A Simple Framework For International Monetary Policy Analysis*

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Abstract

We study the international monetary policy design problem within an optimizaton-based two country model, where each country faces a short run tradeoff between output and inflation. The model is sufficiently tractable to solve analytically. We find that in the Nash equilibrium, the policy problem for each central bank is isomorphic to the one it would face if it were a closed economy. Gains from cooperation arise, however, that stem from the impact of foreign economic activity on the domestic marginal cost of production. While under Nash central banks need only adjust the interest rate in response to domestic inflation, under cooperation they should respond to foreign inflation as well. In either scenario, flexible exchange rates are desirable.

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1 Introduction

The existence of a short run tradeoff between output and inflation is a central obstacle to the smooth management of monetary policy. In the open economy, of course, there are additional complications: Not only must a central bank take account of the exchange rate in this situation, but potentially also the feedback responses of foreign central banks to its policy actions.

In this paper we revisit these classic issues by developing a simple two country model that is useful for international policy analysis. Consistent with a voluminous recent literature, our framework is optimization-based and is sufficiently tractable to admit an analytical solution.[1] In most of this work (particularly the work that is purely analytical), nominal price setting is done on a period by period basis, implying the absence of any kind of meaningful output/inflation tradeoff.[2] We differ by allowing for a short run tradeoff and doing so in a way that does not sacrifice tractability. Thus, we are able to investigate qualitatively the implications of international considerations for monetary policy management without having to abstract from the central problem that the tradeoff poses.

Our framework is essentially a two country version of the small open economy model we developed in Clarida, Gali, Gertler (CGG) (2001), which is in turn based on Gali and Monacelli (2001). In that paper, we showed that under certain conditions, the monetary policy problem was isomorphic to the problem of the closed economy studied in CGG (1999). In this setting, accordingly the qualitative insights for monetary policy management are very similar to what arises for the closed economy. International considerations, though, may have quantitative considerations, as openness does affect the model parameters and thus the coefficients of the optimal feedback policy. In addition, openness gives rise to an important distinction between cpi inflation and domestic inflation. To the extent there is perfect exchange rate pass through, we find that the central bank should target domestic inflation and allow the exchange rate to float, despite the impact of the resulting exchange rate variability on the cpi.

[1] Some examples of this recent literature include, Obstfeld and Rogoff (2000), Corsetti and Pesenti (2000), Kollman, McCallum and Nelson (1999), Devereaux and Engle, Lane, and Chari, Kehoe, and McGrattan (2000).

[2] A recent exception is Benigno and Benigno (2001) who have concurrently emphasized some similar themes as in our paper, though the details of the two approaches differ considerably.

In the two country setting we study here, the monetary policy problem is sensitive to the nature of the strategic interaction between central banks. In the absence of cooperation (the "Nash" case), our earlier "isomorphism" result is preserved. Each country confronts a policy problem that is qualitatively the same as the one a closed economy would face. The two country framework, though, allows us to characterize the equilibrium exchange and illustrate concretely how short run tradeoff considerations enhance the desirability of flexible exchange rates.

The strict isomorphism result, however, breaks down when we allow for the possibility of international monetary coordination. There are potentially gains from cooperation within our framework, though they are somewhat different in nature than stressed in the traditional literature, as they are supply side in nature. In particular, both the domestic natural level of production and domestic natural rate depend on foreign monetary policy behavior. By coordinating policy to take account of these spillovers, central banks can in principle improve welfare. As we show, coordination alters the Nash equilibrium in a simple and straightforward way. Among other things, we show that it is possible to implement the optimal policy under coordination by having each central bank pursue an interest rate feedback rule of the form that was optimal under Nash (a kind of Taylor rule), but augmented to respond to foreign inflation as well as domestic inflation.

In section 2 below we characterize the behavior of households and firms. Section 3 describes the equilibrium. Section 4 describes the policy problem and the solution in the Nash case. In section 5, we consider the case of cooperation. Concluding remarks in section 6. Finally, the appendix (to be added) provides explicit derivations of the welfare functions.

