Resuscitating the Wage Channel in Models with Unemployment Fluctuations*

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Abstract

Higher wages all else equal translate into higher inflation. More rigid wages imply a weaker response of inflation to shocks. This view of the wage channel is deeply entrenched in central banks' views and models of their economy. In this paper, we present a model with equilibrium unemployment which features three distinctive properties. First, using a search and matching model with right-to-manage wage bargaining a proper wage channel obtains. Second, accounting for fixed costs associated with maintaining an existing job greatly magnifies profit fluctuations for any given degree of wage fluctuations, which allows the model to reproduce the fluctuations of unemployment over the business cycle. And third, the model implies a reasonably low elasticity of steady state unemployment with respect to changes in benefits. The calibration of the model implies low profits, but does not require a small gap between the value of working and the value of unemployment for the worker.

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1 Introduction

The channel from wages to inflation plays a key role in explaining aggregate price dynamics. All else equal, increased wages are associated with higher rates of inflation, and a slow adjustment of wages to shocks translates into inflation inertia. This view appears to be shared by central banks around the globe and it is a central feature of their policy models.[1]

At the same time flows in and out of employment take center stage in policy discussions. This in mind, it is surprising that to date there appears to be no model which accounts both for the fluctuations in main labor market variables and for a wage channel to inflation. This paper is meant to fill the gap.

In this paper, we develop a New Keynesian model with search and matching frictions in the labor market, which has three characteristic features: it incorporates a wage channel to inflation, it replicates the fluctuations of unemployment over the business cycle and it implies a reasonable response of unemployment rates to changes in the level of unemployment benefits.

Building on Trigari's (2006) right-to-manage wage bargaining framework (RTM, henceforth), we account for fixed costs associated with maintaining an existing job.[2] These reduce average job-related profits and amplify fluctuations of profits in percentage terms. Since profits are the driving force behind hiring activity, the model can be calibrated to match the cyclical fluctuations of US labor market variables witnessed in the data. At the same time, the model preserves a channel from wages to inflation. We furthermore show that the model replicates second moments of the labor market data without having to rely on an implausibly high elasticity of unemployment with respect to benefits or a high degree of stickiness in wages of new hires.[3]

[1] Central bank models featuring a wage channel include the Federal Reserve Board's FRB/US and SIGMA models, the Bank of England Quarterly Model and the European Central Bank's old and New Area Wide Model.

[2] Some cost items associated with a job are independent of the actual hours worked. These costs are related to part of the supply of work-infrastructure such as IT services, the rental of office space as well as the provision of overhead administrative services or may be related to labor turnover costs. A few authors have pointed out in efficient-bargaining frameworks that the presence of fixed costs or turnover costs makes a firm's net payoff (after paying the fixed costs) more responsive to productivity variation, see the references in Mortensen and Nagypal (2007).

[3] The recent literature on labor market matching has identified these two properties as potential shortcomings of models relying on efficient wage bargaining, which is the bargaining assumption most frequently used. Compare Costain and Reiter (2005) and Mortensen and Nagypal (2007) for a discussion of the elasticity of unemployment with respect to benefits as well as Pissarides (2007) and Haefke, Sonntag, and van Rens (2007) for evidence on wages of new hires.

A wage channel, in our understanding, is present whenever wages have a direct influence on inflationary developments. As Trigari (2006) and Christoffel and Linzert (2005) have shown, this is the case under RTM.[4] The argument is as follows: a job produces a labor good according to production function $y_t^L = h_t^\alpha, \alpha \in (0,1)$, where h_t are hours per worker. Given a bargained wage rate, w_t, and facing a real product price, x_t^L, labor firms set hours along their labor demand curves, so $x_t^L \alpha h_t^{\alpha-1} = w_t$. Price-setting firms acquire the labor good under perfect competition at price x_t^L and produce differentiated wholesale goods, their real marginal costs in equilibrium being $mc_t = x_t^L = \frac{1}{\alpha}\frac{w_t h_t}{y_t^L}$. The behavior of wages thus translates into marginal costs and therefore into the behavior of inflation.

In models of search and matching, unemployment fluctuations are closely linked to labor firms' profits. In our model, these unemployment fluctuations are amplified as follows. Let $\Phi \geq 0$ be the fixed costs associated with a job. Period labor profits, $\Psi_t^L = x_t^L y_t^L - w_t h_t - \Phi$, in equilibrium are given by $\Psi_t^L = \frac{1-\alpha}{\alpha} w_t h_t - \Phi$. In the absence of fixed costs therefore, any 1% increase in profits also means a 1% increase in wages. Since wages do not fluctuate much over the business cycle, profits do not fluctuate sufficiently so as to induce significant fluctuations in hiring activity. We show that as a result unemployment does not fluctuate enough. If fixed costs of maintaining a job exist, however, the share of wages in profits is no longer constant over the business cycle. Let hats denote percentage deviations from steady state and let variables without time index indicate steady state values. Linearizing the latter equation implies that $\widehat{\Psi}_t^L = A\left(\widehat{w}_t + \widehat{h}_t\right)$, where the factor of proportionality $A = \frac{\frac{1-\alpha}{\alpha} wh}{\frac{1-\alpha}{\alpha} wh - \Phi} \geq 1$ links percentage fluctuations in profits to those in the wage per employee.[5] The larger the fixed costs, the larger is A. Therefore even if wages are relatively smooth, percentage job-related profits can fluctuate enough. This reconciles the wage channel with labor market fluctuations.

It appears necessary to relate these results to the majority of literature which uses efficient wage bargaining (EB, henceforth) instead. Hagedorn and Manovskii (2006) clarify that under EB two properties must be met to replicate unemployment fluctuations. First, wages must not move one-to-one with revenue over the cycle. This provided, increases in revenue translate into

[4] Right-to-manage here is taken to mean that firms and workers bargain about the hourly wage rate only. At this wage rate, the firm is free to choose employment along the intensive (hours worked) margin. The work-horse of the literature in contrast is efficient bargaining. There, firms and workers bargain simultaneously about both hours worked and wages.

[5] This refers to job-related profits, Ψ_t^L, only. Economy-wide profits do not need to be proportional to wages.

more than proportional increases in profits. Second, profits in steady state must be small so as to induce sizeable cyclical fluctuations of profits in percentage terms. These conditions carry over to our model with RTM. The per-period fixed cost, $\Phi > 0$, ensures both that steady state profits are small and that wages do not move one-to-one with revenue. In influential papers, Hall (2005) and Shimer (2004) argue that if Nash-bargaining is efficient, smoother wages expose a firm's profits more to cyclical fluctuations in revenue which helps to amplify unemployment fluctuations. With RTM instead the share of wages in revenue is constant over the business cycle, implying that a smoother wage would mean that revenues and profits would fluctuate by *less*. Indeed, absent the fixed costs, in order to achieve vacancy and unemployment fluctuations of a realistic size, wages under RTM would need to be far more *volatile* than they are in the data. Furthermore the argument that sticky wages increase unemployment fluctuations requires in particular that the wages of new hires must be sticky, for which there is only scant empirical support, cp. Pissarides (2007) and Haefke, Sonntag, and van Rens (2007).

Calibrations of matching models under EB similar to the one used in this paper tend to imply a large drop of the unemployment rate when benefits are reduced, see Costain and Reiter (2005) and Mortensen and Nagypal (2007). Instead when we calibrate the RTM model with fixed costs to US data, we obtain an elasticity of the unemployment rate to changes in benefits which is in line with empirical estimates, e.g. Nickell and Layard (1999). The reason is that under RTM job-related profits can be small in steady state while the surplus of workers need not be negligible at the same time.

Apart from the hiring activity, the model adheres to the structure commonly employed in central bank models which follow the New Keynesian approach as in Smets and Wouters (2007). In particular, we show that RTM bargaining lends itself to staggered Calvo type wage-setting which induces real rigidities in the sense of Ball and Romer (1990). We view this structural similarity with the current vintage of policy models, the retention of a channel from wages to inflation and a reasonable empirical success as key requirements to bring models with equilibrium unemployment closer to policy applications at central banks.

The remainder of the paper is structured as follows. We present a New Keynesian model with search and matching frictions in the labor market and staggered right-to-manage bargaining in Section 2. Thereafter, in Section 3, we calibrate the model to US data. Section 4 makes the three points of this paper: First, it shows the existence of the wage channel algebraically and then by means of impulse responses. Second, it highlights the importance of fixed costs for unemployment fluctuations in the model both algebraically and in the calibrated model economy. Third, it illustrates that the model implies a reasonable reaction of the economy's steady state unemployment rate in responses to changes in the level of benefits. A final Section concludes. The Appendix collects the linearized model economy and the steady state.

2 The New Keynesian Model Economy

We incorporate search and matching frictions à la Mortensen and Pissarides (1994) into an otherwise plain New Keynesian business cycle model. In order to make our point most clearly, we abstract from many of the frictions and features typically entertained in the recent empirical New Keynesian literature. In particular, we abstract from capital formation and the various frictions involved. We further abstract from firm-specific production factors and price and wage indexation. The model's production side features competitive factor markets in the only price-setting sector. Wages at the individual labor good firm are set in a Calvo-staggered manner. One time period in the model refers to a calendar time of one month.

2.1 Preferences and Consumers' Constraints

Consumers have time-additive expected utility preferences. Preferences of consumer i can be represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathtt{u} \left(c_{i,t}, c_{t-1}, h_{i,t} \right) \right\}, \tag{1}$$

where E_0 marks expectations conditional on period 0 information and $\beta \in (0,1)$ is the time-discount factor. $\mathtt{u}(c_{i,t}, c_{t-1}, h_{i,t})$ is a standard period utility function of the form

$$\mathtt{u}(c_{i,t}, c_{t-1}, h_{i,t}) = \frac{(c_{i,t} - \varrho c_{t-1})^{1-\sigma}}{1-\sigma} - \kappa^L \frac{(h_{i,t})^{1+\varphi}}{1+\varphi}, \sigma > 0, \varphi > 0. \tag{2}$$

Here, $c_{i,t}$ denotes consumption of member i, c_{t-1} denotes aggregate consumption last period and $h_{i,t}$ are hours worked by member i. κ^L is a positive scaling parameter of disutility of work, $\varrho \in [0,1)$ indicates an external habit motive.

2.1.1 Family Welfare and Budget Constraint

There is a large number of identical families in the economy with unit measure. Each family consists of a measure of $1-u_t$ employed members and u_t unemployed members both with above preferences. The family maximizes the sum of unweighted expected utilities of its individual members,

$$\int_0^1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathtt{u}\left(c_{i,t}, c_{t-1}, h_{i,t}\right) \right\} di. \tag{3}$$

Let $\mathtt{U}\left(c_t, c_{t-1}, u_t, \{h_{i,t}\}\right)$ denote the aggregate per-period utility function of the family:

$$\mathtt{U}\left(c_t, c_{t-1}, u_t, \{h_{i,t}\}\right) := \int_0^1 \mathtt{u}(c_{i,t}, c_{t-1}, h_{i,t}) di, \tag{4}$$

where consumption c_t is the average consumption level of family members and $\{h_{i,t}\}$ is shorthand for the distribution of hours worked. Given its arguments, utility function $\mathtt{U}(\cdot,\cdot,\cdot,\cdot)$ gives the value of period family-utility when consumption spending c_t is optimally distributed among family members. One can show that in our environment additive separability is also preserved for the aggregate utility function:

$$\mathtt{U}\left(c_t, c_{t-1}, u_t, \{h_{i,t}\}\right) = \frac{\left(c_t - \varrho c_{t-1}\right)^{1-\sigma}}{1-\sigma} - \kappa^L \int_0^{1-u_t} \frac{h_{i,t}^{1+\varphi}}{1+\varphi} di. \tag{5}$$

The representative family pools the labor income of its working members, unemployment benefits of the unemployed members and financial income. Its budget constraint is given by

$$c_t + t_t = \int_0^{1-u_t} w_{i,t} h_{i,t} di + u_t b + \frac{D_{t-1}}{P_t} R_{t-1} \epsilon_{t-1}^b - \frac{D_t}{P_t} + \Psi_t, \tag{6}$$

where c_t is a choice variable of the family. t_t are lump-sum taxes per capita payable by the family. $w_{i,t} h_{i,t}$ is the wage per hour times hours worked by individual family member i. b are real unemployment benefits paid to unemployed family members. The family holds D_t units of a risk-free one-period nominal bond (government debt) which pays a gross nominal return $R_t \epsilon_t^b$ in period $t+1$. P_t is the aggregate price-level. ϵ_t^b denotes a serially correlated shock to the risk premium, so $\log(\epsilon_t^b) = \rho_b \log(\epsilon_{t-1}^b) + \zeta_t^b$, where $\rho_b \in [0,1)$ and $\zeta_t^b \overset{iid}{\sim} N(0, \sigma_b^2)$. It drives a wedge between the return on assets held by families and the interest rate controlled by the central bank, see Smets and Wouters (2007). The family owns representative shares of all firms in the economy. Ψ_t denotes real dividend income per member of the family arising from these firms' profits. Dividend income splits into

$$\Psi_t = \Psi_t^C + \int_0^{1-u_t} \Psi_{i,t}^L di, \tag{7}$$

where Ψ_t^C and $\int_0^{1-u_t} \Psi_{i,t}^L di$ are the profits arising in the differentiating industry and in the labor good industry, respectively; see Section 2.2.

