Reconsidering the Natural Rate Hypothesis in a New Keynesian Framework*

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Abstract

This paper formulates a stylized New Keynesian model in which each individual firm can select the frequency of its price adjustments, and we demonstrate that the endogeneity of the contract structure has a dramatic impact on the magnitude of the aggregate effects of steady-state inflation. We start by analyzing the exact nonlinear properties of a benchmark economy with exogenous contract duration, and we show that the long-run Phillips curve is downward sloping even for very low levels of steady-state inflation. We then proceed to analyze economies in which each firm chooses the mean duration of its price contracts in order to maximize its expected profits; with a plausible calibration of the magnitude of menu costs and other structural parameters, this model predicts a relationship between steady-state inflation and the frequency of price adjustment that is remarkably close to the empirical findings of cross-country studies. Furthermore, at moderate inflation rates, steady-state inflation generates relative price distortions that have a non-trivial impact on aggregate output, but this impact wanes and eventually disappears at much higher annual inflation rates because the frequency of price adjustment approaches that of the flexible-price economy. Finally, we generalize the analysis to allow for heterogeneous menu costs and show that the presence of firms with low menu costs and flexible prices creates a strong motivation for firms with high menu costs to select a contract structure with shorter mean duration.

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"Differing steady-state inflation rates will not keep output (or employment) permanently high or low relative to the "natural-rate" levels that would prevail in the absence of nominal price stickiness in the relevant economies". Friedman (1966, 1968)

"There is no monetary policy that can permanently keep output (or employment) above its natural-rate value, not even an ever-increasing (or ever-decreasing) inflation rate". Lucas (1972)

"The agreement of the natural rate hypothesis has seemingly been implicitly overturned via the widespread adoption of the famous Calvo (1983) model of nominal price stickiness". McCallum (2004)

1 Introduction

In this paper, we reconsider the natural rate hypothesis in the context of the recent New Keynesian models. In particular, we formulate a stylized New Keynesian model in which each individual firm can select the frequency of its price adjustments. It is then shown that the endogeneity of contract duration has a dramatic impact on the magnitude of the aggregate effects of steady-state inflation.

Much of New Keynesian models posit that the mean duration of prices is exogenously fixed. Thus, one can easily guess that the absence of endogenous decisions on the timing or time duration of price changes might be responsible for the failure of the natural rate hypothesis in New Keynesian models. In order to see this, we allow firms to choose the arrival rates of their changes while price changes are assumed to be costly. But we still assume that firms do not choose the exact times of price changes.

A cross-country evidence on the relationship between the average inflation and frequency of price changes, reported in Table 1, suggests that the mean duration of prices becomes shorter as average inflation rises, as noted in Golosov and Lucas (2006). In particular, we demonstrate that our modelling framework's theoretic prediction on the relationship between average inflation and the frequency of price adjustment is consistent with the cross-country evidence in Table 1.

We begin by analyzing the exact nonlinear properties of a benchmark economy with exogenous contract duration, and we show that the long-run Phillips curve is downward sloping even for very low levels of steady-state inflation. The reason why we focus on the non-linear properties of the model can be summarized as follows. Much of recent papers on New Keynesian models proceeds with log-linear approximation to equilibrium conditions around a fixed steady-state inflation rate, which provides a reasonable approximation to its dynamics under certain conditions. In particular, the local analysis tends to imply a positive correlation between steady-state inflation and output. But a set of recent papers

Table 1: International Comparison of Frequency of Price Changes

Country	Period	Annual Inflation	Quarterly Frequency	Source
United States	1998-2005	2	0.24	Nakamura and Steinsson (2006)
Euro Area	1995-2000	2	0.26	Dhyne, Alvarez, et. al (2005)
United States	1991-2002	2	0.53	Bils and Klenow (2004)
United States	1998-2003	2	0.65	Klenow and Kryvstov (2005)
Mexico	1995-1996	31	0.70	Gagnon (2005)
Israel	1981-1982	75	0.80	Lach and Tsiddon (1992)
Israel	1978-1979	114	0.85	Lach and Tsiddon (1992)

Note: Monthly frequency (x) in the data is converted into quarterly frequency (q) by using the formula: $q = 1 - (1-x)^3$. In addition, authors' calculation from Golosov and Lucas (2006) for Israel.

such as Guido (2004), Casares (2004), and Burstein (2006) demonstrated that an increase in steady-state inflation may not always decrease the steady-state level of output in New Keynesian models with the Calvo contract. For this reason, our analysis proceeds without having any local approximation.

We then proceed to analyze economies in which each firm chooses the mean duration of its price contracts in order to maximize its expected profits. With a plausible calibration of the magnitude of menu costs and other structural parameters, this model predicts a relationship between steady-state inflation and the frequency of price adjustment that is remarkably close to the empirical findings of cross-country studies.

Furthermore, at moderate inflation rates, steady-state inflation generates relative price distortions that have a non-trivial impact on aggregate output, but this impact wanes and eventually disappears at much higher annual inflation rates because the frequency of price adjustment approaches that of the flexible-price economy. Furthermore, we generalize the analysis to allow for heterogeneous menu costs and show that the presence of firms with low menu costs and flexible prices creates a strong motivation for firms with high menu costs to select a contract structure with shorter mean duration.

Our modelling framework is not new. The optimal duration of contracts has been extensively analyzed in many papers at micro and macro economics levels. In a broad sense, we follow the approach taken in Ball, Mankiw, and Romer (1988), Romer (1990), Ball and Mankiw (1994), and Ireland (1997). The difference from the previous works is that the optimal choice on the mean duration of contracts is analyzed in a completely non-linear structure with exact non-linear solutions. We also explore consequences of heterogenous menu costs on the long-run relationship between inflation and output.

It also has been pointed out that the Calvo pricing model with exogenous contract structures may not perform reasonably in the steady-state analysis for economies with high inflation. For the model requires an upper bound of inflation rates in order to guarantee the internal consistency of the model, given an exogenous mean duration of contract. But allowing for endogenous mean duration of contract would enable one to perform steady-state analysis for economies with high inflation.

The rest of the paper is organized as follows. Section 2 reviews the non-linear steady-state properties of a prototypical New Keynesian macro model. Section 3 presents an endogenous contract structure in which each firm chooses the mean duration of its price contracts in order to maximize its expected profits. Section 4 adds heterogenous menu-costs to the model. Section 5 discusses transition dynamics between states. In section 6, we briefly highlight implication of models with endogenous contract duration for log-linear dynamics. We conclude by summarizing possible directions of future research in section 7.

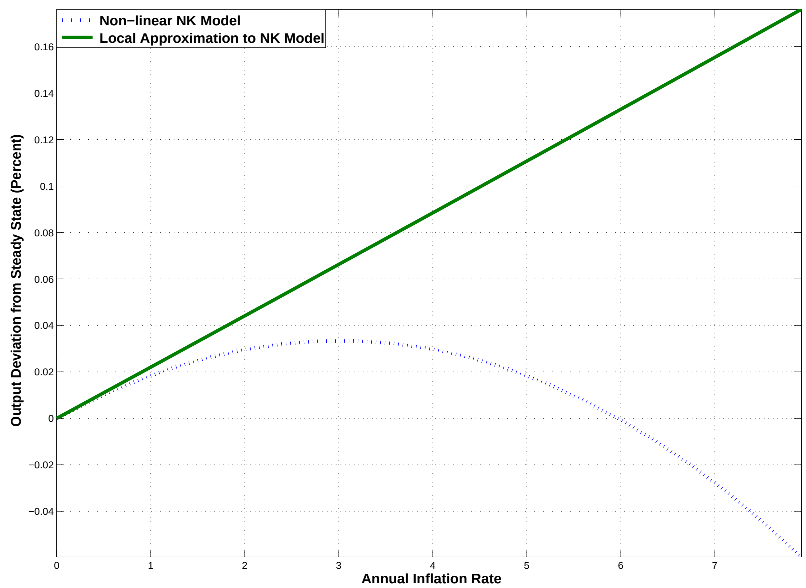
2 Exogenous Contract Structure

The nonlinear New Keynesian model that we review in this section inherits the exogenous contract structure of the prototypical Calvo model in which the average frequency of price changes is represented by an exogenous parameter. Besides, since we focus only on steady-state equilibrium, our analysis proceeds without having any approximation.

Each individual firm faces a downward-sloping demand curve. Given the downward-sloping demand curve, firms set their prices as monopolistic competitors. The demand curves are derived by solving the cost-minimization problems of obtaining composite goods. The composite goods are a product of the aggregation of differentiated goods based on the Dixit-Stiglitz aggregator. The Dixit-Stiglitz aggregator has a constant elasticity of substitution between different goods. In addition, the substitution elasticity becomes the elasticity of demand. We also assume that labor market is perfectly competitive and wages are fully flexible. To the extent that production functions of individual firms exhibit constant returns to scale, it means that the real marginal cost is identical across firms.