2 The Model

The framework is a variant of a Dynamic New Keynesian Model applied to the open economy, in the spirit of much recent literature[3]. There are two countries, home and foreign, that differ in size but are otherwise symmetric. The home country (H) has a mass of households $1-\gamma$, and the foreign country (F) has a mass γ. Otherwise preferences and technologies are the same across countries, though shocks may be imperfectly correlated. Within

[3] See Lane (2000) for a survey.

in each country, households consume a domestically produced good and an imported good. Households in both countries also have access to a complete set of Arrow-Debreu securities which can be traded both domestically and internationally.

Domestic production take place in two stages. First, there is a continuum of intermediate goods firms, each producing a differentiated material input. Final goods producers then combine these inputs into output, which they sell to households. Intermediate goods producers are monopolistic competitors and set nominal prices on a staggered basis. Final goods producers are perfectly competitive. We normalize the number of intermediate goods firms to be the same as the number of households in each country.

Only nominal prices are sticky. As is well known, in the absence of other frictions, with pure forward looking price setting there is no short run tradeoff between output and inflation. To introduce a short run tradeoff in a way that is analytically tractable, we assume that households have a bit of market power in the labor market, and then introduce exogenous variation in this market power as a convenient way to generate cost-push pressures on inflation.

We next present the present the decision problems of households and firms.

2.1 Households

Let C_t be the following index of consumption of home and foreign goods:

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma} \tag{1}$$

and let P_t be the corresponding price index (that follows from cost minimization):

$$\begin{aligned} P_t &= k^{-1} P_{H,t}^{1-\gamma} (P_{F,t})^{\gamma} \\ &= k^{-1} P_{H,t} S_t{}^{\gamma} \end{aligned} \tag{2}$$

where $S_t = \frac{P_{F,t}}{P_{H,t}}$ is the terms of trade and $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$.

The representative household in the home country maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U(C_t, N_t(h)) \tag{3}$$

subject to the sequence of budget constraints

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1}\} = W_t(h)\ N_t(h) + D_t + \Gamma_t + T_t \tag{4}$$

where D_{t+1} is the (random) payoff of the portfolio purchased at t, with $Q_{t,t+1}$ being the corresponding stochastic discount factor, Γ_t is profits, T_t is lump sum taxes, $N_t(h)$ denotes hours of labor, and $W_t(h)$ the household's nominal wage. In addition, the household is a monopolistically competitive supplier of labor and faces the following constant elasticity demand function for its services:

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta_t} N_t \tag{5}$$

where N_t is aggregate employment and W_t the relevant aggregate wage index. The elasticity of labor demand, η_t, is the same across workers, but may vary over time. Further, the parameter η_t, is specifically related to the production technology, as we discuss later.

We specialize the period utility function to be of the form:

$$U(C_t, N_t(h)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi} \tag{6}$$

The first order necessary conditions for consumption allocation and intertemporal optimization are standard:

$$P_{H,t}\ C_{H,t} = (1-\gamma)\ P_t C_t \tag{7}$$

$$P_{F,t}\ C_{F,t} = \gamma\ P_t C_t \tag{8}$$

$$\beta\ (C_{t+1}/C_t)^{-\sigma}\ (P_t/P_{t+1}) = Q_{t,t+1} \tag{9}$$

By using the previous condition and letting R_t denote the (gross) nominal yield on a one-period discount bond (and, hence, $R_t^{-1} = E_t\{Q_{t,t+1}\}$ being the price of the latter) we can derive the Euler equation

$$1 = \beta R_t\ E_t\{(C_{t+1}/C_t)^{-\sigma}(P_t/P_{t+1})\} \tag{10}$$

The first order condition for labor supply reflects the household's market power

$$\frac{W_t(h)}{P_t} = (1 + \mu_t^w)\ N(h)_t^{\phi} C_t^{\sigma} \tag{11}$$

where $\mu_t^w = \frac{1}{\eta_t - 1}$ is the optimal wage markup. In contrast to Erceg, Henderson, and Levin (2000),wages are perfectly flexible, implying the absence of any endogenous variation in the wage markup. On the other hand we allow for exogenous variation in the wage markup arising from shifts in η_t, interpretable as exogenous variation in workers' market power.[4] Note that because wages are flexible, all workers will charge the same wage and have the same level of hours. Thus we can write:

$$\begin{aligned} N_t(h) &= N_t \\ W_t(h) &= W_t \end{aligned} \tag{12}$$

A symmetric set of first order conditions holds for citizens of the foreign country. In particular, given international tradability of state contingent securities, the intertemporal efficiency condition can be written as:

$$\beta\ (C_{t+1}^*/C_t^*)^{-\sigma}\ (P_t^*/P_{t+1}^*)\ (\mathcal{E}_t/\mathcal{E}_{t+1}) = Q_{t,t+1} \tag{13}$$

which combined with the law of one price, which implies $P_t = \mathcal{E}_t\ P_t^*$ all t, (9), and a suitable normalization of initial conditions implies:

$$C_t = C_t^* \tag{14}$$

for all t.[5]

2.2 Firms

2.2.1 Final Goods

Each final goods firm in the home country uses a continuum of intermediate goods to produce output, Y_t, according to the following CES technology:

[4] To be clear, we assume exogenous variation in the wage markup for only for simplicity. In our view this approach provides a convenient way to obtain some of the insights that arise when there is an endogenous markup due to wage rigidity.

[5] As in Corsetti and Pesenti (2000), Benigno and Benigno (2000), the complete asset market equilibrium can be achieved in a simple asset market in which only nominal bonds are traded.

$$Y_t = \left(\int_0^{1-\gamma} Y_t(f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \tag{15}$$

where $Y_t(f)$ is input produced by intermediate goods firm f and where the total mass of different types of intermediate goods is $1-\gamma$. Profit maximization, taking the price of the final good $P_{H,t}$ as given, implies the set of demand equations:

$$Y_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} Y_t \tag{16}$$

as well as the relationship

$$P_{H,t} = \left(\int_0^{\gamma} P_{H,t}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}} \tag{17}$$

2.2.2 Intermediate Goods

Each intermediate goods firm produces output using a technology that is linear in labor input, $N_t(f)$, as follows

$$Y_t(f) = A_t \ N_t(f) \tag{18}$$

where A_t is an exogenous technology parameter. The labor used by each firm is a CES composite of individual household labor, as follows:

$$N_t(f) = \left(\int_0^{1-\gamma} N_t(h)^{\frac{\eta_t-1}{\eta_t}} dh \right)^{\frac{\eta_t}{\eta_t-1}} \tag{19}$$

Aggregating across cost minimizing final goods firms yields the market demand curve for household labor given by equation (11), where the technological parameter η_t is the wage elasticity of hours demand. Because each household charges the same wage and supplies the same number of hours, we can treat the firm's decision problem over total labor demand as just involving the aggregates $N_t(f)$ and W_t. We also assume that each firm receives a subsidy of τ percent of its wage bill.

In addition, intermediate goods firms set prices on a staggered basis as in Calvo (1983), where θ is the probability a firm keeps its price fixed in a given period and $1-\ \theta$, is the probability it changes, where probability draws are i.i.d. over time. Firms that do not adjust price simply adjust output to meet demand (assuming they operate in a region with a non-negative net markup.)

Firms that are able to choose price optimally in period t maximize the following objective:

$$E_t \sum_{j=0}^{\infty} \theta^t Q_{t,t+j} \; Y_t(f) \; \left(\overline{P}_{H,t} - P_{H,t+j} \; MC_{t+j}\right) \tag{20}$$

subject to the demand curve (16), and where MC_t denotes the real marginal cost. Choosing labor to minimize costs conditional on output yields:

$$\begin{aligned} MC_t &= \frac{(1-\tau)(W_t/P_{H,t})}{A_t} \\ &= \frac{(1-\tau)(W_t/P_t)}{A_t}(S_t^{\gamma}/k) \end{aligned} \tag{21}$$

Keep in mind that equation (21) is effectively a first order condition for labor demand.

