end{aligned}$$

Now use the first-order condition for vacancies posted, noting that the Lagrange multiplier of the firms optimization problem is $\mu_{t+1} = \partial \mathcal{J}_{t+1}(n_{t+1})/\partial n_{t+1}$:

$$c'(v_t) = (1-\rho) E_t \beta_{t+1} q(\theta_t)\frac{\partial \mathcal{J}_{t+1}(n_{t+1})}{\partial n_{t+1}}$$

to arrive at the equation for the wage paid to a worker:

$$w_t h_t = \eta mc_t \varepsilon_t^A \alpha n_t^{\alpha-1} h_t^\alpha + (1-\eta)\left(b + \frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n \frac{h_t^{1+\mu}}{1+\mu}\right) + \theta_t \eta c'(v_t).$$

To determine hours chose, once more maximize the Nash product,

$$\left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right)^\eta \left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}\right)^{1-\eta} \tag{29}$$

with respect to hours, to obtains (using the previous result $(1-\eta)\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \eta \frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}$) :

$$\chi \frac{1}{\lambda_t}\varepsilon_t^b \varepsilon_t^n h_t^\mu = mc_t \alpha^2 \varepsilon_t^A n_t^{\alpha-1} h_t^{\alpha-1}$$

or:

$$h_t = \left(\frac{mc_t \alpha^2 \varepsilon_t^A n_t^{\alpha-1}}{\chi \varepsilon_t^n \varepsilon_t^b}\lambda_t\right)^{\frac{1}{1+\mu-\alpha}}$$

Hours are set so that the marginal rate of substitution of labor is equal to the marginal product of labor, just as in the competitive model. Hours variation over the business cycle is depends on the terms in brackets. In right-to-manage setups[31], hours are not determined by bargaining but determined by firms to maximize their profits. This, of course, affects the bargaining outcome, as the surpluses are conditional on firms choosing hours.

2. Why real wage rigidity?

It can be easily illustrated why real wage rigidity helps resolve the 'Shimer puzzle' by using simplified equations from the benchmark model. Ignoring hours and decreasing returns to production, and using A_t for productivity, the job creation condition and wage equation become, respectively:

$$\frac{c}{q(\theta_t)} = (1-\rho)E_t\beta_{t+1}\left[mc_{t+1}A_{t+1} - w_{t+1} + \frac{c}{q(\theta_{t+1})}\right] \tag{30}$$

and

$$w_t = \eta\,(mc_t A_t + \theta_t c) + (1-\eta)\,b.$$

When wages are continuously bargained, as in the Nash bargaining solution, wages respond strongly to changes in A_t. Furthermore, changes in $\theta = v/u$ immediately translate into changes in wages. Firms expected high wages when labor market conditions are tight in the future, will post less vacancies. This mechanism can be overcome by introducing real wage rigidity, as in Hall's (2005) paper.

Alternatively, as in Hagedorn and Manovskii (2006), when b is very high, that is, close to the marginal product of labor, steady state profits are low. A given increase in A_t leads to a lower percentage change in wages than when b is low. Inserting the wage, and linarizing the term in square brackets above (calling it R_{t+1} for return):

$$R_{t+1} = \frac{1}{mc - b - \frac{\eta}{1-\eta}c\theta}\left[mc\left(\widehat{mc}_{t+1} + \widehat{A}_{t+1}\right) - \frac{\eta}{1-\eta}c\theta\widehat{\theta}_{t+1}\right]$$

One can see that the higher b, the larger the coefficient multiplying the brackets, and thus the higher the responsiveness of the return to increases in productivity or revenue. In steady state, the job creation condition is:

$$\frac{c}{q(\theta)}\left[\eta Bp(\theta) + 1\right] = (1-\eta)B[mc - b]$$

where $B = (1-\rho)\beta/(1-(1-\rho)\beta)$. A higher b in the calibration implies a smaller right hand side. This in turn imposes some constraint on the left hand side. One possibility

[31] Trigari, and Christoffel and Linzert.

is a lower c or a lower steady state θ. Either of the two reduces the responsiveness to $\hat{\theta}_{t+1}$, and thus the returns to posting vacancies remain high.