Given these demand and production structures, the Calvo model describes how firms set prices over time. During each period, only a fraction of all sellers, $1-\xi$, can change prices during any period, with others holding their nominal prices fixed at their previous-period values. The probability of price change in a given time interval is constant and

Figure 1: Local Approximation and Non-Linear Relation in Steady State



exogenously determined. The random duration of contract price leads to an infinite-horizon maximization problem facing firm that set their prices. The profit maximization with respect to prices then leads to the equality of the expected present-value of the current and future marginal revenues and the expected present-value of the current and future marginal costs. We also assume that financial markets are complete, so that the inter-temporal marginal rate of substitution is used as a stochastic discount factor for future profit streams. Table 2 summarizes equilibrium conditions of the model described above.

Having described the model, we now discuss the real effect of steady-state inflation. Before proceeding further, it is noteworthy that a set of recent papers have emphasized that numerical results based on log-linear approximation may differ from those of non-linear equilibrium conditions. For example, one can use a quadratic approximation to profit functions of firms together with log-linear equilibrium conditions[1] in order to obtain the following steady-state relationship between inflation and output:

$$y = \frac{\xi(1-\beta)}{(1-\xi)(1-\beta\xi)}\pi, \tag{1}$$

where β is time-discount factor, y is the logarithm of real output and π is the aggregate

[1]Refer to Romer (1999), Kiley (2000), and Devereux and Yetman (2002) for the endogenous frequency of price adjustment in the Calvo pricing model.

Table 2: Equilibrium Conditions

1. Contract Equation

$$\tilde{P}_t^* \sum_{k=0}^{\infty} (\beta\xi)^k E_t[C_{t+k}^{-\sigma} Y_{t+k} (P_{t+k}/P_t)^{\epsilon-1}] = \sum_{k=0}^{\infty} (\beta\xi)^k E_t[C_{t+k}^{-\sigma} Y_{t+k} MC_{t+k} (P_{t+k}/P_t)^{\epsilon}].$$

2. Utility Maximization

$$(1-\alpha) MC_t = C_t^{\sigma} L_t^{\chi+\alpha}.$$

3. Aggregate Price Equation

$$1 = (1-\xi)(\tilde{P}_t^*)^{1-\epsilon} + \xi \Pi_t^{\epsilon-1}.$$

4. Relative Price Distortion

$$\Delta_t = (1-\xi)(\tilde{P}_t^*)^{-\epsilon} + \xi \Pi_t^{\epsilon} \Delta_{t-1}.$$

5. Aggregate and Individual Production Functions

$$C_t = Y_t = (A_t/\Delta_t) L_t^{1-\alpha}, \qquad Y_t(f) = A_t K_t(f)^{\alpha} L_t(f)^{1-\alpha}.$$

$\tilde{P}_t^*$: relative price of contract price, MC_t: real marginal cost, P_t: aggregate price, Y_t: output.
L_t: labor, Δ_t: relative price distortion, C_t: consumption, Π_t: gross inflation, $K_t(f)$: capital of firms.
$L_t(f)$: labor of firms, β: discount factor, ϵ: demand elasticity, α: capital share.
χ: inverse of labor supply elasticity. The aggregate capital is set to be 1 in its competitive rental market.

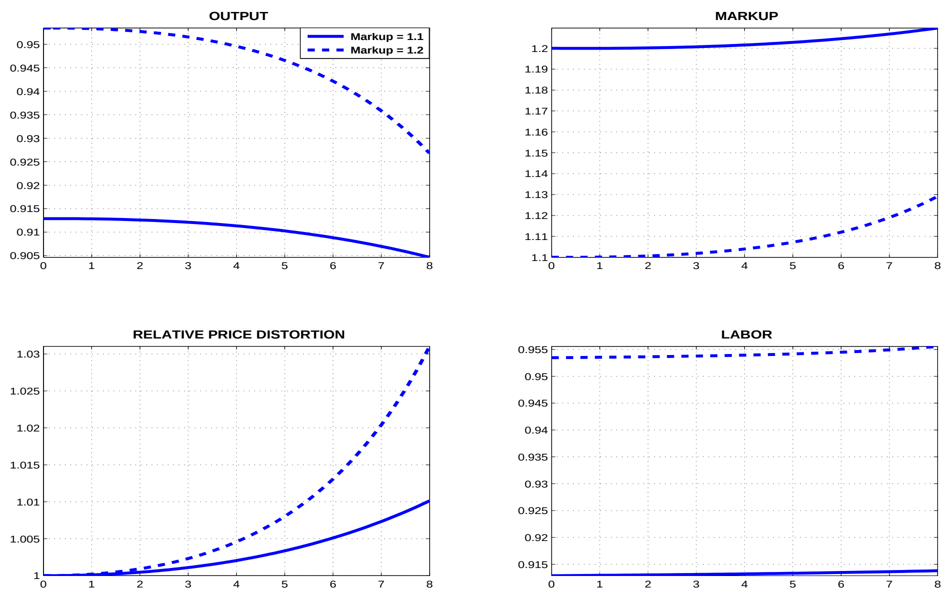
inflation rate. In particular, equation (1) implies that an increase in the rate of inflation leads to an increased level of output. In this sense, local approximation yields a positive correlation between the steady-state inflation and output. But Guido (2004), Casares (2004), and Burstein (2006) demonstrated that an increase in the steady-state rate of inflation can decrease the steady-state level of output in the Calvo model.

Figure 1 compares a log-linear relation between inflation and output with its corresponding exact non-linear relation. The straight line corresponds to the linear relation summarized in equation (1), while the dotted line represents its corresponding non-linear relation. Figure 1 shows that an increase in steady-state inflation leads to an increased level of output so long as inflation remains within a neighborhood of zero inflation rate. This is true for both of the two cases. But in the case of exact non-linear solutions, the steady-state level of output reaches its maximum around three percent of annual inflation rate and then begin to decline as inflation rises. We can thus see that while a positive correlation between steady-state inflation and output holds in a neighborhood of zero inflation rate, it is more likely to have a negative correlation between steady-state inflation and output when inflation is sufficiently high. An important reason for this is that relative price distortion rises as steady-state inflation rises.

Figure 2 depicts the effect of steady-state inflation on output, markup, relative price distortion, and labor.[2] Here, we can find a range of inflation in which the steady-state of

[2] Figures 1 and 2 do not use the same set of parameter values. Figure 1 uses special values of preference

Figure 2: Effects of Steady-State Inflation



output declines, while labor rises with inflation. The difference in the impact of steady-state inflation on labor and output is associated with the fact that the steady-state level of relative price distortion rises with inflation. In particular, relative price distortion is defined as the fraction of output that is foregone because of relative price dispersion. Hence, the steady-state level of output decreases while employment rises as inflation rises. In summary, a rise in the steady-state inflation decreases the steady-state level of output except for a neighborhood of zero inflation.[3]

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parameters to make sure that log-linear and exact non-linear solutions for output hit the origin at zero inflation. For example, substitution elasticity of differentiated goods, denoted by ϵ, is set to be very close to 1 with linear utility of labor and log utility function for consumption. In addition, the assumption of the linear utility in labor leads to the proportionality between the logarithm of output and markup. Besides, the linear relation corresponds to those used in Romer (1990) and Devereux and Yetman (2002). But we do not impose such restrictions in the case of Figure 2, in order to allow for different values of steady-state markup.

[3]See, for example, Goodfriend and King (1997), Woodford (2003) and Guido (2004) for more detailed discussions on the relative price distortion in the recent optimizing sticky price models with staggered price-setting.

3 Endogenous Contract Structure with Homogenous Menu Costs

In this section, we assume that firms choose the probability of their price changes in each period, following Romer (1999), Kiley (2000) and Devereux and Yetman (2002). The difference from theirs is that their optimization problems of choosing the arrival rate of price changes exploit quadratic loss functions of firms, while we do not rely on any approximation to the characterization of the optimization problem.