The solution to the previous problem implies that firms set price equal to a discounted stream of expected future nominal marginal cost.

$$E_t \sum_{j=0}^{\infty} \theta^t Q_{t,t+j} \; Y_t(f) \left(\overline{P}_{H,t} - (1+\mu^p) P_{H,t+j} \; MC_{t+j}\right) = 0 \tag{22}$$

Note that if a firms was able to freely adjust its price each period, it will choose a constant markup over marginal cost, i.e., $\theta = 0$ implies

$$\frac{\overline{P}_{H,t}}{P_{H,t}} = (1+\mu^p) \; MC_t \tag{23}$$

Finally, the law of large number implies that

$$P_{H,t} = \left[\int_0^{\gamma} \theta P_{H,t-1}^{1-\xi} + (1-\theta)\overline{P}_{H,t}^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{24}$$

3 Equilibrium

We begin by characterizing the equilibrium conditional on output. How the model is closed depends on the behavior of prices. We first characterize the flexible price equilibrium, for which an exact solution is available, and then turn to the case of staggered price setting, for which an approximate solution is available.

Goods market clearing in the home and foreign countries implies:

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C^*_{H,t} \tag{25}$$

$$\gamma Y^*_t = (1-\gamma)C_{F,t} + \gamma C^*_{F,t} \tag{26}$$

Using the demand curves for home and foreign goods by home citizens, equations (7) and (8), respectively, as well as the analogues for foreign citizens, as well as the law of one price, we obtain that the cpi real exchange rate is unity:[6]

$$\frac{\mathcal{E}_t P^*_t}{P_t} = 1 \tag{27}$$

and that the trade balance is zero within each country.

$$P_{H,t}Y_t = P_tC_t \tag{28}$$

$$P^*_{F,t}Y^*_t = P^*_tC^*_t \tag{29}$$

In turn, the previous conditions imply an aggregate demand schedule that relates domestic output (per capita) to domestic consumption and the terms of trade, $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$, as follows:

$$Y_t = k^{\ -1} C_t \ S^{\gamma}_t \tag{30}$$

with

$$S_t = \frac{Y_t}{Y^*_t} \tag{31}$$

as well as an aggregate demand schedule. Observe that equations (30), (31) and the consumption euler equation (10) determine domestic output demand, conditional on foreign output and the path of the real interest.

[6] Note that we assume producer currency pricing and complete-pass thorugh. Devereaux and Engle (2000), among others, have emphasized the role of local currency pricing.

On the supply side, given that $\int_0^{1-\gamma} Y_t(f)\, df = A_t \int_{0t}^{1-\gamma} N_t(f)\, df$ and $N_t = \int_{0t}^{1-\gamma} N_t(f)\, df$, aggregate output may be expressed as

$$Y_t = A_t N_t V_t \tag{32}$$

with

$$\begin{aligned} V_t &= [\int_0^{1-\gamma} \frac{Y_t(f)}{Y_t} df]^{-1} \\ &= [\int_0^{1-\gamma} \left(\frac{P_t(f)}{P_t}\right)^{-\xi} df]^{-1} \le 1 \end{aligned}$$

Combining the labor supply and demand relations (11), (21) and the using the aggregate demand schedule (30) to eliminate C_t yields the following expression for real marginal cost:

$$MC_t = (1-\tau)(1+\mu_t^w)\,\frac{k^{\sigma-1}\ N_t^{\phi}\ C_t^{\sigma}\ S_t^{\gamma}}{A_t} \tag{33}$$

$$= (1-\tau)(1+\mu_t^w)\,\frac{k^{\sigma-1}\ N_t^{\phi}\ Y_t^{\sigma_0}\ (Y_t^*)^{-\widehat{\sigma}}}{A_t} \tag{34}$$

where σ_0 is the output elasticity of the marginal utility of an additional unit of domestic output, given by

$$\sigma_0 = \sigma + \gamma(1-\sigma)$$

with

$$\begin{aligned} \widehat{\sigma} &= \sigma_0 - \sigma \\ &= \gamma(1-\sigma) \end{aligned}$$