3. A Complete Description of the Baseline Model and the Steadys State

The Stochastic Model

Euler equation	$\lambda_t = E_t \beta \lambda_{t+1} \left[R_t \frac{P_t}{P_{t+1}} \right],\ \lambda_t = \varepsilon_t^b (c_t - \varsigma C_{t-1})^{-\sigma}$
Production	$Y_t = \varepsilon_t^A n_t{}^{\alpha} h_t^{\alpha}$
Resource Constraint	$Y_t = C_t + c(V_t) + G_t$
Umployment	$u_t = 1 - n_t\ ,\ n_t = (1-\rho)\left[n_{t-1} + M_{t-1} \right],\ M_t = \bar{m} \varepsilon_t^M u_t^{\xi} v_t^{1-\xi}$
Job creation	$\frac{c'(v_t)}{M_t} v_t = (1-\rho) E_t \beta_{t+1} \left[mc_{t+1} \alpha A_t n_{t+1}^{\alpha-1} h_{t+1}^{\alpha} - w_{t+1} h_{t+1} + c'(v_{t+1}) \frac{v_{t+1}}{M_{t+1}} \right]$
Wage	$w_t h_t = \eta mc_t \alpha A_t n_t^{\alpha-1} h_t^{\alpha} + (1-\eta)\left(b + \chi \frac{1}{\lambda_t} \varepsilon_t^n \varepsilon_t^b \frac{h_t^{1+\mu}}{1+\mu} \right) + \theta_t \eta c'(v_t)$
Hours	$h_t = \left(\frac{mc_t \alpha^2 A_t n_t^{\alpha-1}}{\chi \varepsilon_t^n \varepsilon_t^b} \lambda_t \right)^{\frac{1}{1+\mu-\alpha}}$
Inflation	$\psi \left(\pi_t \pi_{t-1}^{-\gamma} \pi^{\gamma-1} - 1 \right) \pi_t \pi_{t-1}^{-\gamma} \pi^{\gamma-1}$
	$= E_t \beta_{t+1} \psi \left(\pi_{t+1} \pi_t^{-\gamma} \pi^{\gamma-1} - 1 \right) \pi_{t+1} \pi_t^{-\gamma} \pi^{\gamma-1} \frac{Y_{t+1}}{Y_t} + (1 - \varepsilon_t^{\epsilon}) + \varepsilon_t^{\epsilon} mc_t,$
Taylor rule	$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi^*} \right)^{r_\pi} \left(\frac{Y_t}{Y^*} \right)^{r_Y} \right]^{1-\rho_r} \varepsilon_t^R$

The Steady State

Marginal utility	$\lambda = (1-\varsigma)^{-\sigma} C^{-\sigma}$
Euler equation	$1 = \beta R \pi^{-1}$
Production	$Y = (nh)^{\alpha}$
Resource Constraint	$Y = C + c(v)$
Employment	$n = \rho^{-1}(1-\rho) M$
Unemployment	$u = 1 - n$
Matching	$M = \bar{m} u^{\xi} v^{1-\xi}$
Job creation	$\frac{c'(v)}{M} v = (1-\rho)\beta \left[\varphi \alpha \frac{Y}{n} - W + c'(v) \frac{v}{M} \right]$
Wage	$wh = \eta \varphi \alpha n^{\alpha-1} h^{\alpha} + (1-\eta)\left(b + \chi \frac{1}{\lambda} \frac{h^{1+\mu}}{1+\mu} \right) + \theta \eta c'(v)$
Hours	$h = \left(\frac{mc \alpha^2 n^{\alpha-1}}{\chi} \lambda \right)^{\frac{1}{1+\mu-\alpha}}$
Inflation	$mc = \frac{\epsilon - 1}{\epsilon}$

$$\begin{aligned}
\ln \varepsilon_t^R &= \rho_R \ln \varepsilon_{t-1}^R + \eta_t^R \\
\ln \varepsilon_t^n &= \rho_n \ln \varepsilon_{t-1}^n + \eta_t^n \\
\ln \varepsilon_t^M &= \rho_M \ln \varepsilon_{t-1}^M + \eta_t^M \\
\ln \varepsilon_t^A &= \rho_A \ln \varepsilon_{t-1}^A + \eta_t^A \\
\ln \varepsilon_t^{\epsilon} &= (1-\rho_{\epsilon}) \ln \varepsilon^{\epsilon} + \rho_{\epsilon} \ln \varepsilon_{t-1}^{\epsilon} + \eta_t^{\epsilon}
\end{aligned}$$