Furthermore, we assume that price changes incur fixed costs ωY_t in each period $t = 0, 1, \cdots, \infty$. The physical costs of price adjustments are assumed to be identical across firms.[4] We also restrict our analysis to the symmetric case in which individual firms choose the same frequency of price adjustments. Specifically, given that other firms choose ξ, firm j choose ξ_j by solving the following maximization problem:

$$\max_{\xi_j} E[\Psi_t(\xi_j, \xi)], \tag{2}$$

where E denotes the unconditional expectation and $\Psi_t(\xi_j, \xi)$ is the expected present-value at period t for firms that choose ξ_j given that other firms choose ξ. In addition, $\Psi_t(\xi_j, \xi)$ can be written as

$$\Psi_t(\xi_j, \xi) = \{\Phi_t(\frac{P^*_{jt}}{P_t}) - \omega Y_t + \beta E_t[(\frac{C_t}{C_{t+1}})^{-\sigma}\{\xi_j \Psi_1(\frac{P^*_{jt}}{P_{t+1}}, \xi_j, \xi) + (1-\xi_j)\Psi_{t+1}(\xi_j, \xi)\}]\}, \tag{3}$$

where P^*_{jt} is the profit maximizing price at period t for firms that choose ξ_j given ξ, $\Psi_t(P^*_{jt}/P_t)$ is the instantaneous profit at period t of firms whose nominal prices is P^*_{jt}, and $\Phi_1(P^*_{jt}/P_{t+1}, \xi_j, \xi)$ is the expected present-value at period $t+1$ of profits conditional on that firms do not change their prices in period $t+1$. Let $\Psi_k(P^*_{jt-k}/P_t, \xi_j, \xi)$ be the expected present-value of profits at period t of firms whose prices at period t is P^*_{jt-k}. The recursive representation of $\Psi_k(P^*_{jt-k}/P_t, \xi_j, \xi)$ is also given by

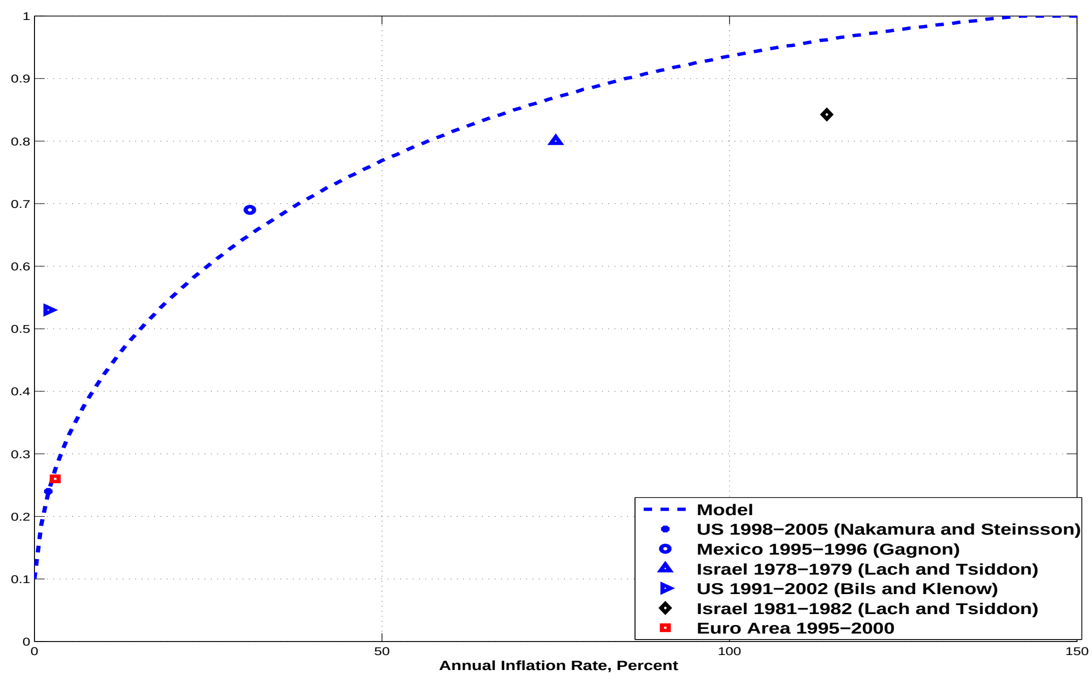
$$\Psi_k(\frac{P^*_{jt-k}}{P_t}, \xi_j, \xi) = \Phi_t(\frac{P^*_{jt-k}}{P_t}) + \beta E_t[(\frac{C_t}{C_{t+1}})^{-\sigma}\{\xi_i \Psi_{k+1}(\frac{P^*_{jt-k}}{P_{t+1}}, \xi_j, \xi) + (1-\xi_j)\Psi_{t+1}(\xi_j, \xi)\}]. \tag{4}$$

The equilibrium frequency of price changes, denoted by ξ, is determined when individual maximization problems, $\max_{\xi_j} E[\Psi_t(\xi_j, \xi)]$ for j, yields the same solution for all firms. The equilibrium condition for ξ can be thus interpreted as a Nash equilibrium condition.

Figure 3 demonstrates the relationship between steady-state inflation and equilibrium probability of price adjustment. The vertical line represents the faction of prices changed in each quarter. The horizontal line denotes the annual inflation rate. Figure 3 indicates that, higher steady-state inflation raises the average frequency of price changes. We also plot several international observations on the relationship between the inflation and frequency

[4] The assumption that all firms have an identical level of menu-costs will be replaced in the next section.

Figure 3: Fraction of Prices Changed in Each Quarter

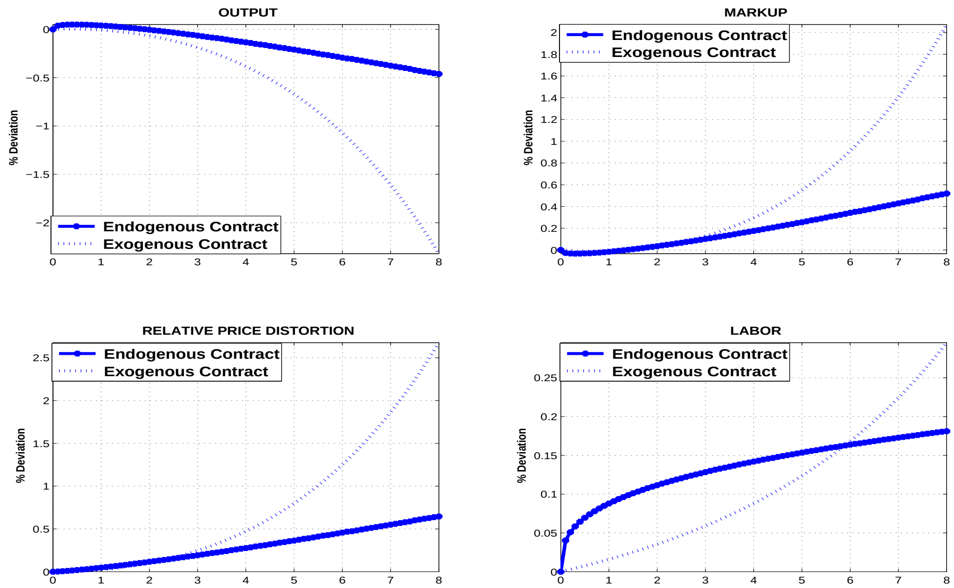


of price adjustments, which are summarized in Table 1. The data reported in Table 1 are comparable to those used in Golosov and Lucas (2006). Golosov and Lucas demonstrated that menu-costs models can replicate the cross-country observations on the relationship between inflation and frequency of price changes. Figure 3 shows that the model analyzed in this paper predicts a relationship between steady-state inflation and the frequency of price adjustment that is consistent with the cross-country data in Table 1. Besides, as noted in the introduction, at moderate inflation rates, steady-state inflation generates relative price distortions that have a non-trivial impact on aggregate output, but this impact wanes and eventually disappears at much higher annual inflation rates because the frequency of price adjustment approaches that of the flexible-price economy.[5]

Figure 4 plots effects of steady-state inflation on output, markup, relative price distortion and labor in models with endogenous and exogenous contract duration. The straight line corresponds to endogenous contract duration, while the dotted line represents exogenous contract duration. The implications of endogenous contract duration for the real

[5]We set $\beta = 0.99$, which corresponds to about 4 % annual real interest rate. We also set $\epsilon = 6$, which corresponds to setting the steady state markup equal to $\mu = 1.2$. The size of menu-cost is set to be 0.6 % of real output. In addition, we assume that one-tenth of all firms change their prices in each period at zero inflation rate.