As will eventually become clear, the implications of international considerations for monetary policy within this framework depend critically on how openness affects marginal cost. We see that when $\sigma > (<)1$, a more open economy has a flatter (steeper) marginal cost curve, and a rise in foreign output increases (lowers) domestic marginal cost. When $\sigma = 1$, the slope of the marginal cost curve is independent of openness and the level of marginal cost is independent of foreign output. Note that if $\widehat{\sigma} = 0$, either due to $\gamma = 0$ or $\sigma = 1$, open economy effects on real marginal costs disappear, and $\sigma_0 = \sigma$.The intuition for this effect is as follows. Consider a rise in foreign output. In equilibrium, some of this will be consumed at home thus boosting C and lowering the shadow value of work: this will tend to raise MC. However, the rise in foreign output will improve the home terms of trade, boosting the purchasing power of the money wage, and this will tend to lower MC. When $\sigma > 1$, the consumption sharing effect will dominate the terms of trade effect, and vice versa. When $\sigma = 1$, the two effects exactly cancel.

Note that we have characterized the values of C_t, S_t, MC_t and N_t conditioned on Y_t, V_t (which captures the dispersion on output and Y_t^*. An analogous set of relations for the foreign country determines C_t^*, $S_t^* (= S_t^{-1})$, MC_t^* and N_t^* conditional on Y_t^*, V_t^* and Y_t . How we close the model depends on the behavior of prices.

3.1 Flexible Price Equilibrium

We consider an equilibrium with flexible prices where the wage markup is fixed at its steady state value $1 + \mu^w$. We focus on this case because we would like to define a measure of the natural level of output that has the feature that cyclical fluctuations in this construct do not reflect variations in the degree of efficiency (hence we shut off variation in the wage markup.) This approach also makes sense if we think of variations in the wage markup as standing in for wage rigidity.

In addition, we make the distinction between the natural flexible price equilibrium that arise when prices are flexible across the world, and the domestic natural equilibrium that corresponds to flexible prices at home, but taking as given foreign output. The distinction between these two concepts becomes highly relevant when we compare the Nash versus cooperative equilibria.

Under flexible prices, all firms set price equal to a constant markup over marginal cost, as equation (11) suggests. Symmetry, further, suggests that all firms choose the same price. Imposing the restriction $\frac{\overline{P}_{H,t}}{P_{H,t}} = 1$ on equation (23) implies that in the flexible price equilibrium, real marginal cost is given by

$$\overline{MC} = \frac{1}{1+\mu^p} \tag{35}$$

where we use the bar to denote the flexible price equilibrium value of a variable. Symmetry of prices further implies all firms choose the same level of output implying, $\overline{V}_t = 1$ and hence from equation (32),

$$\overline{Y_t} = A_t\overline{N_t} \tag{36}$$

Furthermore, using the fact that $\overline{MC}_t = (1+\mu^p)^{-1}$ and fixing the wage markup at its steady state then permits us to use equation (33) to solve for the natural (flexible price) level of output:

$$\overline{Y_t} = \left(\frac{k^{1-\sigma}\ A_t^{1+\phi}(\overline{Y}_t^*)^{\widehat{\sigma}}}{(1-\tau)(1+\mu^w)(1+\mu^p)} \right)^{\frac{1}{\sigma_0+\phi}} \tag{37}$$

where $\overline{Y}_t^*$ is the natural level of foreign output.

We obtained $\overline{Y_t}$ by assuming that prices are flexible across the world. As we discussed earlier, it is also useful to define the domestic natural level of output, $\overline{Y_t}^d$, the level of output corresponding to flexible domestic prices, but taking as given the foreign output level:

$$\begin{aligned} \overline{Y_t}^d &= \left(\frac{k^{1-\sigma}\ A_t^{1+\phi}(Y_t^*)^{\widehat{\sigma}}}{(1-\tau)(1+\mu^w)(1+\mu^p)} \right)^{\frac{1}{\sigma_0+\phi}} \\ &= \overline{Y_t} \cdot (Y_t^*/\overline{Y}_t^*)^{\frac{\widehat{\sigma}}{\sigma_0+\phi}} \end{aligned} \tag{38}$$

Notice that the impact of foreign output on Y_t^* depends on $\widehat{\sigma}$ and σ_0.