2.1.2 The Family's First-order Conditions

The family maximizes welfare function (3) by choosing consumption, c_t, and bond-holdings, D_t, subject to its budget constraint (6). The corresponding Euler equation is given by

$$1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t \epsilon_t^b}{\Pi_{t+1}} \right\}, \tag{8}$$

where marginal utility of consumption is $\lambda_t = (c_t - \varrho c_{t-1})^{-\sigma}$. The optimal consumption plan also satisfies the transversality condition

$$\lim_{j \to \infty} E_t \left\{ \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{D_{t+j}}{P_{t+j}} \right\} = 0, \ \forall t. \tag{9}$$

2.2 Three Sectors of Production: Firms

There are three sectors of production. Firms in the first sector produce a homogenous intermediate good, which we shall call the "labor good". These firms need to find exactly one worker in order to produce. They take hours worked as their sole input into production. In the model, searching for a worker is a costly and time-consuming process due to matching frictions. Once a firm and a worker have met, they infrequently Nash-bargain over the hourly wage rate. Given this wage rate, the firm decides in each period how many hours of work it wants to hire. In other words, we entertain the right-to-manage framework of Trigari (2006). Wages in the labor sector are sticky à la Calvo (1983). Firms and workers cannot rebargain their nominal hourly wage rate in every period. This feature is deeply entrenched in New Keynesian macro-economic models in use at central banks, see for example Smets and Wouters (2005) and Edge, Kiley, and Laforte (2007).[6] Labor goods are sold to a wholesale sector in a perfectly competitive market. Firms in the wholesale sector take the intermediate labor good as their sole input and produce differentiated goods using a constant-returns-to-scale production technology. Subject to price-setting impediments à la Calvo, they sell under monopolistic competition to a final retail sector.[7] Retailers bundle differentiated goods into a homogenous consumption/investment

[6] As argued in the introduction, sticky wages in our model environment are not instrumental in generating labor market fluctuations. The contrary is the case under efficient bargaining.

basket, y_t. They sell this final good to consumers and to the government at price P_t. We next turn to a detailed description of the respective sectors. In the following, subscript index j will refer to wholesale good firm/product j. Subscript i will refer to labor good firm/firm-worker match i.

2.2.1 Retail Firms

The retail sector operates in perfectly competitive factor markets. It takes wholesale goods of type $j \in [0,1]$, labeled $y_{j,t}$, and aggregates all these varieties into the homogenous final good, y_t, according to

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1. \tag{10}$$

The cost-minimizing expenditure, P_t, needed to produce one unit of the final good is given by

$$P_t = \left(\int_0^1 P_{j,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \tag{11}$$

where $P_{j,t}$ marks the price of good $y_{j,t}$. P_t coincides with the consumer/GDP price index. The demand function for each single good $y_{j,t}$ is given by

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t. \tag{12}$$

ϵ is thus the own-price elasticity of demand for differentiated goods.

2.2.2 Wholesale Firms

Firms in the wholesale sector have unit mass and are indexed by $j \in [0,1]$. Firm j produces variety j of a differentiated good according to

$$y_{j,t} = y_{j,t}^{L,d}. \tag{13}$$

Here $y_{j,t}^{L,d}$ denotes its demand for the intermediate labor good which it can acquire in a perfectly competitive market at real price x_t^L. Real period profits of firm j, $\Psi_{j,t}^C$, are given by

$$\Psi_{j,t}^C = \frac{P_{j,t}}{P_t} y_{j,t} - y_{j,t}^{L,d} x_t^L.$$

[7] Following most of the literature we part the markup pricing decision from the labor demand decision. Kuester (2007) highlights that search and matching frictions in principle make labor a temporarily firm-specific factor of production. When price-setting and labor market activity are conducted in the same sector, real rigidities arise even under efficient bargaining. Sveen and Weinke (2007) and Thomas (2007) confirm this result in slightly different setups.

The first term gives wholesale firm revenues, the second term marks real payments for the labor good.

We follow Calvo (1983), Yun (1996) in assuming that in each period a random fraction $\omega \in [0,1)$ of firms cannot reoptimize their price.[8] Those firms which reoptimize their price in period t face the problem of maximizing the value of their enterprise by choosing their sales price, $P_{j,t}$, taking into account the pricing frictions, demand function (12) and production function (13). Assuming that firms at least break even ex ante, realizing that for any given demand the optimal factor input choice leads to marginal costs which are independent of the production level, the price-setting problem simplifies to

$$\max_{P_{j,t}} E_t \left\{ \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left[\frac{P_{j,t}}{P_{t+s}} - mc_{t+s} \right] y_{j,t+s} \right\}. \tag{14}$$

Here mc_t are real marginal costs

$$mc_t = x_t^L. \tag{15}$$

$\beta_{t,t+s} := \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the equilibrium stochastic discount factor. The typical reoptimizing wholesale firm's first order condition for price-setting is:

$$E_t \left\{ \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left[\frac{P_t^*}{P_{t+s}} - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \right] y_{j,t+s} \right\} = 0, \tag{16}$$

where P_t^* marks the optimal price. Total real profits of the wholesale (Calvo) sector are $\Psi_t^C = \int_0^1 \Psi_{j,t}^C dj$, where

$$\Psi_{j,t}^C = \left\{ \frac{P_{j,t}}{P_t} - mc_t \right\} y_{j,t} \tag{17}$$

denotes the period profits of firm j. These profits accrue to the representative family.

2.2.3 Labor Good Firms

The labor good is homogenous. Each firm in this sector consists of one and only one worker matched with an entrepreneur. In period t there is thus a mass $(1 - u_t)$ of operative labor firms. Match i can produce amount $y_{i,t}^L$ of the labor good using hours worked according to

$$y_{i,t}^L = z_t h_{i,t}^{\alpha}, \ \alpha \in (0,1). \tag{18}$$

[8] For expositional clarity, we abstract from indexation as in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), where firms which cannot re-optimize index their price to realized inflation, $\Pi_{t-1} := \frac{P_{t-1}}{P_{t-2}}$.

z_t is a labor sector-wide technology shock, which follows an AR(1) process:

$$\log(z_t) - \log(z) = \rho_z(\log(z_{t-1}) - \log(z)) + \zeta_t^z,$$

where $\rho_z \in [0,1)$ and $\zeta_t^z \overset{iid}{\sim} N(0, \sigma_z^2)$. The technology shock is identical over the different matches. Note that in our notation subscript index i denotes match/worker/labor firm i, while subscript index j pertains to wholesale firms.

2.3 Labor Market – Matching, Bargaining and Vacancy Posting

We now turn to the specification of the labor market in our model. We first describe the matching technology and then focus on the bargaining and vacancy posting decisions.

2.3.1 Matching Firms and Workers

The matching process is governed by a Cobb-Douglas matching technology

$$m_t = \sigma_m(u_t)^{\xi}(v_t)^{1-\xi}, \ \sigma_m > 0, \ \xi \in (0,1). \tag{19}$$

Here m_t is the number of new matches of workers with firms, v_t is the number of job vacancies. A searching firm finds a worker in period t with probability $q_t = \frac{m_t}{v_t}$. An unemployed worker will find a job with probability $s_t = \frac{m_t}{u_t}$.

In the U.S., according to Hall (2005), most of the variation of employment over the business cycle is explained by variations in vacancy posting while the separation rate appears to be rather stable. We therefore assume that separations occur with a constant, exogenous probability $\vartheta \in (0,1)$ in each period.[9] New matches in t, m_t, become productive for the first time in $t+1$. As a consequence of these assumptions, the employment rate $n_t := 1 - u_t$ evolves according to

$$n_t = (1-\vartheta)n_{t-1} + m_{t-1}. \tag{20}$$

[9] This view is not uncontended in the literature. For example, Fujita and Ramey (2007) argue that separation rates are correlated with business cycle indicators. In addition, they contend the view that variations in the separation rate are negligible for explaining variations in unemployment relative to variations in hiring activity. A similar point is made by Pissarides (2007) and Elsby, Michaels, and Solon (2007). In the current paper, we follow most of the literature, not least for the sake of clarity of exposition and abstract from endogenous separation decisions.

2.3.2 Wage Bargaining

Due to a fixed cost of posting a vacancy, κ, and decreasing returns-to-scale at the individual labor firm level, formed matches entail economic rents. Firms and workers bargain about their share of the overall match surplus. We follow den Haan, Ramey, and Watson (2000) in assuming that the family takes the labor supply decision for its workers. We start by describing the gain of a representative family from having an additional member i in employment.

The value (to the family) of a worker who is employed and receives nominal wage $W_{i,t}$ is

$$\begin{aligned} V_t^E(W_{i,t}) \quad = \quad & \tfrac{W_{i,t}}{P_t} h_{i,t} - \kappa^L \tfrac{h_{i,t}^{1+\varphi}}{(1+\varphi)\lambda_t} \\ & + E_t\left\{\beta_{t,t+1}(1-\vartheta)\left[\gamma V_{t+1}^E(W_{i,t}) + (1-\gamma)V_{t+1}^E(W_{t+1}^*)\right]\right\} \\ & + E_t\left\{\beta_{t,t+1}\vartheta U_{t+1}\right\}. \end{aligned} \tag{21}$$

The value of a worker in employment depends on his wage income, for which both the nominal wage, $W_{i,t}$, and the hours worked, $h_{i,t}$, matter. The final term in the first row pertains to the utility loss from working. An employed worker retains his job with probability $1-\vartheta$. In the next period, if he stays employed, he faces a probability γ that he will not be able to rebargain the nominal wage, in which case his value is $V_{t+1}^E(W_{i,t})$. Or he is able to rebargain, in which case his value reflects the optimal rebargained wage in $t+1$: $V_{t+1}^E(W_{t+1}^*)$. With probability ϑ he will be unemployed next period. The value of a worker when unemployed is given by

$$\begin{aligned} U_t \quad = \quad b \quad & + E_t\left\{\beta_{t,t+1} s_t \left[\gamma V_{t+1}^E(W_t) + (1-\gamma)V_{t+1}^E(W_{t+1}^*)\right]\right\} \\ & + E_t\left\{\beta_{t,t+1}(1-s_t)U_{t+1}\right\}. \end{aligned} \tag{22}$$

Here b are real unemployment benefits. An unemployed worker has a chance of s_t of finding a new job. In that case, he enters the same Calvo scheme as the average currently employed worker. With probability $(1-\gamma)$ he can bargain over the wage in $t+1$, with probability γ he will start working at the average nominal hourly wage rate of existing contracts in t, W_t. These assumptions ensure a sufficient degree of homogeneity across workers, which is needed to keep the model tractable.