Figure 4: Effects of Steady-State Inflation (Endogenous and Exogenous Contracts)

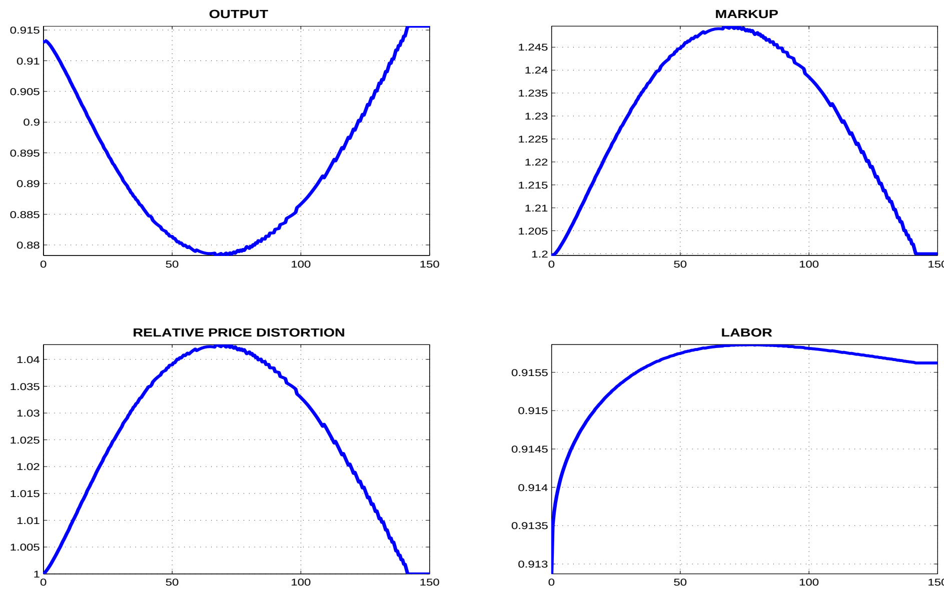


effect of steady-state inflation can be summarized as follows. First, the output effect of inflation becomes dramatically small for high rates of inflation, relative to models with exogenous contract duration. Second, there is a threshold value of the steady-state inflation rate that determines whether or not the natural rate hypothesis holds. More specifically, there is a range of inflation in which output declines as inflation rises. In this range of inflation, models with endogenous contract duration also imply negative correlations between steady-state inflation and output for a certain range of inflation. But the natural rate hypothesis holds for sufficiently high inflation. Moreover, even with the introduction of endogenous contract structure, we can still find a small range of inflation around zero inflation in which yields a positive relationship between inflation and output.

In order to confirm the presence of the threshold value of inflation discussed above, Figure 5 plots output, markup, relative price distortion and labor for an extended range of inflation rate. Figure 5 indicates that markup and relative price distortion return to their values at zero inflation in the end as inflation rises. The steady-state level of output, however, differs from its level at the inflation rate that makes the natural rate hypothesis hold. The difference arises because of menu-costs. Specifically, the level of output at zero inflation is smaller than the level of output at the inflation rate that makes the natural rate hypothesis hold.[6]In relation to these results, it should be noted that there is a range

[6]The first-order condition for labor supply at a deterministic steady-state can be written as $Y =$

Figure 5: Effects of Steady-State Inflation (Endogenous Contracts with Homogenous Menu Costs)



of high positive rates of inflation in which inflation raises output. In this region, markup and relative price distortion decline.

Finally, it has been pointed out that the Calvo pricing model with exogenous contract structures may be of limited use in the steady-state analysis for economies with high inflation. As noted in the introduction, the reason for this is that the internal consistency of the model yields upper bounds of inflation rate. For example, notice that relative prices of lagged contract prices keep falling with a positive inflation rate, while the price level is an weighted average of infinite number of prices. Hence, the condition that an infinite number of different prices must be consistent with a positive relative price of contract price at a positive rate of inflation turns out to be binding under a certain range of inflation given fixed values of parameters. In particular, the definitions of the price level and relative price distortion lead to the following inequalities $\xi\Pi^{\epsilon-1} < 1$ and $\xi\Pi^{\epsilon} < 1$, respectively.[7] The two equations thus act as constraints for steady-state inflation when ξ, β, and ϵ are exogenously determined. However, when ξ is endogenously determined, they are not biding constraints even for high inflation.

$(MC/(\Delta^{\chi}(1-s_F)^{\sigma}))^{1/(\sigma+\chi)}$, where s_F is the share of the aggregate menu-costs in real output.

[7] The inequalities of $\xi\beta\Pi^{\epsilon} < 1$ and $\xi\beta\Pi^{\epsilon-1} < 1$ result from the fact that the expected present-value of profits must not blow up in an equilibrium and the profit-maximization problem in the Calvo pricing model is an infinite-horizon maximization problem.

4 Endogenous Contract Structure with Heterogenous Menu Costs

Having discussed the endogenous contract structure with homogenous menu-costs, the analysis of this section moves onto the case in which individual firms have different levels of menu-cost.

4.1 Conceptual Issues

We begin with issues associated with the specification of heterogenous menu-costs. Recent manu-costs models take into account some heterogenous aspects of individual firms for several different reasons. First, Danziger (1999) and Gertler and Leahy (2005) use the independence of idiosyncratic productivity shocks to make price adjustments staggered, while the multiple shock are random walks so the variances of relative prices grow linearly over time. Golosov and Lucas (2006) also show that it is important to include idiosyncratic shocks in menu-cost models, in order to match the empirical evidence that a significant fraction of firms make price adjustment under low inflation. What these papers have in common regarding the specification of menu-costs is that menu costs are identical across individual firms. It thus means that the presence of idiosyncratic shocks plays an important role in generating staggered price-setting of firms over time.

Second, Dotsey, King and Wolman (1997) allow for random shocks to fixed costs of price adjustments. Thus, the staggered price-setting behavior of firms in their model depends on random shocks to fixed costs of price adjustment, rather than idiosyncratic productivity shocks. The presence of random shocks to menu-costs, however, means that the size of menu-costs may not be a characteristic of each individual firm. In relation to this, we assume that each individual firm can have a different level of menu-costs, which can be viewed as a characteristic of an individual firm. The different levels in the fixed cost of price adjustment can generate interactions of firms in terms of their pricing behaviors. For example, firms would have incentives to exploit their low menu-costs or reduce high menu-costs.

A set of recent papers have combined time-dependent pricing models with state-dependent pricing models. Ireland (1997) and Devereux and Siu (2005) include fixed price-adjustment costs into optimizing sticky price models with the Taylor-type staggered price-setting, though their specifications of menu costs are not the same. For example, Devereux and Siu posit that fixed costs of price adjustment are stochastic and firms do not observe realized values of price adjustment costs when they set their prices, whereas Ireland assumes that fixed costs of price adjustment are identical across individual firms. In the model analyzed in this paper, however, there is no random shock to the fixed cost of price adjustment though each firm has a different level of price-adjustment cost. As a result, different price-setting behaviors of firms in this paper take place because of the heterogeneity of price adjustment costs across firms as well as the Calvo pricing mechanism.

In particular, we permit firms to choose the arrival rates of their price changes. In relation to this, one may point out that modelling an endogenous choice on the exact time duration of contracts would be more realistic. But we simply restrict our analysis to the Calvo pricing structure, which has been widely used in recent New Keynesian macro models. Another reason why we maintain such a time-dependent structure is associated with a set of recent empirical works on the cause of price stickiness. More explicitly, a set of recent empirical works rely on the interview method as a way of finding out about the cause of price rigidity, following Blinder (1998). In order to apply the interview method, he explained selected theories of sticky prices to managers in face-to-face interviews and assumed that they would recognize the line of reasoning when it came close to their way of thinking. A candidate theory of sticky prices included in the interviews is *explicit and implicit* contracts. For example, firms have contractual arrangements with their customers, in which they guarantee to offer the product at a specific price. An explanation why firms might engage in such agreements is that they want to build up long-run customer relationships. This should discourage customers from shopping elsewhere, stabilizing the firms future sales. Customers are attracted by a constant price because it helps to minimize transaction costs (e.g. shopping time). Thus, customers focus on the long-run average price rather than on the spot price. In relation to contract theory, Claudia Kwapil, Josef Baumgartner and Johann Scharler (2005) report that explicit contracts are indeed widely used by Austrian firms.

The second issue is a trade-off between the imposition of restrictions on the optimal choice problem and the computational complexity. The most completely unrestricted optimization problem is to let each firm choose a different frequency of price changes depending on the magnitude of its menu-cost. But such a completely unrestricted optimization problem raises computational complexity, though it is preferable. In order to yield a tractable approach, we thus assume that there are only a discrete number of pricing mechanisms available, each of which corresponds to a different frequency of price changes. The simplest example that we consider is the case in which only two pricing mechanisms are available for firms. More explicitly, one is the prototypical Calvo pricing mechanism with an optimized probability of price changes and the other is a Calvo contract with indexation.

Finally, we need to specify how to decide the mean duration of contract in the face of heterogenous menu-costs. The issue is that firms with a different level of menu-costs may want to choose a different level of mean duration of their price changes, if thy are allowed to choose their own optimal mean durations. However, in order to maintain a certain level of analytic tractability, we exploit a kind of median voter theorem.[8] Specifically, the median

[8] A median vote theorem posits that in a majority election, if voters can all be represented along one dimension, then politicians seeking to win the election will realize that the pivotal voter required to win

firm among firms with sticky-price contracts is required to choose the mean duration of the sticky-price contract. Given the mean duration of the sticky-price contract, individual firms decide on whether to choose sticky-price or flexible-price contracts.