3.2 Equilibrium Dynamics under Sticky Prices

We now express the system with sticky prices as a loglinear approximation about the steady state that determines behavior conditional on a path for the nominal interest rate.

From equation (30), aggregate demand is given by

$$y_t = c_t + \gamma s_t \tag{39}$$

where, from the euler equation (10), aggregate consumption evolves according to

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \gamma E_t\{\Delta s_{t+1}\}) \tag{40}$$

where π_{t+1} is the rate of domestic inflation from t to $t+1$ and where, from equation (31), the terms of trade is given by

$$s_t = y_t - y_t^* \tag{41}$$

On the supply side, the first order approximation to the aggregate production function, (32) implies:

$$y_t = a_t + n_t \tag{42}$$

Further, combining the log-linearized optimal price setting rule (22) with the price index (24) yields

$$\pi_t = \delta mc_t + \beta E_t\{\pi_{t+1}\} \tag{43}$$

where $\delta = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Let $x_t = y_t - \overline{y}_t^d$ denote the domestic output gap, i.e., the gap between output and the domestic natural level. Then from the loglinearized version of the expression for marginal cost (33) and the production function, (42), we obtain[7]

$$mc_t = (\sigma_o + \phi)\ x_t + \mu_t^w \tag{44}$$

where from equation (38):

$$\overline{y}_t^d = \frac{1+\phi}{\sigma_0 + \phi}\ a_t + \frac{\widehat{\sigma}}{\sigma_0 + \phi}\ y_t^* \tag{45}$$

It is straightforward to collapse the system into an "IS curve" and "AS" that determine x_t and π_t conditional on the path of r_t:

$$x_t = E_t\{x_{t+1}\} - \sigma_o^{-1}\ E_t\{r_t - \pi_t - \overline{rr}_t^d\} \tag{46}$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \lambda x_t + u_t \tag{47}$$

[7] From the loglinearized version of the expression for marginal cost (33) and the production function, (42), we obtain $mc_t = \mu_t^w + (\sigma_0 + \phi)\ y_t - \widehat{\sigma} y_t^* - (1+\phi)\ a_t$. Combining this expression with equation (45) then yields equation (44).

with $\lambda = \delta(\sigma_o + \phi)$.and $u_t = \delta\widehat{\mu}_t^w$, and where $\overline{rr}_t^n$ is the domestic natural real interest rate, given by:

$$\overline{rr}_t^n = \sigma_0 \ E_t\{\Delta\overline{y}_{t+1}^d\} - \widehat{\sigma} \ E_t\{\Delta y_{t+1}^*\} \tag{48}$$

An analogous set of equations holds for the foreign country, with $\sigma_0^* = \{\sigma + (1-\gamma)(1-\sigma)\}$.and thus $\widehat{\sigma}^* = (1-\gamma)(1-\sigma)$.

As we discussed in Clarida, Gali and Gertler (2001), the form of the system is isomorphic to that of the closed economy. Open economy effects enter in two ways: first, through the impact of the parameter σ_o which affects both the interest elasticity of domestic demand (equal to σ_o^{-1} and the slope coefficient on the output gap $\lambda = \delta(\sigma_o + \phi)$); and second via the impact of foreign output on the natural real interest rate. In the special case of log utility (implying $\sigma_o = \sigma$ and $\widehat{\sigma}$) , the open economy effects disappear and the system becomes perfectly identical to a closed economy.

Finally, we obtain a simple expression linking the terms of trade to movements in the output gap:

$$\begin{aligned} s_t &= x_t + \overline{y}_t^d - y_t^* \\ &= x_t + \overline{s}_t^d \end{aligned} \tag{49}$$

where $\overline{s}_t$ is the domestic natural level of the terms of trade.

4 Welfare and Optimal Policy: the Non Co-operative Case

In this section we analyze the problem of a policymaker that seeks to maximize the utility of domestic households, while taking as given foreign policy and outcomes. First, we assume that the fiscal authority chooses a subsidy rate τ that maximizes the utility of the domestic household in a zero inflation steady state (i.e., in the absence of cost-push shocks), while taking as given foreign variables. As shown in the appendix, the subsidy must satisfy

$$(1-\tau)(1+\mu^w)(1+\mu)(1-\gamma) = 1 \tag{50}$$

and for the foreign country

$$(1-\tau^*)(1+\mu^w)(1+\mu)\gamma = 1 \tag{51}$$

We refer to the resulting steady state as the Nash steady state.