Let $\Delta_t(W_{i,t}) := V_t^E(W_{i,t}) - U_t$ denote the family's surplus from having a worker in employment at wage $W_{i,t}$ rather than having him unemployed. A few steps of algebra show that

$$\begin{aligned}\Delta_t(W_{i,t}) \quad = \quad & \tfrac{W_{i,t}}{P_t} h_{i,t} - b - \kappa^L \tfrac{(h_{i,t})^{1+\varphi}}{(1+\varphi)\lambda_t} \\ & + E_t \left\{ \beta_{t,t+1} (1-\vartheta) \gamma \left[V^E_{t+1}(W_{i,t}) - V^E_{t+1}(W^*_{t+1}) \right] \right\} \\ & - E_t \left\{ \beta_{t,t+1} s_t \gamma \left[V^E_{t+1}(W_t) - V^E_{t+1}(W^*_{t+1}) \right] \right\} \\ & + E_t \left\{ \beta_{t,t+1} (1-\vartheta - s_t) \Delta_{t+1}(W^*_{t+1}) \right\}. \end{aligned} \tag{23}$$

Firms are worthless when they separate from a worker. The market value of a labor firm matched to a worker who receives nominal wage $W_{i,t}$ is given by

$$J_t(W_{i,t}) \quad = \quad \Psi^L_t(W_{i,t}) + (1-\vartheta) E_t \left\{ \beta_{t,t+1} \left[\gamma J_{t+1}(W_{i,t}) + (1-\gamma) J_{t+1}(W^*_{t+1}) \right] \right\}. \tag{24}$$

Here $\Psi^L_t(W_{i,t})$ are real per-period profits of the firm when the nominal wage rate is $W_{i,t}$ and $h_{i,t}$ is the firm's labor input:

$$\Psi^L_t(W_{i,t}) = x^L_t z_t h^\alpha_{i,t} - \frac{W_{i,t}}{P_t} h_{i,t} - \Phi.$$

x^L_t is the competitive price for the labor good in real terms, $\Phi \geq 0$ denotes a per-period fixed cost of production. The second term in (24) reflects that firms which survive until the next period are subject to Calvo staggering: Only with a certain probability, $1-\gamma$, will they be able to rebargain their wage.

For those firms which bargain in a given period, nominal wages are determined by means of Nash-bargaining over the match surplus:

$$\arg\max_{W_{i,t}} \left[\Delta_t(W_{i,t}) \right]^\eta \left[J_t(W_{i,t}) \right]^{1-\eta} \quad \Rightarrow W^*_t \tag{25}$$

where $\eta \in (0,1)$ denotes the family's bargaining power. This optimization takes into account that in each period, each firm sets hours worked optimally according to the usual marginal profit condition by which the marginal value product of labor is equated to the real wage rate

$$x^L_t z_t \alpha h^{\alpha-1}_{i,t} = \frac{W_{i,t}}{P_t}. \tag{26}$$

The first-order condition for the wage can then be written as

$$\eta J^*_t \underbrace{\left. \frac{\partial \Delta(W_{i,t})}{\partial W_{i,t}} \right|^* W^*_t}_{:=\delta^W_t} = (1-\eta) \Delta^*_t \underbrace{\left. \frac{-\partial J(W_{i,t})}{\partial W_{i,t}} \right|^* W^*_t}_{:=\delta^F_t}. \tag{27}$$

2.3.3 Vacancy Posting Decision

Free entry into the vacancy posting market drives the value of a vacancy to zero. In equilibrium therefore real vacancy posting costs κ equal the discounted expected value of a firm, so

$$\kappa = q_t E_t \left\{ \beta_{t,t+1} \left[\gamma J_{t+1}(W_t) + (1-\gamma) J_{t+1}(W^*_{t+1}) \right] \right\}. \tag{28}$$

The term in square brackets reflects our assumption that newly started jobs face the same Calvo rigidities as incumbent jobs. This is similar to the assumptions made in Gertler and Trigari (2006), who appeal to wage structures in multi-worker firms. Namely, with probability $(1-\gamma)$ the firm-worker pair can reset its wage. With the remaining probability, the wage is set to the average wage rate prevailing in the previous period.

2.4 Government: Monetary and Fiscal Policy

The monetary authority controls the one-month risk-free interest rate on nominal bonds, R_t. The empirical literature (see, e.g. Clarida, Galí, and Gertler, 2000) finds that simple generalized Taylor-type rules of the form

$$\begin{aligned} \log(R_t) \quad = \quad & \log\left(\frac{\overline{\Pi}}{\beta}\right)(1-\phi_R) + \phi_R \log(R)_{t-1} \\ & +(1-\phi_R)\left[\frac{\phi_\pi}{12}\log\left(\frac{\Pi^a_{t-1}}{\overline{\Pi}^{12}}\right) + \frac{\phi_y}{12}\log\left(\frac{y_t}{y}\right)\right] + \log(\epsilon^{money}_t), \end{aligned} \tag{29}$$

once linearized are a good representation of monetary policy in recent decades. Here $\Pi^a_t = \frac{P_t}{P_{t-12}}$ is year-on-year inflation, and $\overline{\Pi} = 1$ is the month-on-month gross target inflation rate. $\phi_R \in [0,1), \phi_\pi > 1$ and $\phi_y \geq 0$ are response coefficients to lagged interest rates, inflation and output, respectively. $\log(\epsilon^{money}_t) = \zeta^{\text{money}}_t \overset{iid}{\sim} N(0, \sigma^2_{\text{money}})$ is an iid log-normal shock to the monetary policy stance.

Government spending, g_t, is exogenous and evolves according to:

$$\log(g_t) - \log(\overline{g}) = \rho_g(\log(g_{t-1}) - \log(\overline{g})) + \zeta^{\text{g}}_t,$$

where $\rho_g \in [0,1)$ and $\zeta^g_t \overset{iid}{\sim} N(0,\sigma^2_g)$. $\overline{g}$ is the government's long-run target level for government expenditure. The government budget constraint is given by

$$t_t + \frac{D_t}{P_t} = u_t b + \frac{D_{t-1}}{P_t} R_{t-1} + g_t. \tag{30}$$

The government generates revenue from lump-sum taxes. It also earns income through new debt issues, $\frac{D_t}{P_t}$. On the expenditure-side appear unemployment benefits (the term involving b), debt repayment and coupon as well as government spending. We assume that fiscal policy is Ricardian.

2.5 Market Clearing

The aggregate retail good is used for private and government consumption. In addition, vacancy posting activity requires resources and so do the fixed costs of producing labor goods. Total demand is thus given by

$$y_t = c_t + g_t + \kappa v_t + n_t \Phi. \tag{31}$$

Market clearing in the retail market requires that above demand of retail goods equals total supply, which is given by $y_t = \left[\int_0^1 (y_{j,t})^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$.

For each firm j in the wholesale sector, its supply $y_{j,t} = y_{j,t}^{L,d}$, must be matched by the corresponding demand $y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} y_t$ in order to clear the wholesale market.

The total demand for the labor good is given by $y_t^L = \int_0^1 y_{j,t}^{L,d} dj$, where $y_{j,t}^{L,d}$ marks demand for the labor good by individual wholesale firm j. Market clearing requires that total demand for the labor good equals the supply of the labor good which is given by $y_t^L = z_t \int_0^{1-u_t} h_{i,t}^{\alpha} di$.

3 Calibration to the US

We calibrate the model to the US using data from 1964:q1 to 2006:q3. The sample start coincides with the samples used by Gertler and Trigari (2006) and Krause and Lubik (2007). All data are taken from the Federal Reserve Bank of St. Louis' database FRED II except for the Help Wanted Advertising Index which was obtained from the Conference Board. We use the Hodrick-Prescott filter with a conventional filter weight of 1,600 to extract the business cycle component from the data in logs.

As to the underlying data, output is measured by nominal output in the business sector divided by the GDP deflator. Total hours worked are hours worked in the business sector. Total wages are the compensation in the business sector divided by the GDP deflator, and wages per employee are obtained by dividing the former by the number of employees in the business sector. Real hourly wages are measured by the real compensation per hour in the business sector, again obtained by dividing the nominal quantity by the GDP deflator. Vacancies are measured by the Conference Board's index of Help-Wanted Advertising. We use the civilian unemployment rate among those 16 years old and older. The inflation rate is the (quarter-on-quarter) GDP inflation rate. The interest rate is the quarterly average of the FED Funds rate. We note that both the interest rate and the inflation rate are not annualized in the figures reported below.

The model runs at a monthly frequency in order to be able to match stocks and flows in the US labor market. The calibrated parameter values and the targets are summarized in Table 1. Turning first to preferences, the time-discount factor, β, is chosen so as to match an annual real rate of 2.45%. The curvature of disutility of work, $\varphi = 2$, follows the estimates of Domeij and Flodén (2006). The coefficient of relative risk aversion, $\sigma = 1.5$, follows the estimates in Smets and Wouters (2007). Habit persistence, ϱ, is set to a value of 0.7, in line with Smets and Wouters (2007). Scaling parameter κ^L is set so as to meet our target for hours worked per employee of $h = \frac{1}{3}$.

Turning to the labor good sector and the labor markets, we set $\alpha = 0.99$, implying only mildly decreasing returns to hours worked per worker. We set the elasticity of matches with respect to unemployment to $\xi = 0.5$, which is in the range of reasonable values suggested by Petrongolo and Pissarides (2001). The bargaining power is set to a conventional value of $\eta = 0.5$. The monthly separation rate of $\vartheta = 0.03$ follows Shimer (2005). The average contract duration for wages is set to 5 months, which amounts to the same contract duration which we use for prices. The degree of nominal rigidity of wages is $\gamma = 0.8$. This is roughly consistent with panel data when not adjusting for possible reporting errors, cp. Gottschalk (2005). The same author, however, also shows that the duration of wage contracts considerably increases when eliminating possibly spurious statements. Doing so, the hazard rate of a wage change peaks at 12 months, leading us to consider also more wage rigidity in the impulse responses reported in Section 4.

As regards the labor market steady state, we target an unemployment rate of $u = 0.0588$ in line with the data average and a quarterly probability of finding a worker of 70%. The latter figure follows den Haan, Ramey, and Watson (2000) and implies a monthly job filling probability of $q = 0.33$. The efficiency parameter of the matching function is set to $\sigma_m = 0.398$ in order to match the above two assumptions regarding the labor market steady state. With the same target, the cost of posting a vacancy is set to $\kappa = 0.0051$. The technology parameter is set to $z = 3.152$, which ensures that output is equal to unity in steady state. All steady state values reported can thus be interpreted as ratios to GDP.

We calibrate the fixed costs to $\Phi = 0.0092$. This amounts to 0.86% of steady state output of

Table 1: Parameters and their calibrated values

Parameter	Value	Explanation; Target/Reference
Preferences		
β	0.998	time-discount factor; matches annual real rate of 2.5 percent.
φ	2	labor supply elasticity of 0.5; Domeij and Flodén (2006).
σ	1.50	risk aversion; Smets and Wouters (2007).
ϱ	0.70	external habit persistence; Smets and Wouters (2007).
κ^L	372.31	scaling factor to disutility of work; targets $h = 1/3$.
Bargaining and Labor Good		
α	0.99	labor elasticity of production; close to constant returns to scale.
ξ	0.50	elasticity of matches w.r.t. unempl.; Petrongolo and Pissarides (2001).
η	0.50	bargaining power of workers; conventional value.
ϑ	0.03	monthly rate of separation; Shimer (2005).
γ	0.80	avg. duration of wages of 5 mths; same stickiness as for prices.
σ_m	0.398	efficiency of matching; reconciles m with target for u, q.
κ	0.0051	vacancy posting costs; reconciles m with target for u, q.
z	3.1526	technology; targets output $y = 1$.
Φ	0.0092	fixed cost associated with labor; targets $std(\widehat{u}_t)$.
Wholesale Sector		
ϵ	11	markup; conventional price-markup of 10 percent over marginal costs.
ω	0.80	Calvo stickiness of prices; avg. duration of 5 months; Bils and Klenow (2004).
Government		
ϕ_π	1.50	response to inflation; conventional Taylor rule.
ϕ_y	0.50	response to output; conventional Taylor rule.
ϕ_R	$0.85^{\frac{1}{3}}$	interest rate smoothing; 0.85 at quarterly frequency.
$\overline{g}$	0.347	government spending; targets consumption-GDP ratio of 0.65.
b	0.3825	unemployment benefits; targets replacement rate $\frac{b}{wh} = 0.4$.
Correlation of Shocks and Size of Innovations		
ρ_g	0.89	autocorr. of government spending; 0.79 in quarterly data.
ρ_z	0.82	autocorr. of technology shock; 0.67 in quarterly data.
		(identified using the model's resource constraint).
ρ_b	0.95	autocorr. of premium shock; 0.90 in quarterly terms.
σ_{money}	0.043	standard deviation of innovation to Taylor rule; data.
σ_g	0.674	std. dev. of innov. to gov. spending; match std. dev. (0.87) in qtrly data.
σ_z	0.571	std. dev. of innov. to tech. shock; match std. dev. of techn.(0.69) in qtrly data.
σ_b	0.102	std. dev. of innov. to premium shock; targets $std(\widehat{y}_t)$.