4.2 Occasional Choice of Pricing Mechanism

In each period $t = 0, 1, \cdots, \infty$, only a fraction of all firms are allowed to choose a new pricing mechanism after exogenous disturbances at period t are realized. What is meant by pricing mechanism can be interpreted as a set of procedures specifying how firms set their prices. We also assume that a contract reflects a particular pricing mechanism. It thus means that a choice of a pricing mechanism is equivalent to that of a type of contract. Moreover, there are multiple pricing mechanisms available for firms. In our paper, we posit that firms are allowed to choose a pricing mechanism on an occasional basis. Besides, we also assume that each firm has a particular level of fixed price adjustment cost, denoted by $\omega\ Y_t$, and the density function of ω is $f(\omega)$.

More specifically, the time duration of a pricing mechanism is random as is done in the Calvo model. During each period, a fraction of firms, 1 - ξ, are allowed to choose a new pricing mechanism, while the other fraction are not. Besides, there are two types of contracts available for firms. The first-type contract does not permit any price adjustments until the next new contract, which corresponds to the prototypical Calvo-type contract. The second-type is a Calvo-type pricing mechanism with full indexation.

Having described the specifications of price contracts, we move onto the discussion on the choice of a contract. The choice of a contract is associated with selection of a mean duration of price. Consider the choice problem at period t of a pricing mechanism facing firms whose fixed cost of price adjustment is ω. Let $V_t(\omega)$ be the expected present-value at period t of current and future profit streams for firms that are allowed to renew their pricing mechanisms. It is noteworthy that $V_t(\omega)$ depends on the size of fixed price adjustment cost, which differs across firms. However, it does not necessarily mean that profit maximizing prices in each period depend on the size of fixed adjustment cost, which will be seen.[9]

The optimizing choice at period t of a pricing mechanism can be formalized as follows:

$$V_t(\omega) = \max\{V_{st}(\omega), V_{it}(\omega)\}, \tag{5}$$

where $V_{st}(\omega)$ and $V_{it}(\omega)$ denote the payoffs of Calvo-type pricing mechanisms with and without indexation, respectively. It is important to note that the expected present-value

the election is that voter who lies at the median of the voter distribution.

[9]Danziger (1999) posits that each individual firm has a cost of price adjustment, which is proportional to its demand. With the proportionality assumption, prices can be affected by the magnitude of costs of price adjustment. But we take a conventional specification of menu-costs, which assumes that costs of price adjustment are additively separable in sales revenue and direct production costs.

at period t of profit streams that can be obtained from each pricing mechanism has a recursive representation. For example, the payoff of the conventional Calvo-type pricing mechanism is given by

$$V_{st}(\omega) = \Phi_t(\tilde{P}^*_{st}) - \omega Y_t + \beta E_t[(\frac{C_t}{C_{t+1}})^{-\sigma}\{\xi V_{s,1}(\tilde{P}^*_{st}\Pi^{-1}_{t+1}, \omega) + (1-\xi)V_{t+1}(\omega)\}], \quad (6)$$

where $\Phi_t(\tilde{P}^*_{st})$ is the instantaneous profit at period t of firms whose relative price is $\tilde{P}^*_{st}$, and $V_{s,1}(\tilde{P}^*_{st}\Pi^{-1}_{t+1}, \omega)$ is the expected present-value at period $t+1$ of profits conditional on that firms do not change their prices in period $t+1$. Let $V_{s,k}(P^*_{st-k}/P_t, \omega)$ be the expected present-value of profits at period t of firms whose prices at period t is P^*_{st-k}. The recursive representation of $V_{s,k}(P^*_{st-k}/P_t, \omega)$ is then given by

$$V_{s,k}(\frac{P^*_{st-k}}{P_t}, \omega) = \Phi_t(\frac{P^*_{st-k}}{P_t}) + \beta E_t[(\frac{C_t}{C_{t+1}})^{-\sigma}\{\xi V_{s,k+1}(\frac{P^*_{st-k}}{P_{t+1}}, \omega) + (1-\xi)V_{t+1}(\omega)\}]. \quad (7)$$

We now assume that there exists a threshold level at period t of menu cost that makes firms indifferent between the two pricing mechanisms, which is denoted by ω_{st}.[10] Here, since one-period profit does not depend on the size of fixed cost, the profit-maximizing prices in each period does not depend on the size of fixed adjustment cost. Then, firms choose a non-indexed sticky-price contract when $\omega > \bar{\omega}_{st}$ or an indexed contract when $\omega \leq \bar{\omega}_{st}$. It is noteworthy that since prices do not depend on costs of price adjustments, the cutoff points of menu-costs, denoted by ω_{st}, are affected by only aggregate variables as long as real marginal cost is independent of individual output levels. Table 3 summarizes equilibrium conditions for the aggregate share of menu-costs, price-level equation, and relative price distortion.

We continue to describe how firms choose their probability of price changes. It is likely that firms with a different level of menu-costs want to choose a different level of mean duration of their price changes. In the presence of heterogenous menu-costs, it would be the most desirable to let each individual firm choose the optimal mean duration of its own prices reflecting its fixed cost of price adjustment. But it would lead us to lose track of analytic tractability. In order to get around such a problem, we choose a median firm for those firms that are willing to be involved in a type of sticky-price contract. Since every firm is ex-ante identical except menu-costs, the median firm of sticky-price contract group corresponds to the firm whose menu-cost is the median level of menu costs among firms that choose sticky-price firms. As noted earlier, we let the median firm choose the mean duration of sticky-price contacts as a result of the firm's profit maximization. Having selected a mean duration for sticky-price contracts, we permit individual firms to opt for

[10] When the indexed contract is replaced by the flexible price pricing mechanism, we can use the same optimization technique. In this case, the payoff from flexible-price pricing mechanism can be written as $V_{ft}(\omega) = \Phi(\frac{P^*_{ft}}{P_t})$ - ωY_t + $E_t[\ q_{t,t+1}\ \{\ \xi V_{ft+1}(\omega) + (1-\xi)\ V_{t+1}(\omega)\ \}]$.

Table 3: Costs of Price Adjustments, Price-Level, and Relative Price Distortion

1. Share of Menu Costs in Output
$$s_{Ft} = (1-\xi)(\tilde{\omega} - \xi \int_{\tilde{\omega}_{st-1}}^{\infty} \omega f(\omega) d\omega) + \xi s_{Ft-1}.$$

2. Fraction of Indexed Contracts
$$\gamma_t = \xi\ \gamma_{t-1} + (1-\xi)(1-\phi_t)$$

3. Aggregate Price Equation: $1 = U_{it} + U_{st}$
$U_{it} = (1-\xi)(1-\phi_t)(\tilde{P}^*_{it})^{1-\epsilon} + \xi(\Pi_t/\Pi)^{\epsilon-1} U_{it-1}$, $U_{st} = (1-\xi)\phi_t(\tilde{P}^*_{st})^{1-\epsilon} + \xi \Pi_t^{\epsilon-1} U_{st-1}$.

4. Relative Price Distortion: $\Delta_t = \Delta_{it} + \Delta_{st}$
$\Delta_{it} = (1-\xi)(1-\phi_t)(\tilde{P}^*_{it})^{-\epsilon} + \xi(\Pi_t/\Pi)^{\epsilon}\Delta_{it-1}$, $\Delta_{st} = (1-\xi)\phi_t(\tilde{P}^*_{st})^{-\epsilon} + \xi \Pi_t^{\epsilon}\Delta_{st-1}$.

s_{Ft}: share of menu-costs in real output, $\tilde{P}^*_{it}$: relative price of indexed-contract price
ϕ_t: measure of firms that choose Calvo contracts at period t, γ_t: total measure of indexed contracts
Δ_t: relative price distortion, $\tilde{\omega}$: mean of menu-costs

either sticky-price or flexible-price contracts. A formalization of the choice problem of the median firm is given in the next section. The profit maximization problem of the median firm proceeds with the assumption of deterministic steady-states because our primary focus of this paper lies in the steady-state analysis.

4.3 Determination of Equilibrium Frequency of Price Changes

Consider an value of ϕ^*, which represents an equilibrium value for the fraction of firms with sticky prices. Given a value of ϕ^*, there is an interval $(\underline{\omega}, \overline{\omega})$ of menu costs for firms with sticky prices and there is a value ω_m for the median firm in the group of firms that choose sticky prices. For a given steady-state gross inflation Π, the following two conditions must be satisfied for (ϕ^*, ξ^*) to be a Nash equilibrium of the economy.