4. Data

We take the series for the job separation and the job finding rate from Shimer (2005). They are quarterly averages of monthly rates. Shimer calculates two different series for the job separation and job finding rate. The first two are available from 1948 up to 2004. He uses available data from the Bureau of Labor Statistics for employment, unemployment, and unemployment duration to calculate the *instantaneous* rate at which workers move from employment to unemployment and vice versa. The two rates are computed under the assumption that workers move between employment to unemployment, and therefore abstracts from workers' labor force participation decisions. Hence, they are an approximation to the true underlying labor market rates. Starting from 1967:2, Shimer also uses the monthly Current Population Survey public microdata to directly calculate the flow of workers that move in and out of the three possible labor market states (employment, unemployment, and out of the labor force). With this information he calculates the instantaneous rates at which workers move in and out each state. This yields an exact instantaneous rate at which workers move from employment to unemployment and from unemployment to employment.

We also compare the results by using two data sets of two recent studies that have modified and extended Shimer's original calculation. We first use the hazard rates series from Fujita and Ramey (2006). The series are available at monthly frequency and cover the period of January 1976 through December 2005. We compute the quarterly averages of monthly rates. These authors correct by potential margin error –inconsistency in the stock-flow identities– in the CPS. In addition, these authors for time aggregation problems. Elsby et al. (2007) also propose some refinements of the correction methods used by Shimer's analysis based on publicly available data from the CPS, and as Fujita and Ramey redesign the analysis and correct for time aggregation bias. Interestingly, Elsby et al. (2007) also distinguish employment-to-unemployment flows stemming from job loss and from job leaving, and they show that these two flows have very different cyclical properties. Thus, we use their disaggregated analysis of unemployment where we distinguish three categories: job losers, job leavers, and labor force entrants.

We use the index of help-wanted advertisements released by the Conference Board as an approximation for vacancies (`HW`). We also use the stock of unemployed –16 years and over– from the BLS, and the Unemployment Index equals to $\frac{U(t)}{U(June87)}$, which is consistent with HW Index. We construct the quarterly averages of monthly rates that are available starting at January 1951.

Finally, our measure of marginal costs corresponds to the Nonfarm Business Sector. The data are drawn from FRED®II database and the variables correspond to: real output (OUTNFB), the output deflator (IPDNBS), the aggregate number of hours worked (HOANBS), and the compensation per hour (COMPNFB), respectively. Real consumption corresponds to the sum of real non-durable (PCNDGC96) and services (PCESVC96), respectively. Finally, CNP16OV is the civilian non institutional population.

5. Additional Shocks-Impulse Responses

Here we show the impulse responses for the labor supply, matching function, and markup shock, in the baseline model.