Notes: The Table reports calibrated parameter values. The model is calibrated to the US using data from 1964:q1 to 2006:q3; see the main text for details. As to the shocks, the government spending and technology shocks are estimated using quarterly data. The autocorrelation coefficients and the standard deviation of the respective innovation at a monthly frequency were chosen such that the resulting series would imply the same first-order autocorrelation coefficient and the same standard deviation as the quarterly estimates if the monthly series were to be time-aggregated to a quarterly frequency. See the main text for details.

an individual labor firm being absorbed by fixed costs or, alternatively, 0.95% of the value of its revenue. In choosing this number, we target the degree of fluctuations in unemployment in data.[10]

The markup in the wholesale sector is set to a conventional value of 10%, implying $\epsilon = 11$. Following Bils and Klenow (2004) the average contract duration of prices is set to 5 months, so $\omega = 0.80$.

We rely on a monthly adaptation of a standard Taylor rule (i.e. a long-run response to inflation with $\phi_\pi = 1.5$ and to output with $\phi_y = 0.5$), with the coefficient on interest rate smoothing being set to $\phi_R = 0.85^{1/3}$. This roughly corresponds to a quarterly interest rate smoothing coefficient of 0.85 which is standard in the literature. In order to determine the steady state level of "government spending", we target a consumption output ratio of 65% which is the data average over the sample period. We take $c/(c+g)$ as the model counterpart of this ratio. By this and equation, (31) $\overline{g} = 0.347$ (some resources are also used for vacancy posting costs and for job-related fixed costs). We target a steady state replacement rate of $\frac{b}{wh} = 0.4$, a conventional value which is used for example in Shimer (2005) and which is close to the evidence reported in Engen and Gruber (2001). Our target for the replacement rate implies $b = 0.382$.

The resulting steady state for some of the model variables is reported in Table 2. As argued, profits in the labor sector are small. As a result, the value of labor firms, J, amounts to only 1.5% of monthly output. The surplus of workers is an order of magnitude larger, $\Delta = 0.3589$. This has implications for the elasticity of unemployment with respect to benefits, on which Section 4.4 will comment.

Returning to the calibration, the technology process is modeled as an AR(1) process, so $\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \zeta_t^z$, where a hat denotes percent deviation of the corresponding series from steady state and $\zeta_t^z \overset{iid}{\sim} N(0, \sigma_z^2)$. We first use the model's inverted production function $\widehat{z}_t = \widehat{y}_t - \left(\alpha \widehat{h}_t + \widehat{n}_t\right)$ to identify the time series for the technology shock from the data as follows. Time-aggregation implies that $\widehat{z}^q_{t_q} = \widehat{y}^q_{t_q} - \left(\alpha \widehat{h}^q_{t_q} + \widehat{n}^q_{t_q}\right)$, where superscript q denotes quarterly aver-

[10] To obtain empirical evidence for the size of overhead labor costs, Ramey (1991) uses the proportion of non-productive workers in total manufacturing employment as a proxy. Using BLS data from 1985 to 2006 the proportion of non-productive workers varies between 27 and 30 percent. Basu (1996) argues that even higher values are plausible if more general overhead costs would be taken into account. These numbers, though indicative of possibly substantial fix costs, are not directly interpretable as parameter Φ in our calibration but rather constitute upper bounds for this parameter. In our model, Φ indicates costs which are fixed with respect to hours worked per employee. The measures just cited define fixed costs more broadly and also include costs which to a certain extent are fixed with respect to the number of employees, for example.

Table 2: Steady State

Variable	Value	Description
y	1	output.
c	0.6439	consumption.
wh	0.9562	wage per employee.
u	0.0588	unemployment rate.
v	0.0854	vacancies (as share of labor force).
s	0.4802	probability of finding a job within a month.
q	0.3306	probability of finding a worker within a month.
$b/(wh)$	0.40	unemployment insurance replacement rate.
$\kappa v/y \cdot 100$	0.0432	percent share of output lost to vacancy posting.
$\Phi/(x^L z h^\alpha)$	0.0095	share of a labor firm's revenue lost to fixed costs.
Ψ^C/y	0.0909	profit share (wholesale sector) in total output.
$\Psi^L n/y$	0.0005	profit share (labor sector) in total output.
J	0.0153	value of a labor firm.
Δ	0.3589	surplus of the worker from working.

Notes: Steady state for some variables implied by the calibration in Table 1.

ages and time index t_q indicates that one time step is one quarter. So for example $\widehat{y}^q_{2005:q1} = \frac{1}{3}\left(\widehat{y}_{2005:m1} + \widehat{y}_{2005:m2} + \widehat{y}_{2005:m3}\right)$. The monthly autocorrelation parameter, ρ_z, and the standard deviation of the innovation, σ_z, are then obtained as follows. We estimate an AR(1) process on the quarterly average of $\widehat{z}_t$, i.e. $\widehat{z}^q_{t_q} = \rho^q_z \widehat{z}^q_{t_q-1} + \zeta^{z,q}_{t_q}$, by ordinary least squares. Here $\widehat{z}^q_{t_q-1}$ denotes the average of the technology shock during the previous quarter. We then choose ρ_z and σ_z such that the first-order autocorrelation of the quarterly average of this monthly technology process matches the counterparts in the estimated quarterly process.

Government spending is represented by an AR(1) process estimated on the HP(1,600) filtered government consumption data for the sample period (detrended by the GDP deflator). Just as with the technology shock, we adjust the autocorrelation parameter and the standard deviation of the innovation in the model in such a way, that the monthly series for government spending once aggregated to quarterly numbers would fit the serial correlation and standard deviation as estimated from the quarterly data.

The standard deviation of the monetary policy shock is obtained as follows. We obtain the residual (plus a constant term) by using actual data in Taylor rule (29). We use monthly observations of the Federal funds rate and its one month lagged value. The one month lagged year-on-year GDP inflation rate in that formula is proxied for by year-on-year CPI inflation, which is available at a monthly frequency. The deviation of output from steady state in each month of the sample is proxied by the seasonal component of hp(14,400) filtered data of the index of industrial production. The standard deviation, σ_{money}, is computed as the standard deviation of the residual in (29) such obtained.

Finally, the standard deviation of the risk premium shock is set such that the standard deviation of the output series in our model coincides with the standard deviation of hp-filtered output in the data. This implies $\sigma_b = 0.102$. The serial correlation of the risk premium shock is set to 0.95, which translates into an autocorrelation of a quarterly aggregate of this shock of around 0.9.

Table 3 shows the second moments of endogenous variables as implied by the model, namely unconditional standard deviations, the contemporaneous correlation with output as well as the serial correlation coefficients. This information can be compared to the moments implied by the data which are given in brackets. The model captures both the standard deviations and

Table 3: Second Moments of the Model

Variable	Meaning	std		std to std(y)		corr with y		AR(1)	
$\widehat{y}_t$	output	1.91	(1.92)	1.00	(1.00)	1.00	(1.00)	0.92	(0.86)
$\widehat{R}_t$	nominal rate	0.41	(0.41)	0.21	(0.21)	0.02	(0.37)	0.89	(0.83)
$\widehat{\Pi}_t$	inflation	0.23	(0.29)	0.12	(0.15)	0.21	(0.15)	0.36	(0.47)
$\widehat{h}_t + \widehat{n}_t$	total hours	2.08	(1.74)	1.09	(0.91)	0.89	(0.88)	0.87	(0.92)
$\widehat{w}_t + \widehat{h}_t + \widehat{n}_t$	total compensation	2.00	(1.90)	1.05	(0.99)	0.95	(0.85)	0.87	(0.91)
$\widehat{w}_t + \widehat{h}_t$	compens. per empl.	1.37	(0.90)	0.72	(0.47)	0.90	(0.49)	0.82	(0.81)
$\widehat{h}_t$	hours per worker	1.49	(0.49)	0.78	(0.25)	0.79	(0.72)	0.82	(0.77)
$\widehat{w}_t$	hourly compensation	0.43	(0.85)	0.22	(0.45)	0.09	(0.11)	0.88	(0.78)
$\widehat{u}_t$	unemployment	10.93	(11.01)	5.74	(5.74)	-0.98	(-0.87)	0.91	(0.92)
$\widehat{v}_t$	vacancies	13.77	(13.15)	7.23	(6.85)	0.88	(0.90)	0.76	(0.91)

Notes: The Table reports summary statistics of the model and compares those to the data (values in brackets). All statistics refer to the variables being measured at a quarterly frequency. Model variables are averaged/aggregated over the quarter so as to bring their measurement in line with the data. The data are hp(1,600) filtered. The third column reports the standard deviation of the series, the fourth its standard deviation relative to that of GDP. The fifth column shows the cross-correlation with GDP. The final column reports first-order autocorrelation coefficients. Note that these refer to the autocorrelation measured quarter on quarter. The computations for the data are performed on the sample from 1964:q1 to 2006:q3.

the overall co-movement in the data. The compensation per employee is still a bit more volatile than in the data, while real hourly wages are not volatile enough. As a consequence, total hours worked and, especially, hours per worker fluctuate more than their counterparts in the data. Most importantly, however, the model reproduces the substantial fluctuations in unemployment and vacancies in the data. This happens despite the tight link that, in the right-to-manage model, exists between labor profits and the relatively smooth wages per employee which are only about 3/4 as volatile as output in percentage terms.

Using this calibration, we next turn to illustrating the wage channel to inflation, the importance of the per-period fixed costs in labor firms and the implications of the model for the response of unemployment to changes in benefits.

4 Wage Channel, Unemployment Fluctuations and Benefits

In the Introduction we identified three main features of our model: (a) that the model contains a proper wage channel, (b) that it reproduces the fluctuations of unemployment over the business cycle and (c) that it implies a reasonable elasticity of steady state unemployment with respect to changes in benefits.

This Section analyzes the three features in detail: Subsection 4.1 presents key equations of the model to illustrate the model's wage channel and to explain the mechanism which induces unemployment fluctuations. A wage channel, in our understanding, is present whenever wages and the wage-setting process have a direct influence on inflation. In our model this materializes itself primarily in two observations which we corroborate in Subsection 4.2: First, a higher degree of wage rigidity induces a weaker response of inflation to aggregate shocks. Second, higher wages all else equal translate into higher inflation. Subsection 4.2 also clarifies in which respect these defining characteristics of the wage channel are present under right-to-manage bargaining (RTM) but not under efficient wage bargaining (EB). Moving to point (b) above, Table 3 already showed that the model can reproduce the fluctuations of unemployment over the business cycle under a suitable calibration. Subsection 4.3 makes clear that the value of fixed costs is crucial for this result. Finally, Subsection 4.4 examines by how much unemployment would rise in the long-run if unemployment benefits were to rise in our model environment.

4.1 Wage Channel and Unemployment Fluctuations – Key Equations

For expositional clarity, in this Subsection we abstract from wage rigidity and set the wage stickiness parameter γ to zero. In this case, all firms pay the same wage rate and all workers work the same number of hours. This allows us to drop superscript * and subscript index i in the following exposition.