The first-condition is that given a value of ϕ^*, ξ^* must maximize the expected present-value of profits for median sticky-price firms, which have menu cost ω_m.

$$\xi^* = \arg\max_{\xi} V_s(\xi, \omega_m;\ \phi^*, \xi^*, \Pi). \tag{8}$$

The second-condition is that given ϕ^* and ξ^*, firms with menu cost $\underline{\omega}$ must be indifferent between indexed and non-indexed sticky-price contracts:

$$V_s(\xi^*, \underline{\omega};\ \phi^*, \xi^*, \Pi) = V_i(\xi^*, \underline{\omega};\ \phi^*, \xi^*, \Pi). \tag{9}$$

It is noteworthy that the interval $(\underline{\omega}, \overline{\omega})$ can differ from the support of menu costs ($\omega_{\min}$, $\omega_{\max}$). For example, the case in which all firms choose indexed contracts takes place when the threshold value of menu costs is greater than the upper bound of the support of menu costs, so that $\underline{\omega} > \omega_{\max}$. The case in which all firms choose non-indexed contracts corresponds to $\underline{\omega} \leq \omega_{\min}$.

Furthermore, the maximum level of gross inflation that has all firms choose non-indexed sticky-price contracts is defined as

$$\Pi^{\star} = \max\{\Pi \mid \underline{\omega}(\Pi) \leq \omega_{\min}\}. \tag{10}$$

The minimum level of gross inflation that has all firms choose indexed sticky-price contracts is defined as

$$\Pi^{\circ} = \min\{\Pi \mid \underline{\omega}(\Pi) \geq \omega_{\max}\}. \tag{11}$$

The reason why we define two threshold values of inflation is that they help to understand interactions between two groups of firms, which will be seen. In addition, Π^{*} corresponds to the maximum inflation rate for the region in which $\phi^{*} = 1$ holds. Π° is the minimum level of inflation for the region in which the natural rate hypothesis holds with $\phi^{*} = 0$.

It would be desirable to elaborate the difference between sections 3 and 4 in terms of the definition of the equilibrium frequency of price changes. The definition of section 3 can be rewritten in the light of the corresponding definition of section 4:

$$\xi^{*} = \arg\max_{\xi} V_s(\xi, \omega_m;\ \phi^{*} = i, \xi^{*}, \Pi), \tag{12}$$

for $i = 0$ or 1. It indicates that the definition of section 3 can be interpreted as the case in which the aggregate fraction of sticky-price contract is discrete. The reason for this si that all firms choose an identical type of contract at the same time, so that the aggregate fraction of each type of contract takes a value of one or zero in section 3.

4.4 Calibration

In this section, we begin with a brief discussion on how we assign numbers to structural parameters in order to explore quantitative implications of our set-up. In particular, the staggered price-setting behavior of firms depends on the heterogeneity of price adjustment costs across firms as well as the Calvo pricing mechanism. However, some recent menu-cost models posit that the magnitude of price adjustment costs is identical across firms. We thus briefly discuss the specification and calibration of some recent state-dependent pricing models.

In order to set the parameters of the distribution function of menu costs, we rely on microeconomic empirical studies on the menu costs. For example, Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) report that the share of menu costs in revenue is 0.4 % when they measure menu costs for a large U.S industrial manufacturer and its customers. However, in order to match the cross-country evidence in Table 1, we choose a value of s_F slightly higher than 0.004. Specifically, we set $s_F = 0.006$, where s_F denotes the steady state share of menu costs in output. Besides, the model implies that the steady-state share of menu costs in real output is given by

$$s_F = (1-\xi)\int_{\omega_s}^{\infty} \omega f(\omega)d\omega + \int_{0}^{\omega_s} \omega f(\omega)d\omega. \tag{13}$$

Hence, $s_F = 0.006$ gives rise to a restriction on the parameters of the distribution function of ω. Since the share of menu costs in real output depends on the steady state inflation, it is necessary to choose a steady state inflation rate in order to calibrate parameters of the distribution of ω. We set the steady state annual inflation rate equal to 2 percent.[11]

The demand elasticity is set to be either $\epsilon = 11$ or $\epsilon = 6$, which corresponds to assuming that steady state markup is $\mu = 1.1$ or $\mu = 1.2$. The time-discount factor is set to be $\beta = 0.99$, which corresponds to 4 % annual real interest rate. In addition, we assume a logarithmic utility function for consumption and a quadratic function for labor.

4.5 Results

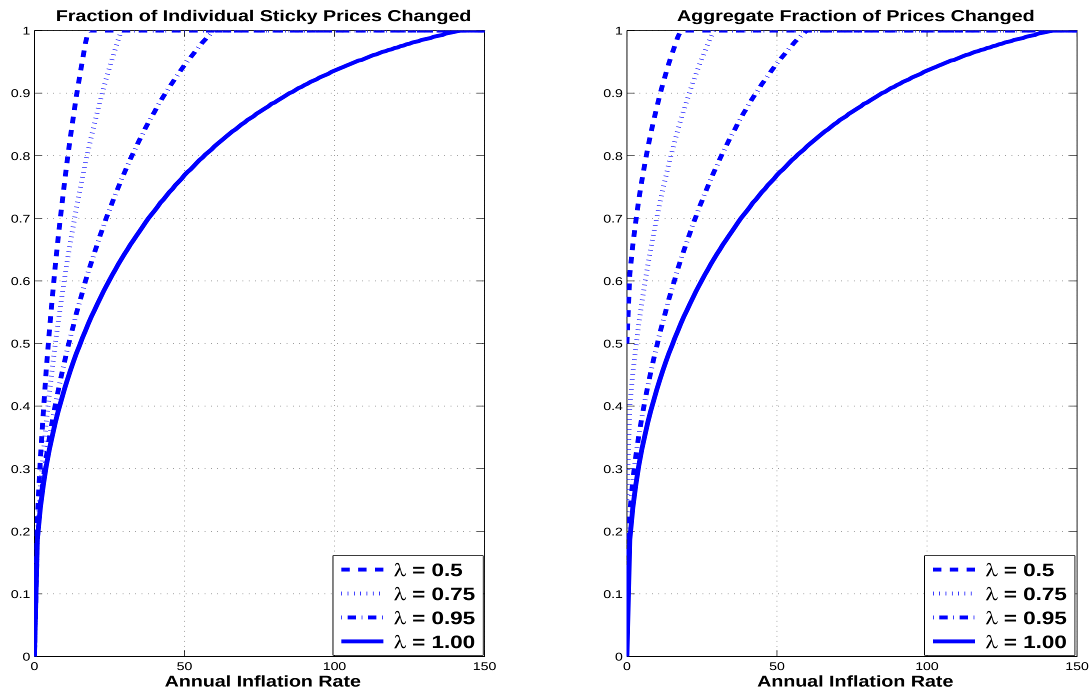
The key difference from the previous section is that menu-costs are different across firms. In particular, the introduction of heterogenous menu-costs tends to raise the equilibrium frequency of price changes. In order to have some intuition about this result, firms are divided into two groups; one is firms with non-zero menu-costs and the other is firms with zero menu-costs. In order to formalize the analysis, we let λ denote the fraction of firms with non-zero menu-costs. We then vary the value of λ within a unit interval in order to see how changes in the magnitude of firms with zero menu-costs affect the decisions of firms with non-zero menu-costs.

Figure 6 demonstrates that the natural rate hypothesis begins to hold at a lower rate of inflation in the presence of heterogenous menu-costs than it does in the case of homogenous menu-costs. The mechanism behind this is that the presence of heterogenous menu-costs generates interactions among firms in terms of the choice of price contracts, thereby leading to faster adjustments in the equilibrium frequency of price changes. In relation to this, the left panel in Figure 6 shows that as the size of λ decreases, non-zero menu-costs firms tend to choose a shorter duration of sticky-price contracts. It thus implies that changes in the fraction of firms with zero menu-costs have significant impacts on the decisions of firms with non-zero menu-costs firms.

The reason for this rapid adjustment can be elaborated as follows. First, a fraction of firms with low menu costs begin to deviate from non-indexed sticky-price contracts at a lower rate of inflation than firms with homogenous menu costs do. Second, the presence of such firms then affects the aggregate variables including the aggregate real marginal cost. Moreover, a rise in the fraction of firms that choose indexed contracts would lead

[11] In the case of homogenous menu-costs, $s_F = (1-\xi)\tilde{\omega}$, where $\tilde{\omega}$ denotes the identical level of menu costs.