Figure 10: Impulse responses to a labor supply shock

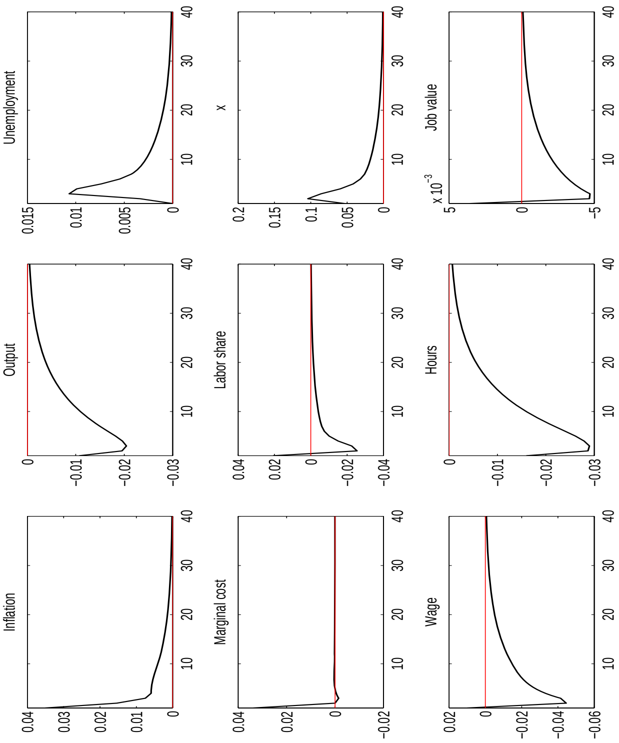


Figure 11: Impulse responses to match efficiency shock

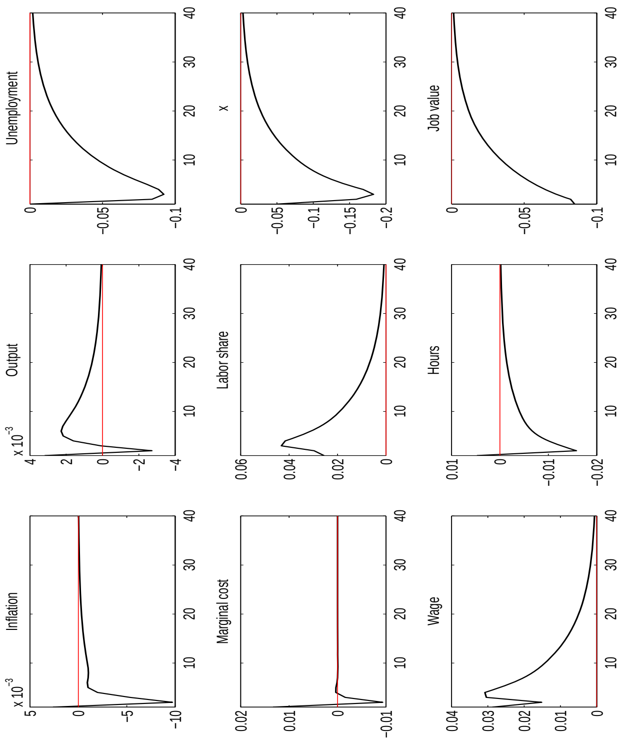
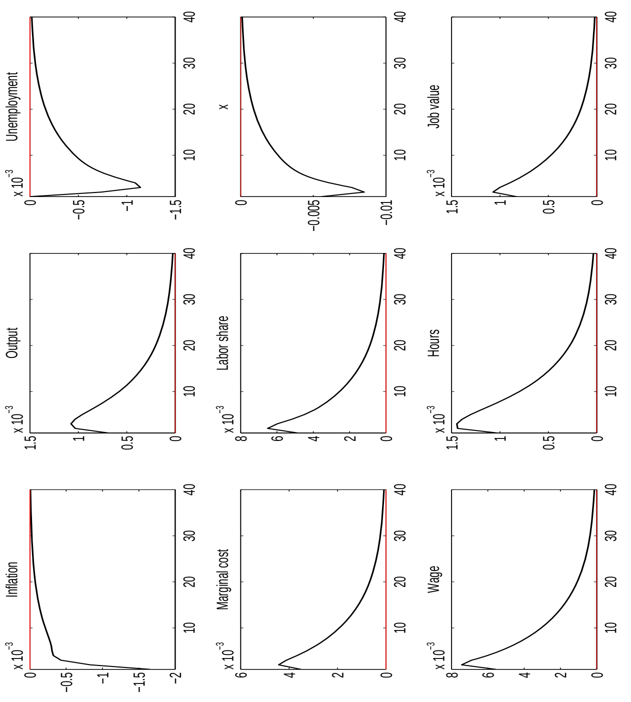


Figure 12: Impulse responses to a markup shock



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