Under RTM, workers and firms bargain about the hourly wage rate only. At this wage rate a labor firm faces a perfectly elastic labor supply. The first-order condition for hours worked equates the marginal value product of labor and the real hourly wage rate:[11]

$$x_t^L \alpha z_t h_t^{\alpha-1} = w_t.$$

Since the marginal cost of a price-setting firm is $mc_t = x_t^L$, rewriting above equation yields

$$mc_t = \frac{1}{\alpha}\frac{w_t h_t}{y_t^L}. \tag{32}$$

Equation (32) implies that higher wages all else equal induce higher inflation and that stickiness in wages all else equal translates into stickiness of the marginal costs of price-setting firms. This stickiness translates into a muted response of inflation to shocks (when compared to a model with more flexible wages) via the New Keynesian Phillips curve. Wages and anything affecting the wage-setting process thereby have a direct effect on inflation.

We next clarify the relation between the introduction of a period-by-period fixed cost associated with jobs, Φ, and the fluctuations of unemployment over the business cycle. Under the assumption of no wage rigidity, vacancy posting condition (28) simplifies to

$$\kappa = q_t E_t \left\{\beta_{t,t+1} J_{t+1}\right\}. \tag{33}$$

Using this in the definition of the market value of the firm (the simplified version of (24)) yields an expression for J_t which depends on contemporaneous variables only. Putting this expression back into (33) yields

$$\frac{\kappa}{q_t} = E_t\left\{\beta_{t,t+1}\left[\Psi_{t+1}^L + \frac{\kappa}{q_{t+1}}\right]\right\}.$$

[11] Under efficient bargaining, a firm and a worker jointly bargain over the wage and hours worked: $\arg\max_{w_t,h_t} [\Delta_t]^\eta [J_t]^{1-\eta}$. The corresponding first-order conditions under efficient bargaining are as follows: for the wage: $\eta J_t = (1-\eta)\Delta_t$, and for hours worked: $x_t^L \alpha z_t h_t^{\alpha-1} = \kappa^L \frac{h_t^\varphi}{\lambda_t}$. Under EB hours are set so as to equate the marginal value product of labor and the worker's marginal rate of substitution between leisure and consumption. Wages therefore do not play a direct role in influencing marginal costs of price-setting firms.

Linearizing this around the steady state, one obtains

$$-\widehat{q}_t = E_t\left\{\widehat{\beta}_{t,t+1}\right\} + [1-(1-\vartheta)\beta]\, E_t\left\{\widehat{\Psi}^L_{t+1}\right\} - \beta(1-\vartheta)E_t\left\{\widehat{q}_{t+1}\right\}.$$

There is, neglecting fluctuations in the pricing kernel, a one-to-one relationship between percentage fluctuations in expected per-period labor profits, $E_t\left\{\widehat{\Psi}^L_{t+1}\right\}$, and percentage fluctuations in the probability of finding a worker, $\widehat{q}_t$. The more per-period profits react to the business cycle, the more will the vacancy posting activity react to the business cycle – and thus the more will unemployment react.

Fixed costs in period profits amplify fluctuations in labor profits in percentage terms. The revenue of a labor firm is given by $x^L_t z_t h^\alpha_t$. Using the first-order condition for hours worked, $\alpha x_t z_t h^\alpha_t = w_t h_t$, the share of *revenue* of the firm that is paid to labor is given by $\alpha \in (0,1)$ and thus constant over the business cycle. As a result, per-period profits of a labor firm can be expressed as

$$\Psi^L_t = \frac{1-\alpha}{\alpha} w_t h_t - \Phi. \tag{34}$$

Once fixed costs associated with a job are positive, labor costs are still proportional to revenue as in the previous literature, e.g. Trigari (2006), but they no longer are proportional to *profits*. Percentage fluctuations in profits can then be larger than percentage fluctuations in wages. In particular, linearizing (34) around steady state gives

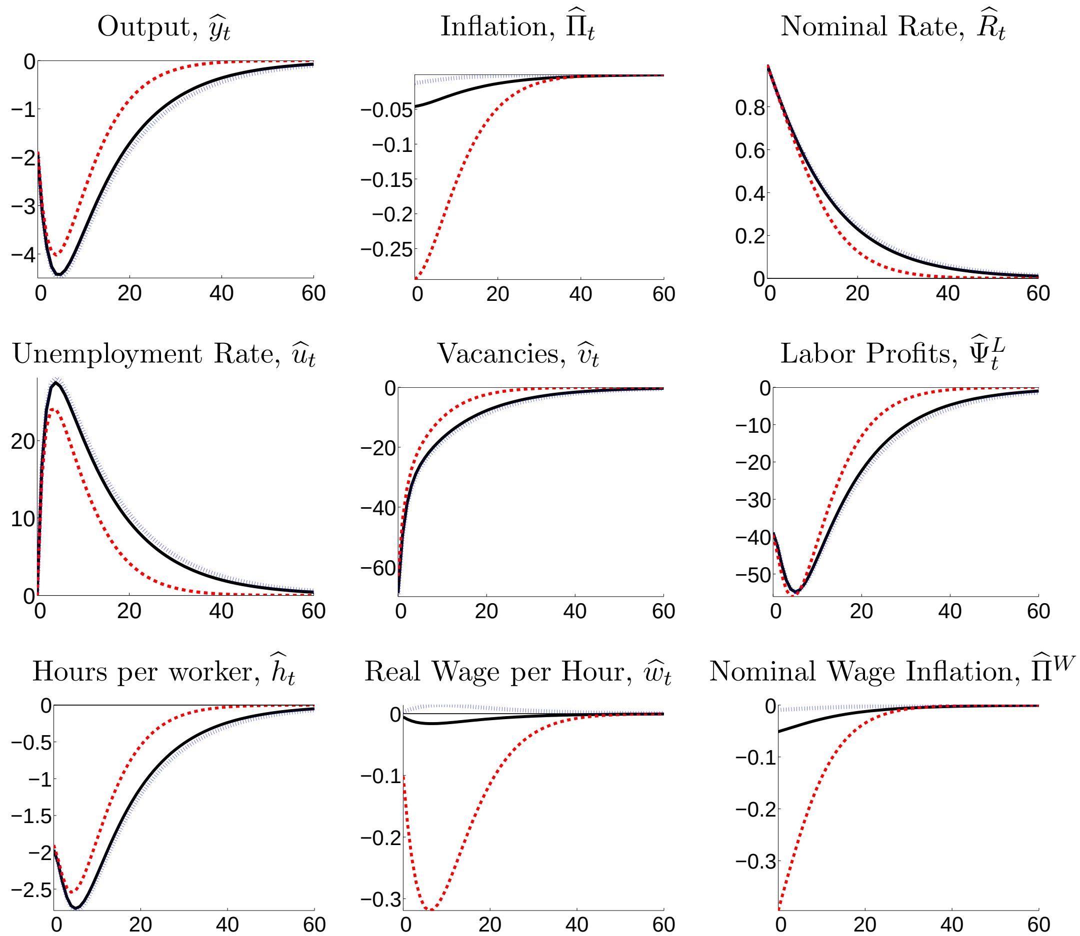
$$\widehat{\Psi}^L_t = A\left(\widehat{w}_t + \widehat{h}_t\right). \tag{35}$$

In percentage terms, fluctuations in labor profits are linked to percentage fluctuations in wages per employee by a factor of proportionality $A = \frac{\frac{1-\alpha}{\alpha}wh}{\frac{1-\alpha}{\alpha}wh-\Phi}$ which is larger than unity if $\Phi > 0$. For any given level of fluctuations in wages per employee, labor profits associated with a job will be the more volatile in percentage terms, the more the fixed costs consume of a firm's revenue, i.e. the lower the labor firm's steady state profit is. With a suitable choice of calibration for the size of fixed costs Φ, unemployment rates exhibit the desired amplitude over the business cycle. In our calibration $A = 19.7239$.

4.2 The Wage Channel – Simulations

We next turn to graphically illustrate the wage channel. Figure 1 shows impulse responses to a monetary policy shock for different degrees of nominal wage rigidity. All graphs refer to variables measured at the monthly frequency implied by the model. The black solid line marks

Figure 1: Impulse Responses to a Monetary Policy Shock – the Effect of Wage Rigidity

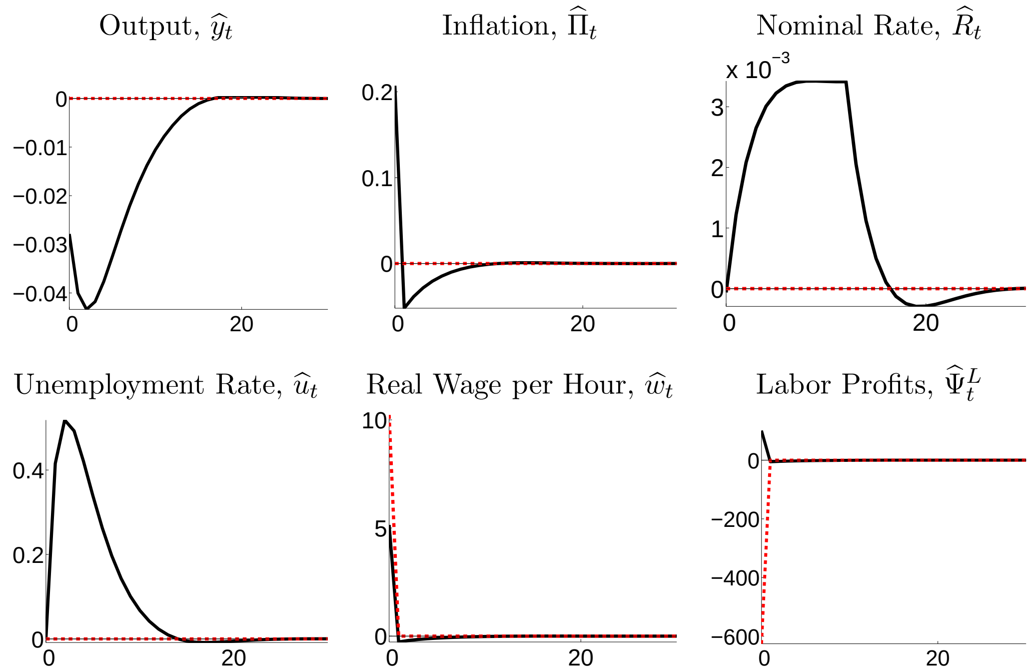


Notes: The Figure shows percentage responses (1 in the plots corresponds to a 1% increase over the respective steady state value) of endogenous variables to a one percent monetary policy shock. All variables are measured at a monthly frequency. A time period in the graphs is one month. The black solid line marks the calibrated benchmark model (the average contract duration is 5 months). The red dashed line shows the case of lower wage rigidity (the average contract duration is 2 months). The blue dotted line corresponds to the case of higher wage rigidity (the average contract duration is 12 months). The wage rigidity in the model is a rigidity in nominal hourly wage rates. Nominal wage inflation is defined as $\widehat{\Pi}_t^W := (\widehat{W_t/W_{t-1}})$.

the baseline calibration which features a wage rigidity parameter of $\gamma = 0.8$ implying an average wage duration of 5 months. The red dashed line shows impulse responses in an economy with lower wage rigidity ($\gamma = 0.5$, so wages are optimized on average every second month). A blue dotted line reports the impulse responses when $\gamma = 11/12$, which implies an average wage duration of 12 months. The real wage rate, $\widehat{w}_t$, falls less strongly in response to a monetary tightening when nominal wages are more rigid. As a consequence also inflation falls less sharply – illustrating one of the defining properties of the wage channel. The same is not true under EB, as Krause and Lubik (2007) show.[12]

Figure 2 illustrates that in our model with RTM all else equal higher wages *directly* induce higher inflation while again this is not the case under EB. In the simulation underlying the Figure the

Figure 2: Increase in Bargaining Power – Right-to-manage vs. Efficient Bargaining



Notes: The Figure shows percentage responses (1 in the plots corresponds to a 1% increase above steady state) of endogenous variables to an increase in the worker's bargaining power. The workers' bargaining power increases from 0.5 to 0.6 under the baseline calibration. The shock is white noise. All variables are measured at a monthly frequency. A time period in the graphs is one month. A black solid line refers to the benchmark model under right-to-manage bargaining. The red dashed line reports impulse responses for the same model but with efficient bargaining. For comparability, both models do not feature any nominal wage rigidity ($\gamma = 0$). The calibration for EB uses the same targets as the calibration for RTM in Table 1. See Table 5 in the Appendix for the steady state and the parameters under EB.