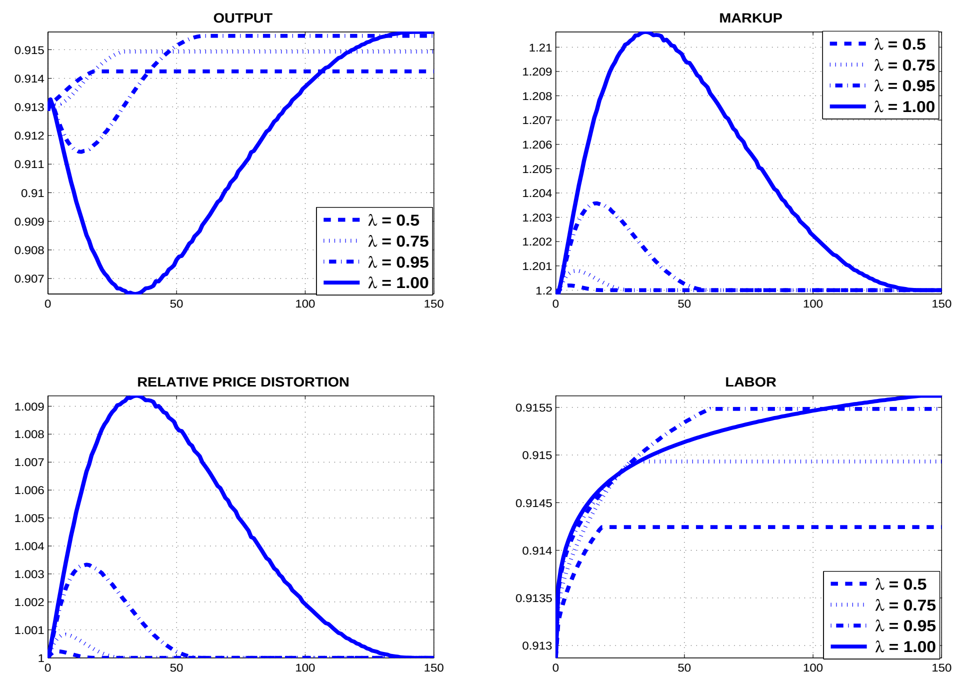
Figure 6: Effects of Heterogenous Menu Costs on Price Changes



to an increased influence on the decision making of firms with high menu costs on their pricing behaviors. For example, as more firms choose indexed contracts, firms with high menu-costs can have incentives to change their prices more frequently than they might do in the absence of those firms that choose indexed contracts. Consequently, we can tell that the externality in the pricing behaviors of individual firms acts as an important mechanism to reduce time-intervals between price changes in models with heterogenous menu-costs relative to those in models with homogenous menu-costs.

Figure 7 plots the real effect of steady-state inflation. It has been demonstrated in section 3 that allowing for endogenous contract structures reduces the real effect of steady-state inflation. Moreover, we have seen that the equilibrium frequency of price changes is higher in models with heterogenous menu-costs than in models with homogenous menu-costs. It therefore means that the real effect of inflation is smaller in models with heterogenous menu-costs than in models with homogenous menu-costs. Besides, one may wonder how much the introduction of heterogenous menu-costs contributes to the reduction of the real effect of inflation. As shown in Figure 4, we have seen a dramatic reduction in the size of the real effect of inflation after an endogenous contract structure is taken into account. Considering that the real effect of inflation becomes small in models with ho-

Figure 7: Effects of Steady-State Inflation (Endogenous Contracts with Heterogenous Menu Costs)



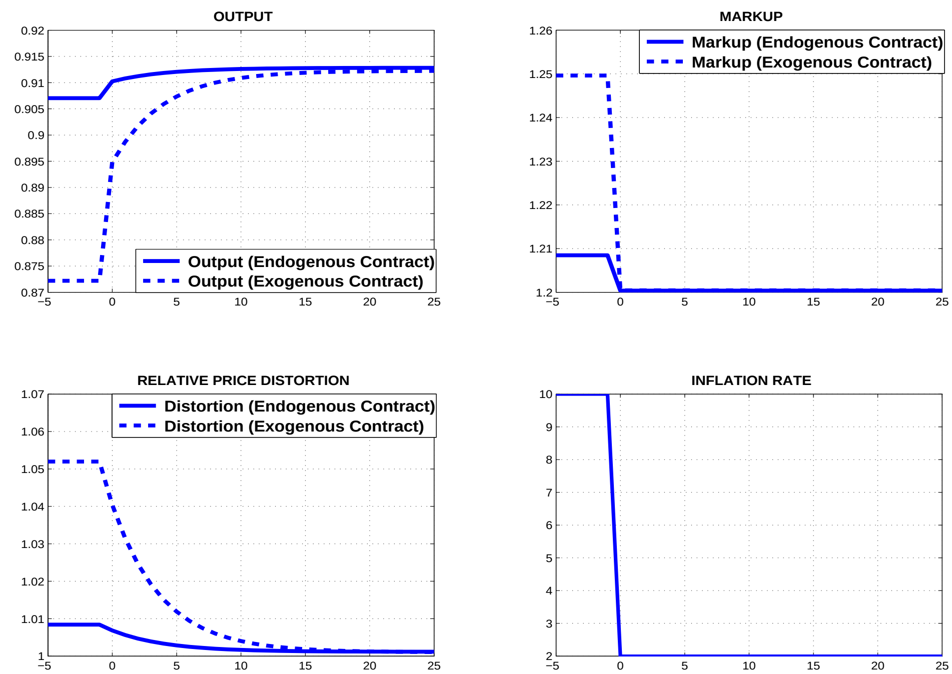
mogenous menu-costs, the introduction of heterogenous menu-costs may have a smaller effect on the real effect of inflation than does the introduction of endogenous contract structures. However, comparing mean durations of price adjustments in models with and without heterogenous menu costs, the effect of heterogenous menu-costs on the average frequency of price changes is significant.

5 Transition Dynamics

Having shown that the pricing behavior of firms varies as the steady-state rate of inflation changes, we discuss time-paths of equilibrium prices and output that would arise when the government begins a permanent decline in the steady state inflation at period 0 without having any prior announcement about it. We also proceed our analysis with the assumption that agents have perfect foresight.

We focus on models with homogenous menu-costs. The key reason why we do this is that it is comparable to the Calvo model with exogenous contract structures. In addition, such a disinflation policy generates immediate adjustments in the pricing behaviors of firms, which permit more analytical results. In order to see this, notice that after the

Figure 8: Transition Dynamics



disinflation policy is announced, the relative price of a new contract price turns out to be

$$\tilde{P}_t^* = \left(\frac{1 - \xi(\Pi_e)\Pi_e^{1-\epsilon}}{1 - \xi(\Pi_e)}\right)^{\frac{1}{1-\epsilon}}, \tag{14}$$

for $t = 0, 1, \cdots, \infty$. But the immediate adjustment in the pricing behaviors of firms does not necessarily mean the absence of output variations over the transition period. In other words, the fact that the real marginal cost is constant over time does not necessarily lead to a constant level of output during the transition period. The reason for this is that relative price distortion can vary over time.[12] In particular, it is the case that disinflation lowers relative price distortion in the long-run. It means that output can rise as relative price distortion goes down. As a result, the analysis implies that unanticipated disinflation does not always generate output declines in the initial periods. However, it does not mean that output gap is positive over the transition periods.

[12] Specifically, the utility maximization condition for labor supply can be written as $(\Delta_t Y_t)^{\chi} = C_t^{-\sigma} MC_t$. Table 2 implies that the real marginal cost is constant over time when $\sigma = 1$. Hence, to the extent that labor supply elasticity is infinite so that $\chi = 0$, we see that consumption and output are constant over time from period 0 onward. But, output and relative price distortion vary over time when $\chi > 0$.

6 Implications for Log-Linear Dynamics

In this section, we briefly discuss implications of our analysis for the first-order dynamics of the aggregate inflation. In so doing, we begin with the discussions on the role of indexation in the specification of the Phillips curve equation. We do this for the following reason. The steady-state analysis employed in the previous sections can be applied to both indexed sticky-price and flexible-price contracts, if prices are fully indexed. But if we move onto stochastic economies, the difference between indexed and flexible-price contracts becomes more apparent. Besides, since trend inflation has significant effects on the log-linear dynamics, it would be interesting to see the effect of allowing for endogenous contract duration on the log-linear dynamics.

6.1 Exogenous Contract Duration

The Calvo model assumes that only a fraction of all sellers can make price adjustments during any period, with others holding their nominal prices fixed at their previous-period values. In the Calvo model, log-linearizing the optimization conditions of sellers around the steady state with zero inflation rate leads to the prototypical New Keyenesian Phillips curve equation:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - \bar{y}_t), \tag{15}$$

where π_t is the logarithmic difference of price levels at periods t and $t-1$, y_t is the logarithm of real output, y_t is the logarithm of the natural-rate output, β is a discount factor satisfying $0 < \beta < 1$ and κ is the slope of the Phillips curve equation. The Phillips curve equation then implies that it is possible to maintain a constant value of the output gap permanently by setting $\pi_t = \pi$ from period t onward. The natural rate hypothesis thus fails in the prototypical New Keynesian Phillips curve equation.