[12] The reason being that under EB marginal costs are related to the marginal rate of substitution of the worker between consumption and leisure and not to the wage rate, cf. also footnote 11.

bargaining power unexpectedly rises from $\eta = 0.5$ to $\eta = 0.6$ in $t = 0$. The bargaining power is reset to its steady state level, $\eta = 0.5$, in all following periods. We abstract from wage rigidity. Under both bargaining schemes the rise in the bargaining power of workers triggers a sharp increase in hourly wages. Under RTM this immediately translates into a rise in inflation, just as equation (32) would have suggested (black solid line). This response is absent under EB (red dashed line) where movements in wages, unless they affect the hiring decisions of firms, affect nothing else but for the distribution of the joint surplus of workers and firms. In above scenario, the bargaining power rises only in period $t = 0$, so there is no effect on future wages and profits. The vacancy posting decisions today and future unemployment are therefore not affected under EB. As a consequence, even though wages in $t = 0$ rise sharply under EB there is no effect on inflation.[13]

4.3 The Role of Fixed Costs – Simulations

Figure 3 shows impulse responses to a monetary policy shock for various values of the job-related fixed cost, Φ (with wage ridigity "switched on" again). The calibrated model is shown as a black solid line. The red dashed line shows the case without fixed costs.

While the responses of output, interest rates and inflation are hardly affected by the size of Φ, the response of the unemployment rate is dampened by a factor of two and a half when no fixed costs are present.[14] Much of the response of employment instead shifts towards a reduction at the intensive margin (hours worked per employee fall by more). As an intermediate case, Figure 3 also shows impulse responses when the factor of proportionality is halfway between the two cases just shown. This case sets $\Phi = 0.0087$, implying $A = 10$ (green dashed-dotted line).[15] In line with the intuition underlying equation (35), unemployment reacts by more than in the complete absence of fixed costs but still a long way less than in our model calibrated to the US data.

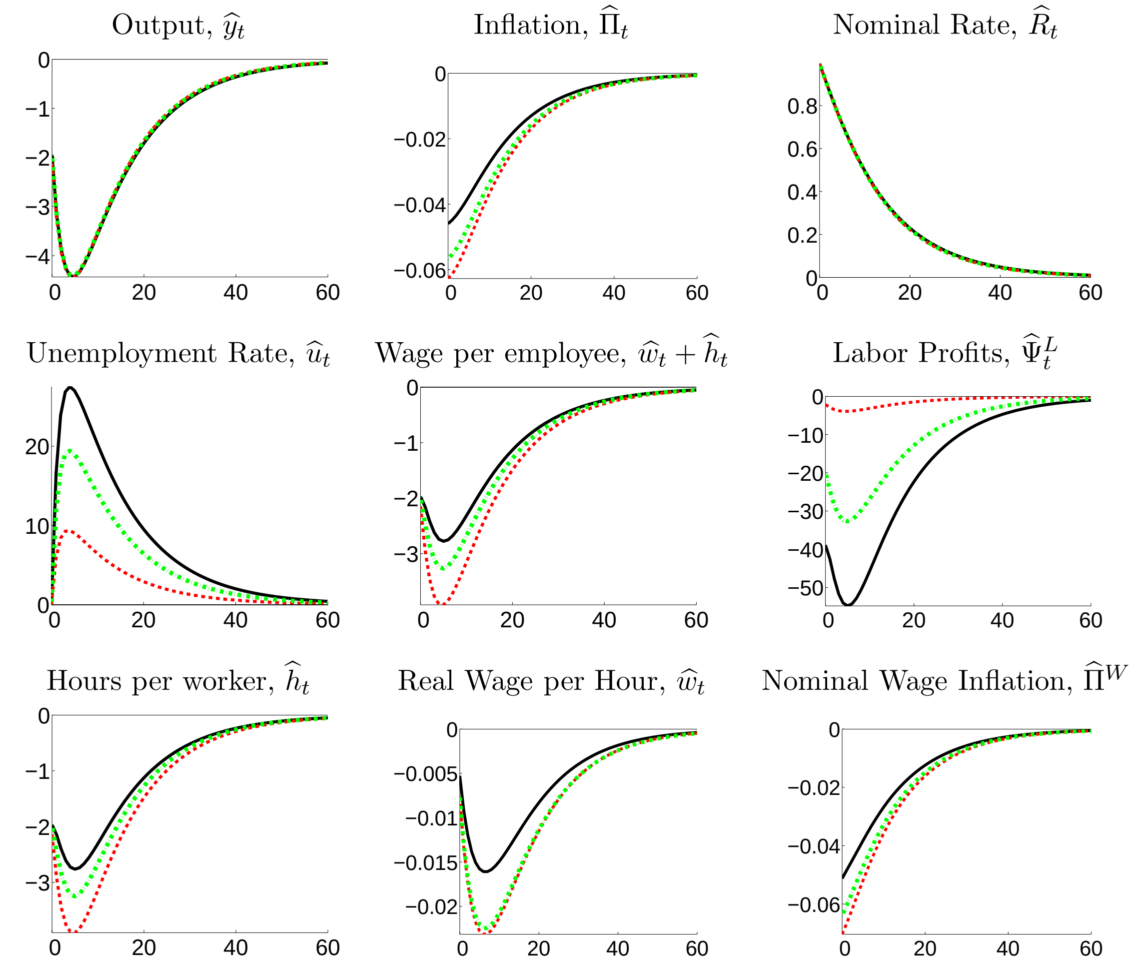
Table 4 corroborates this result. The model underlying this Table relies on RTM and the

[13] Appendix A.3 reports details for the steady state and the linearized model underlying the impulse responses with EB.

[14] As in the previous Figures, the response of the unemployment rate, $\widehat{u}_t$, corresponds to the percent increase of the unemployment rate above its steady state ($\equiv \frac{u_t - u}{u} \cdot 100$.)

[15] The reason for the non-linear increase of A in fixed costs lies in the strong non-linearity of factor $A = \frac{\frac{1-\alpha}{\alpha}wh}{\frac{1-\alpha}{\alpha}wh - \Phi} \geq 1$ as fix costs Φ rise towards their upper bound.

Figure 3: Impulse Responses to a Monetary Policy Shock - the Effect of Fixed Costs



Notes: The Figure shows percentage responses (1 in the plots corresponds to a 1% increase over the respective steady state value) of endogenous variables to a one percent monetary policy shock. All variables are measured at a monthly frequency. A time period in the graphs is one month. The black solid line marks the calibrated benchmark model (fixed costs $\Phi = 0.0092$). The calibration implies a factor of proportionality $A = 19.72$. The red dashed line shows the case of no fixed costs, so $A = 1$. The green dashed-dotted line corresponds to the intermediate case with fixed costs $\Phi = 0.0087$ implying a factor of proportionality of $A = 10$.

same calibration as used so far but does not account for job-related fixed costs, so $\Phi = 0$. Similar to Table 3 it compares the second moments in the model under this calibration to their counterparts in the data. The standard deviation of output, the contemporaneous correlation of unemployment and vacancies with output as well as the serial correlation properties of output, unemployment rates and vacancies are hardly affected when removing job-related fixed costs. However, in the absence of fixed costs the right-to-manage model fails to reproduce the *amplitude* of fluctuations of both unemployment and vacancies over the business cycle by a wide margin (cp. column "std"). We conclude that the fixed costs are instrumental for amplifying the effect of shocks on unemployment in the model.

Table 4: Second Moments of the Model - No Fix Costs

Variable	Meaning	std		std to std(y)		corr with y		AR(1)	
$\widehat{y}_t$	output	1.89	(1.92)	1.00	(1.00)	1.00	(1.00)	0.92	(0.86)
$\widehat{u}_t$	unemployment	3.48	(11.01)	1.84	(5.74)	-0.97	(-0.87)	0.90	(0.92)
$\widehat{v}_t$	vacancies	4.43	(13.15)	2.35	(6.85)	0.84	(0.90)	0.74	(0.91)

Notes: Same as Table 3 except that the model does not feature period-by-period fixed costs ($\Phi = 0$, so $A = 1$). The Table reports summary statistics of the model and compares those to the data (values in brackets). Refer to Table 3 for details.

4.4 The Elasticity of Unemployment with Respect to Benefits

When calibrating the textbook search and matching model with EB to low steady state profits associated with jobs, the resulting model generates reasonably strong variations of unemployment over the business cycle, e.g. Hagedorn and Manovskii (2006). Under EB, such a calibration additionally implies that workers are close to indifferent between taking up work and entering unemployment. Small changes in benefits can then have a large effect on the incentives to work for a given wage. As a consequence, these calibrations tend to imply a large drop of the unemployment rate when benefits are reduced. The latter observations have lead some authors, notably Costain and Reiter (2005) and Mortensen and Nagypal (2007), to question the underlying mechanism which leads to the amplification of unemployment fluctuations over the cycle. This Subsection shows that the RTM model with fixed costs as calibrated in Section 3 is not subject to the same criticism.

Under both RTM and EB, the first-order condition for the bargained wage can be expressed as a suitably modified surplus sharing rule:

$$\eta J_t^* \delta_t^W = (1-\eta)\Delta_t^* \delta_t^F .$$

Here δ_t^W gives the *rise* in worker surplus when hourly wages rise, while δ_t^F gives the *fall* in the firm's profits when the wage rises. Under EB, $\delta_t^W = \delta_t^F$ in every period and especially in steady state. As argued above, since $\eta J = (1-\eta)\Delta$, EB implies that whenever the value of labor firms is small (as it needs to be to achieve sufficient fluctuations of unemployment) the worker's surplus needs to be small, too.

This is not the case under RTM, where typically the worker's gain and the firm's loss from a wage increase are not of the same size. For the RTM bargaining scheme one can show that in steady state

$$\eta J \left[\frac{\alpha}{\alpha-1} - \frac{1}{\alpha-1} \frac{mrs}{w} \right] = (1-\eta)\Delta.$$

Thus if in steady state the worker's marginal rate of substitution between leisure and consumption exceeds the wage rate, the term in square brackets exceeds unity. As a result, a higher value obtains for the worker's surplus, Δ, than under EB – for any value of a labor firm, J, and for any parametrization for the bargaining power of workers.[16] In particular, for the calibration of the RTM model to US data discussed in Section 3 the model yields the following values in the modified surplus sharing rule

$$0.5\, J \underbrace{\left[\frac{.99}{.99-1} - \frac{1}{.99-1} \frac{3.5123}{2.8687} \right]}_{=\,23.43} = 0.5\, \Delta.$$

In that calibration, labor firms' profits in steady state', Ψ^L, are just a small 0.05% percent of the value of their revenue. This implies that the value of a job to a labor firm in steady state is as low as $J = 0.0153$, or 1.5% of a month's output. The job, however, is valued much more by the worker. The worker's surplus is an order of magnitude larger than the value of the labor firm, with $\Delta = 0.3589$ corresponding to roughly 38% of a worker's wage per month.

As a result, an increase in benefits does not cause a dramatic rise in steady state unemployment rates: increasing the replacement rate, $\frac{b}{wh}$, from 40% to 41% in the calibrated economy with RTM raises the unemployment rate in the long-run by 1.11% (from 5.88 pp. to 5.95 pp.). This is an order of magnitude lower than the values reported for EB by Costain and Reiter (2005). For purposes of comparison, Nickell and Layard (1999) find that this semi-elasticity of the unemployment rate with respect to the replacement rate is 1.3, while Costain and Reiter (2005) favor a value of 2.[17] We view this property of the model as a further argument in favor of using

[16] The condition $mrs > w$ is not special to RTM but also holds for the calibration with EB used in Figure 2, cf. Table 5 in the Appendix. In fact, under EB, as $\alpha \to 1$, $mrs > w$ becomes a necessary condition for positive ex-post profits of labor firms and thus for the existence of an equilibrium with positive hiring costs.

RTM instead of EB in models with unemployment fluctuations.[18]

5 Conclusions and Outlook

The current paper has presented a New Keynesian model with search and matching frictions which (a) in many elements is similar in structure to policy-models without equilibrium unemployment. Most importantly, the model implies a wage channel to inflation, which is one of the central features of policy-models used at central banks. The model (b) is empirically successful in reproducing the pronounced fluctuations of unemployment over the business cycle. Towards this aim, the model accounts for fixed costs associated with maintaining an existing job. Job-related fixed costs amplify the effect of wages (smooth) on profits (need to be volatile) in the model. While our calibration relies on low profits associated with jobs in steady state, it does not at the same time demand a small gap between the value of working and the value of unemployment for the worker. The model presented in this paper therefore, (c) implies reasonable comparative statics in the labor market: Steady state unemployment does not change tremendously when unemployment benefits rise.

The model is based on Trigari's (2006) right-to-manage formulation and shares some properties with the recent literature using search and matching frictions but efficient wage bargaining, cp. Hagedorn and Manovskii (2006). Namely, in order to reproduce the size of unemployment fluctuations over the business cycle, steady state profits of labor firms need to be small and wages must not move one-to-one with revenue. Yet, this is just how far the similarities go. Most notably, the unemployment fluctuation mechanism does not rely on smooth wages or on a high outside option of the worker. It does, especially, not require that entry-wages do respond little to the business cycle. This is important since sticky wages of new hires have received only limited empirical blessing recently, see Pissarides (2007) and Haefke, Sonntag, and van Rens (2007).

[17] We are not the first to highlight the qualitative differences of the right-to-manage and the efficient bargaining approaches when it comes to the effect of structural reforms, see for example Blanchard and Giavazzi (2003).

[18] Inflation and thus demand and unemployment behave fairly differently under right-to-manage and efficient bargaining. The size of unemployment fluctuations, however, is needed to identify the size of fixed costs associated with jobs. The different business cycle behavior of the model under the two alternative bargaining schemes therefore makes it difficult to compare the two approaches directly in a New Keynesian model. In a companion note to the current paper, we use an RBC setup to compare the implied response of unemployment benefits to changes in replacement rates under right-to-manage and efficient bargaining. See Christoffel and Kuester (2007) for details.

The combination of right-to-manage bargaining and fixed costs thus brings back to life the wage channel in a model with a realistic degree of unemployment fluctuations and a realistic response of unemployment to changes in the benefit level as our calibration to US data illustrates. With the wage channel alive and well, we believe, it is time to explore the inclusion of this mechanism in a larger-scale policy model – and to check the robustness of policy advice derived under the alternative bargaining schemes.

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A Steady State and Linearized Economy

This Section of the appendix presents the steady state of our model economy with RTM bargaining, and the equilibrium conditions linearized around steady state. For completeness, we also present the steady state equations and linearized equilibrium conditions for the efficient bargaining model used for Figure 2.

A.1 Steady State

We turn to present the steady state of the model economy with RTM bargaining. Variables indexed by superscript * have the same steady state as non-indexed variables, so e.g. $J^* = J$. Nominal rate:

$$R = \frac{\Pi}{\beta}.$$

Inflation (quarter-on-quarter):

$$\Pi = 1.$$

Inflation (year-on-year):

$$\Pi^a = 4\Pi.$$

Marginal utility of consumption:

$$\lambda = (c - \varrho c)^{-\sigma}.$$

Marginal cost and price of labor good:

$$mc = x^L = \frac{\epsilon - 1}{\epsilon}.$$

Matches:

$$m = \sigma_m u^\xi v^{1-\xi}.$$

Employment:

$$\vartheta n = m.$$

Unemployment:

$$u = 1 - n.$$

Probability of finding a worker:

$$q = \frac{m}{v}.$$

Probability of finding a job:

$$s = \frac{m}{u}.$$

Wage bargaining first-order condition:

$$\eta J \delta^W = (1 - \eta)\Delta\delta^F. \tag{36}$$

$$\delta^F = \frac{1}{1 - \beta(1 - \vartheta)\gamma} wh. \tag{37}$$

$$\delta^W = \frac{1}{1 - \beta(1 - \vartheta)\gamma} h \left[\frac{-\alpha}{1 - \alpha} w - \frac{-1}{1 - \alpha} mrs\right]. \tag{38}$$

Hours first-order condition:

$$w = x^L z \alpha h^{\alpha - 1}. \tag{39}$$

Definition marginal rate of substitution:

$$mrs = \frac{\kappa^L h^\varphi}{\lambda}.$$

Value of labor firm:

$$J = \frac{1}{1 - \beta(1 - \vartheta)} \left[\frac{1 - \alpha}{\alpha} wh - \Phi\right]. \tag{40}$$

Surplus of representative family:

$$\Delta = \frac{1}{1 - \beta(1 - \vartheta - s)} \left[wh - b - mrsh\frac{1}{1 + \varphi}\right].$$

Vacancy posting - zero profit condition:

$$\kappa = q\beta J.$$

Resource constraint:

$$y = c + g + \kappa v + \Phi n.$$

Production:

$$y = nzh^{\alpha}.$$

Period profit of a labor firm:

$$\Psi^L = x^L zh^{\alpha} - wh - \Phi.$$

Period profit of a goods differentiation firm:

$$\Psi^C = (1 - mc)y.$$

A.2 Linearized Model Economy

This Subsection presents the linearized model economy.
Consumption Euler equation

$$\widehat{\lambda}_t = E_t \left\{ \widehat{\lambda}_{t+1} + \widehat{R}_t + \widehat{\epsilon}_t^b - \widehat{\Pi}_{t+1} \right\},$$

where $\widehat{\lambda}_t = -\frac{\sigma}{1-\varrho} \left(\widehat{c}_t - h_c \widehat{c}_{t-1} \right)$.
New Keynesian Phillips curve

$$\widehat{\Pi}_t = \beta E_t \left\{ \widehat{\Pi}_{t+1} \right\} + \frac{(1-\omega)(1-\omega\beta)}{\omega} \widehat{mc}_t,$$

where $\widehat{mc}_t = \widehat{x}_t^L$.
Matching

$$\widehat{m}_t = \xi \widehat{u}_t + (1 - \xi) \widehat{v}_t.$$

Employment stock

$$\widehat{n}_t = (1 - \vartheta) \widehat{n}_{t-1} + \frac{m}{n} \widehat{m}_{t-1}.$$

Link employment to unemployment

$$\widehat{n}_t = -\frac{u}{1-u} \widehat{u}_t.$$

Probability of finding a worker

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t.$$

Probability of finding a job

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t.$$

Bargaining first-order condition for the wage rate

$$\widehat{J}_t^* + \widehat{\delta}_t^W = \widehat{\Delta}_t^* + \widehat{\delta}_t^F. \tag{41}$$

Aggregate hours index (from hours first=order conditions)

$$\widehat{x}_t^L + \widehat{z}_t + (\alpha - 1)\widehat{h}_t = \widehat{w}_t. \tag{42}$$

Evolution of aggregate real wage

$$\widehat{w}_t = \gamma\left[\widehat{w}_{t-1} - \widehat{\Pi}_t\right] + (1-\gamma)\widehat{w}_t^*. \tag{43}$$

Law of motion of $\widehat{\delta}_t^F$:

$$\begin{aligned}\widehat{\delta}_t^F \quad = \quad & [1-\beta(1-\vartheta)\gamma]\left[\tfrac{-\alpha}{1-\alpha}\widehat{w}_t^* + \tfrac{1}{1-\alpha}\left(\widehat{x}_t^L + \widehat{z}_t\right)\right] \\ & + \beta(1-\vartheta)\gamma E_t\left\{\tfrac{-\alpha}{1-\alpha}\left[\widehat{w}_t^* - \widehat{w}_{t+1}^* - \widehat{\Pi}_{t+1}\right] + \widehat{\delta}_{t+1}^F + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right\}.\end{aligned} \tag{44}$$

Law of motion of $\widehat{\delta}_t^W$:

$$\begin{aligned}\delta^W\widehat{\delta}_t^W \quad = \quad & \tfrac{-\alpha}{1-\alpha}wh\left[\tfrac{-\alpha}{1-\alpha}\widehat{w}_t^* + \tfrac{1}{1-\alpha}\left(\widehat{x}_t^L + \widehat{z}_t\right)\right] \\ & -\tfrac{-1}{1-\alpha}mrsh\left[\tfrac{(-1)(1+\varphi)}{1-\alpha}\widehat{w}_t^* - \widehat{\lambda}_t + \tfrac{1+\varphi}{1-\alpha}\left(\widehat{x}_t^L + \widehat{z}_t\right)\right] \\ & +\tfrac{\beta(1-\vartheta)\gamma}{1-\beta(1-\vartheta)\gamma}\left[\left(\tfrac{\alpha}{1-\alpha}\right)^2 wh - \tfrac{(1+\varphi)}{(1-\alpha)^2}mrs\,h\right]E_t\left\{\widehat{w}_t^* - \widehat{w}_{t+1}^* - \widehat{\Pi}_{t+1}\right\} \\ & +\beta(1-\vartheta)\gamma\delta^W E_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{\delta}_{t+1}^W\right\}.\end{aligned} \tag{45}$$

Evolution of $\widehat{J}_t^*$:

$$\begin{aligned}J\widehat{J}_t^* \quad = \quad & \tfrac{wh}{\alpha}\left[-\alpha\widehat{w}_t^* + \widehat{x}_t^L + \widehat{z}_t\right] \\ & +\tfrac{\beta(1-\vartheta)\gamma}{1-\beta(1-\vartheta)\gamma}whE_t\left\{\widehat{w}_{t+1}^* + \widehat{\Pi}_{t+1} - \widehat{w}_t^*\right\} \\ & +\beta(1-\vartheta)JE_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{J}_{t+1}^*\right\}.\end{aligned} \tag{46}$$

Evolution of $\widehat{\Delta}_t^*$:

$$\begin{aligned}\Delta\widehat{\Delta}_t^* \quad = \quad & wh\tfrac{1}{1-\alpha}\left[-\alpha\widehat{w}_t^* + \widehat{x}_t^L + \widehat{z}_t\right] \\ & -\tfrac{1}{1+\varphi}mrsh\left[\tfrac{1+\varphi}{1-\alpha}\left(-\widehat{w}_t^* + \widehat{x}_t^L + \widehat{z}_t\right) - \widehat{\lambda}_t\right] \\ & +\tfrac{\beta(1-\vartheta)\gamma}{1-\beta(1-\vartheta)\gamma}\left[\tfrac{\alpha}{1-\alpha}wh - \tfrac{1}{1-\alpha}mrsh\right]E_t\left\{\widehat{w}_{t+1}^* + \widehat{\Pi}_{t+1} - \widehat{w}_t^*\right\} \\ & -\tfrac{\beta\gamma s}{1-\beta(1-\vartheta)\gamma}\left[\tfrac{\alpha}{1-\alpha}wh - \tfrac{1}{1-\alpha}mrsh\right]E_t\left\{\widehat{w}_{t+1}^* + \widehat{\Pi}_{t+1} - \widehat{w}_t\right\} \\ & +(1-\vartheta-s)\beta\Delta E_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{\Delta}_{t+1}^*\right\} \\ & -\beta\Delta s\widehat{s}_t.\end{aligned} \tag{47}$$

Vacancy posting equation

$$\begin{aligned}-\tfrac{\kappa}{q}\widehat{q}_t \quad = \quad & \tfrac{\beta\gamma}{1-\beta(1-\vartheta)\gamma}whE_t\left\{\widehat{w}_{t+1}^* + \widehat{\Pi}_{t+1} - \widehat{w}_t\right\} \\ & +\beta JE_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{J}_{t+1}^*\right\}\end{aligned} \tag{48}$$

Market clearing

$$y\,\widehat{y}_t = c\,\widehat{c}_t + g\,\widehat{g}_t + \kappa\,v\,\widehat{v}_t + \Phi\,n\,\widehat{n}_t.$$