Meanwhile, indexation plays an important role in recovering the natural rate hypothesis in the New Keynesian Phillips curve. For example, suppose that firms in the Calvo model index their prices to steady state inflation in the period when they are not allowed to re-optimize their prices. The introduction of such a simple indexation replaces the prototypical New Keynesian Phillips curve with the following equation:

$$\pi_t - \pi = \beta(E_t[\pi_{t+1}] - \pi) + \kappa(y_t - \bar{y}_t). \tag{16}$$

The Phillips curve equation (16) then implies that setting $\pi_t = \pi$ from period t onward no longer results in maintaining a constant value of the output gap permanently. The New Keynesian Phillips curve equation (16) thus satisfies Friedman's weaker version of the natural rate hypothesis.

Furthermore, suppose that firms index their prices to the expected inflation conditional on their lagged information set instead of using steady state inflation. Specifically, the full indexation to expected inflation leads to the following Phillips curve equation:

$$\pi_t = E_{t-1}[\pi_t] + \kappa(y_t - \bar{y}_t). \tag{17}$$

The key difference between equations (16) and (17) is whether steady-state inflation or expected inflation is used for indexing prices. The resulting difference in terms of their implications for the effectiveness of monetary policy is remarkable. For the use of expected inflation for indexing prices in the Calvo pricing model yields a variant of the Lucas's aggregate supply curve. A policy implication of equation (17) is that monetary policy affects real output, to the extent which it generates unexpected changes in inflation. In other words, only unexpected changes in the monetary policy that generate unexpected changes in the current inflation rate can influence the real output. It is thus evident that a variant of the Lucas's curve (17) satisfies the natural rate hypothesis.

Firms can use lagged inflation to update their prices:

$$\pi_t - \delta\pi_{t-1} = \beta(E_t[\pi_{t+1}] - \delta\pi_t) + \kappa(y_t - \bar{y}_t), \tag{18}$$

where δ measures of the degree of indexation. For example, the full indexation corresponds to $\delta = 1$, while an incomplete indexation satisfies $0 < \delta < 1$. In particular, a permanent shift in the steady state inflation does not have any permanent effect on output gap in the long-run in the case of full indexation to lagged inflation, which has been used in many New Keynesian macro models in order to increase inflation persistence.[13] In summary, it has been shown that indexation is an important mechanism to make the natural rate hypothesis hold in the New Keynesian Phillips curve. But the difficulty associated with indexation is that while indexation requires all firms to change prices in each period, many microeconomic empirical studies indicate that many individual firms tend to fix their prices for more than one quarter.

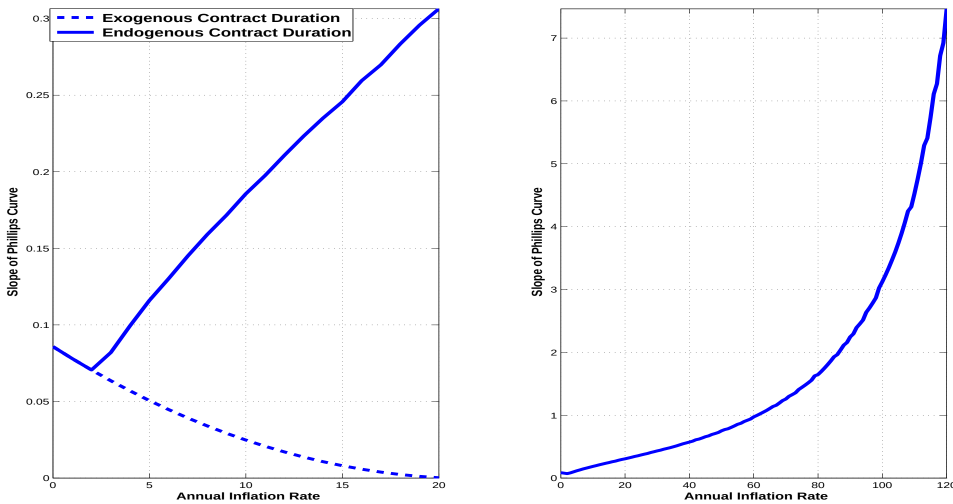
6.2 Endogenous Contract Duration

Having discussed implications of indexed contracts, we now discuss consequences of flexible-price contracts with endogenous contract duration on the slope of Phillips curve. In order to do so, we follow the approach of Ascari (2004) that log-linearize equilibrium conditions around steady states with non-zero inflation rate in order to analyze the effect of trend inflation on the NK Phillips curve. Specifically, we have the following Phllips curve equation:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda_{mc}(\Pi)\hat{m}c_t + \lambda_u(\Pi)u_t, \tag{19}$$

[13] The recent dynamic indexation includes not only a full indexation to lagged inflation as is done in Woodford (2003), Christiano, Eichenbaum and Evans (2005), and Levin, Onaski, Williams and Williams (2005) but also the optimal indexation of Calvo, Celasun, and Kumhopf (2003).

Figure 9: Slope of the NK Phillips Curve in Models with Exogenous and Endogenous Contract Duration



where u_t is an endogenous disturbance term that depends on current and future inflation and output and λ_{mc} and λ_u are defined as

$$\lambda_{mc}(\Pi) = \frac{(1-\xi(\Pi)\Pi^{\epsilon-1})(1-\beta\xi(\Pi)\Pi^{\epsilon})}{\xi(\Pi)\Pi^{\epsilon-1}}, \ \lambda_u(\Pi) = \beta(1-\Pi)(1-\xi(\Pi)\Pi^{\epsilon-1}). \tag{20}$$

Before proceeding further, we can see that the coefficient of the real marginal cost is subject to similar constraints described in section 3. More explicitly, the steady-state inflation rate must satisfy $\beta\xi\Pi^{\epsilon} < 1$ when inflation is positive. When inflation is high, this constraint can be binding in the case of exogenous contract duration. For example, when we set $\beta = 0.99$, $\xi = 0.75$, and $\epsilon = 11$, the constraint implies that the annual steady-state inflation rate should be less than 10.98 %. In the case of $\epsilon = 6$, the upper limit becomes 20.35 %.

Figure 9 plots changes in $\lambda_{mc}(\Pi)$ attributable to Π. Figure 9 indicates that the effect of trend inflation on the slope of the Phillips curve becomes dramatically changed as we introduce endogenous contract duration in the model. First, models with exogenous contract duration imply that the size of $\lambda_{mc}(\Pi)$ is reduced as inflation rises. This result is in line with Ascari (2004). Second, models with endogenous contract duration imply that $\lambda_{mc}(\Pi)$ rises with the aggregate inflation. The slope of the Phillips curve therefore is likely to rise as trend inflation rises. Figure 9 demonstrates that there is a range of inflation around zero inflation rate in which the slope of the Phillips curve drops down in the model with endogenous contract duration. The key reason for this is that at least one-fourth of all firms change their prices in each period at zero inflation rate. Furthermore, as shown in the right panel, we can see that the size of the slope parameter becomes large as inflation approaches the threshold value in which the natural rate hypothesis holds.

7 Directions For Future Research

We have demonstrated that a modelling framework with endogenous selection on the mean duration of contract leads the natural rate hypothesis to hold when steady state inflation is high, while it fails with exogenous contract structure. Besides, allowing for heterogenous menu costs results in a shorter average mean duration of contract through interactions of firms it generates.

In this paper, our analysis has proceeded with non-stochastic cases, in order to focus on economic environments directly relevant for the natural rate hypothesis, though it can be extended to stochastic economies. As a future study that follows the current analysis, we can extend our analysis to the real effect of monetary policy in stochastic economies. In particular, the modelling framework described above can be used to augment the prototypical New Keynesian Phillips with an endogenous disturbance term that depends on the optimizing selection of pricing mechanisms. Besides, since the parameter values of the Phillips curve can be affected by monetary policy as well as exogenous shocks, allowing for the endogenous slope of the Phillips curve reduces the extent of which the Lucas critique is applied to the exogenous determination of mean duration of contract in the prototypical Calvo model.

It also should be mentioned that our modelling framework can be easily incorporated into recent empirical DSGE models such as Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), and Levin, Onaski, Williams, and Williams (2005). These models include Calvo-type pricing structure, while the mean duration of prices and timing of price changes in their models are exogenously determined. In relation to this, an advantage of the modelling strategy adopted in this paper is to make analytically tractable solutions of dynamic equilibrium models with complicated decision problems on pricing mechanism, thereby improving the flexibility of model specification.[14] Although the modelling framework leads to a tractable non-linear solution of the model, we can also use log-linear approximation around a steady state with low inflation in order to analyze effects of exogenous shocks as well as policy changes. It then facilitates the estimation of the model analyzed in this paper.

[14] We have combined elements of time-dependent and state-dependent pricing models to allow firms to change their pricing mechanisms occasionally, taking into account fixed costs of price adjustments. Thus, our modelling framework builds on the works of Ball, Mankiw and Romer (1988), Ball and Mankiw (1994), Ireland (1997), Willis (2002), and Devereux and Siu (2005).

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