Aggregate production

$$\widehat{y}_t = \widehat{z}_t + \alpha\widehat{h}_t + \widehat{n}_t.$$

Average period profits in labor good sector

$$\widehat{\Psi}_t^L = \frac{\frac{1-\alpha}{\alpha}wh}{\frac{1-\alpha}{\alpha}wh - \Phi}\left[\widehat{w}_t + \widehat{h}_t\right]. \tag{49}$$

Average period profits in differentiating sector

$$\Psi^C\widehat{\Psi}_t^C = (1-mc)y\,\widehat{y}_t - y\,mc\,\widehat{mc}_t.$$

Taylor rule

$$\widehat{R}_t = \gamma_R\widehat{R}_{t-1} + (1-\gamma_R)\left[\frac{\gamma_\Pi}{12}\widehat{\Pi}_{t-1}^a + \frac{\gamma_y}{12}\widehat{y}_t\right] + \widehat{\epsilon}_t^{money}.$$

Law of motion of the shocks

$$\widehat{\epsilon}_t^b = \rho_b\,\widehat{\epsilon}_{t-1}^b + \zeta_t^b, \quad \zeta_t^b \overset{iid}{\sim} N\left(0,\sigma_b^2\right).$$

$$\widehat{z}_t = \rho_z\,\widehat{z}_{t-1} + \zeta_t^z, \quad \zeta_t^z \overset{iid}{\sim} N\left(0,\sigma_z^2\right).$$

$$\widehat{g}_t = \rho_g\,\widehat{g}_{t-1} + \zeta_t^g, \quad \zeta_t^g \overset{iid}{\sim} N\left(0,\sigma_g^2\right).$$

$$\widehat{\epsilon}_t^{\text{money}} = \zeta_t^{\text{money}}, \quad \zeta_t^{\text{money}} \overset{iid}{\sim} N\left(0,\sigma_{\text{money}}^2\right).$$

A.3 Efficient Bargaining

The steady state equations and the linearized equilibrium conditions under EB largely coincide with the ones under RTM bargaining with the exception of the following equations.

A.3.1 Steady State under Efficient Bargaining

The wage-bargaining first-order condition (36) is replaced by

$$\eta J = (1-\eta)\Delta.$$

reflecting that $\delta^F = \delta^W$ under efficient bargaining. As a consequence, we drop the steady state equations governing these terms, (37) and (38), from the steady state of the model.
The first-order condition for hours worked (39) changes to

$$mrs = x^L z\alpha h^{\alpha-1}.$$

The equation for the value of the firm (40) reads as

$$J = \frac{1}{1-\beta(1-\vartheta)}\left[x^L zh^\alpha - wh - \Phi\right].$$

The remaining equations are identical with the ones under RTM. The steady state in the calibration of the EB model underlying Figure 2 is as follows.

Table 5: Steady State with Efficient Bargaining

Variable	Value	Description
y	1	output.
c	0.6354	consumption.
g	0.3424	government consumption.
mrs	2.8687	marginal rate of substitution between leisure and cons.
w	2.8240	real hourly wage rate.
wh	0.9413	wage per employee.
u	0.0588	unemployment rate.
v	0.0854	vacancies (as share of labor force).
s	0.4802	probability of finding a job within a month.
q	0.3306	probability of finding a worker within a month.
$b/(wh)$	0.40	unemployment insurance replacement rate.
$\kappa v/y \cdot 100$	1.3563	percent share of output lost to vacancy posting.
$\Phi/(x^L z h^\alpha)$	0.0095	share of a labor firm's revenue lost to fixed costs.
Ψ^C/y	0.0909	profit share (wholesale sector) in total output.
$\Psi^L n/y$	0.0145	profit share (labor sector) in total output.
J	0.4813	value of a labor firm.
Δ	0.4813	surplus of the worker from working.

Notes: Steady state for the efficient bargaining version of the model used in Figure 2. The targets are the same as in Table 1 and so are most parameters. The exceptions being the following: $\gamma = 0$, $\sigma_m = 0.3984$, $\kappa = 0.1588$, $\kappa^L = 310.24$.

A.3.2 Linearized Model Economy under Efficient Bargaining

The linearized model economy under EB largely coincides with the one under RTM bargaining with the exception of the following equations. As said, for efficient bargaining we abstract from wage rigidity by assumption. The wage-bargaining first-order condition (41) is replaced by

$$\widehat{J}_t^* = \widehat{\Delta}_t^*,$$

reflecting that $\widehat{\delta}_t^F = \widehat{\delta}_t^W$. As a consequence, we drop the equations governing these terms, (44) and (45), from the model.

The first-order condition for hours worked (42) changes to

$$\widehat{x}_t^L + \widehat{z}_t + (\alpha - 1)\widehat{h}_t = \varphi \widehat{h}_t - \widehat{\lambda}_t.$$

Equation (43) which linked newly bargained wages to aggregate wages is redundant – by assumption in the EB model variant all wages are bargained in every period.

The equation for the value of the firm (46) does not depend on wage stickiness anymore, in addition it does not use the first-order condition for hours worker to simplify terms anymore. It reads as

$$\begin{aligned} J\widehat{J}_t^* \quad = & \quad x^L z h^\alpha \left[\widehat{x}_t^L + \widehat{z}_t + \alpha \widehat{h}_t\right] - wh\left[\widehat{w}_t + \widehat{h}_t\right] \\ & +\beta(1-\vartheta) J E_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{J}_{t+1}^*\right\}. \end{aligned}$$

Similarly, the equation for the worker surplus (47) changes to

$$\begin{aligned} \Delta\widehat{\Delta}_t^* \quad = & \quad wh\left[\widehat{w}_t + \widehat{h}_t\right] - \tfrac{mrsh}{1+\varphi}\left[(1+\varphi)\widehat{h}_t - \widehat{\lambda}_t\right] \\ & +(1-\vartheta-s)\beta\Delta\, E_t\left\{\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{\Delta}_{t+1}^*\right\} \\ & -\beta\Delta s\widehat{s}_t. \end{aligned}$$

The vacancy posting equation (48) has the same form as under RTM (when setting $\gamma = 0$). The equation for average period profits in the labor good sector changes from (49) to

$$\Psi^L\widehat{\Psi}_t^L = x^L z h^\alpha\left[\widehat{x}_t^L + \widehat{z}_t + \alpha\widehat{h}_t\right] - wh\left[\widehat{w}_t + \widehat{h}_t\right].$$

B List of Variables and Symbols

Functions:

- $\mathtt{u}(\cdot)$: per-period utility function of individual worker.
- $\mathtt{U}(\cdot)$: per-period welfare function of the family.

Endogenous variables (Roman letters):

- $c_{i,t}$: consumption of consumer i
- c_t : aggregate/average consumption.
- D_t : nominal government debt in t issued as one period nominal bonds.
- $h_{i,t}$: hours worked by consumer/worker i.
- h_t : average hours worked per employee.
- $J_t(W_{i,t})$: real market value of a firm the worker of which earns nominal hourly wage $W_{i,t}$.
- m_t : number of matches in t (productive from $t+1$ onwards).
- mc_t : real marginal costs in the wholesale (differentiating) industry.
- n_t : employment (rate) in period t (number of employees).
- $P_{j,t}$: (nominal) price of differentiated wholesale good j.
- P_t : aggregate price level in t.
- P_t^* : price chosen by wholesale firms which reoptimize in period t.

- q_t : probability of finding a worker.
- R_t : gross nominal interest rate (in monthly terms).
- s_t : probability of finding a job.
- t_t : lump-sum taxes payable.
- u_t : unemployment rate in period t.
- U_t : value to the family of having a marginal worker unemployed in period t.
- v_t : open vacancies in t.
- $V_t^E(W_{i,t})$: value to the family of having marginal worker i employed in period t at nominal wage $W_{i,t}$.
- $W_{i,t}$: nominal hourly wage earned by consumer/worker i.
- W_t^* : nominal hourly wage chosen by a firm-worker match that renegotiates wages in t.
- $w_{i,t}$: hourly real wage earned by consumer i.
- w_t : hourly real wage rate
- x_t^L : real price of one unit of the labor good.
- y_t^L : output of labor good in average labor good firm.
- $y_{i,t}^L$: supply of labor goods by firm-worker match i.
- $y_{j,t}^{L,d}$: demand for labor goods by differentiated wholesale good firm j.
- $y_{j,t}$: output of differentiated wholesale good of type j, $j \in [0,1]$.
- y_t : output of final consumption good (used for private and government consumption, and for fixed costs and vacancy posting costs).

Endogenous variables (Greek letters):

- $\beta_{t,t+s}$: stochastic discount factor, discounting real income from $t+s$ to t.
- $\Delta(W_{i,t})$: surplus to the family of having marginal worker i employed in period t at nominal wage $W_{i,t}$; $\Delta(W_{i,t}) := V_t^E(W_{i,t}) - U_t$.
- λ_t : marginal utility of consumption in t.
- Π_t : month-on-month inflation rate (not annualized).
- Π_t^a : year-on-year inflation rate.
- Ψ_t : real dividend income of the family/consumer in period t.

- $\Psi_{j,t}^C$: real per-period profits of wholesale firm j.
- Ψ_t^C : real per-period profits in differentiating sector (with Calvo price rigidity).
- $\Psi_{i,t}^L$: real per-period profit in labor good firm i.
- $\Psi_t^L(W_{i,t})$: real per-period profit of a labor good firm i which pays nominal wage rate $W_{i,t}$.
- Ψ_t^L : real per-period profit in average labor good firm.

Shocks and innovations:

- ϵ_t^b : risk-premium shock.
- $\epsilon_t^{\text{money}}$: iid monetary policy shock (log-normal).
- g_t : shock to real government spending, AR(1).
- z_t : technology shock in labor sector, AR(1).
- ζ_t^b : innovation to risk premium shock, $\zeta_t^b \overset{iid}{\sim} N(0, \sigma_b^2)$.
- ζ_t^{money} : monetary policy shock, $\zeta_t^{\text{money}} \overset{iid}{\sim} N(0, \sigma_{\text{money}}^2)$.
- ζ_t^g : innovation to government spending shock, $\zeta_t^g \overset{iid}{\sim} N(0, \sigma_g^2)$.
- ζ_t^z : innovation to technology shock, $\zeta_t^z \overset{iid}{\sim} N(0, \sigma_z^2)$.

Parameters (Roman letters):

- A : factor of proportionality between percentage profits and compensation.
- b : real replacement income.
- $\overline{g}$: long-run target level for government consumption, $\overline{g} \geq 0$.

Parameters (Greek letters):

- α : elasticity of output w.r.t. hours worked.
- β : time-discount factor, $\beta \in (0, 1)$.
- γ : probability that a labor good firm/a firm-worker match cannot bargain its wage in the period, $\gamma \in [0, 1)$.
- ϵ : own-price elasticity of demand for differentiated goods, $\epsilon > 1$.
- η : bargaining power of workers, $\eta \in (0, 1)$.
- ϑ : probability of separation of a match, $\vartheta \in (0, 1)$.
- κ : real cost of posting a vacancy once, $\kappa > 0$.

- κ^L : scaling parameter of disutility of work.
- ξ : elasticity of matches w.r.t. unemployment, $\xi \in (0,1)$.
- $\overline{\Pi}$: target rate for month-on-month inflation.
- Φ : real per-period fixed cost in labor good firm, $\Phi \geq 0$.
- ϕ_R : response of interest rate to lagged interest rate in Taylor rule, $\phi_R \in [0,1)$.
- ϕ_π : response of interest rate to inflation rate in Taylor rule, $\phi_\pi > 1$.
- ϕ_y : response of interest rate to output in Taylor rule.
- φ : inverse of Frisch-elasticity of labor supply, $\varphi > 0$.
- ϱ : degree of external habit persistence, $\in [0,1)$.
- σ : coefficient of relative risk-aversion, $\sigma > 0$.
- σ_m : efficiency of matching, scaling parameter in matching functions, $\sigma_m > 0$.
- ω : probability that a wholesale firm cannot update its price in the period, $\omega \in [0,1)$.