Let $\widetilde{x}_t \equiv y_t - \overline{y}_t$ denote the gap between output and its natural level, $\overline{y}_t$ and $\widetilde{x}_t^*$ the corresponding value for the foreign country. As we show in the appendix, a second order approximation of the utility of the domestic consumer around the (world) flexible price equilibrium is given by:

$$\mathbb{W}^H \equiv -\frac{\Lambda}{2}E_o \sum_{t=0}^{\infty}\beta^t \; [\pi_t^2 + \alpha\; \widetilde{x}_t^2 + 2\alpha\frac{\widehat{\sigma}}{\sigma_0+\phi}\widetilde{x}_t\widetilde{x}_t^*] \tag{52}$$

where $\Lambda \equiv \frac{\xi}{\delta}$ and $\alpha \equiv \frac{(\sigma_0+\phi)\delta}{\xi} = \frac{\lambda}{\xi}$. In the Nash equilibrium, the home country takes the foreign output gap as exogenous.

It is convenient to express the objective function in terms of the domestic output gap, $x_t^d (\equiv y_t - \overline{y}_t^d)$, as follows:

$$\mathbb{W}^H \equiv -\frac{\Lambda}{2}E_o \sum_{t=0}^{\infty}\beta^t \; [\pi_t^2 + \alpha\; x_t^2] + F_t \tag{53}$$

with

$$F_t \equiv \frac{\Lambda}{2}E_o \sum_{t=0}^{\infty}\beta^t 2\alpha(\frac{\widehat{\sigma}}{\sigma_0+\phi})^2\widetilde{x}_t^{*2}$$

and where the relation between the domestic output gap, x_t and the overall output gap $\widetilde{x}_t$ is given by

$$x_t = \widetilde{x}_t + (\frac{\widehat{\sigma}}{\sigma_0+\phi})\widetilde{x}_t^* \tag{54}$$

and where an analogous relation holds for the foreign country.

Given that the home central bank takes foreign output as exogenous, the objective function is of the same form to that for a closed economy, though with the goal in terms of domestic inflation, as opposed to overall cpi inflation. In particular, the central bank minimizes a loss function that is quadratic in the domestic output gap and domestic inflation, with a weight α on the output gap.[8] In addition, the parameter α differs from the closed

[8] See, e.g., Woodford (2000). As in the closed economy, the presence of domestic inflation in the objective function reflects the costs of resource misallocation due to relative

economy counterpart only to the extent σ_o differs from σ. Otherwise, the objective function (including the underlying parametric specification of α) is perfectly identical to what would arise in the closed economy.

Given that the IS and AS curves also have the same form as in the closed economy, the overall policy problem is completely isomorphic to the closed economy, as we found in Clarida, Gali and Gertler (2001). In particular, in the absence of commitment, the central bank will choose x_t and π_t each period to maximize equation (53) subject to the aggregate supply curve given by equation (47), taking expectations of the future as given.

The solution satisfies:

$$x_t = -\frac{\lambda}{\alpha}\,\pi_t \quad (55)$$

$$= -\xi\;\pi_t \quad (56)$$

Substituting this optimality condition into ((47) and solving forward yields reduced form solutions for the domestic inflation and the domestic output gap in terms of the cost push shock, we obtain

$$\pi_t = \psi\; u_t \quad (57)$$

$$x_t = -\xi\psi u_t \quad (58)$$

$\psi = [(1-\beta\rho) + \lambda\xi]^{-1}$. Similar expressions hold for the foreign economy.

Several points are worth emphasizing. First, the policy response is identical in form to the case of the case of the closed economy (see Clarida, Gali and Gertler, 1999). There is a lean against the wind response to domestic inflation, as suggested by the optimality condition given by (55). In addition, in the absence of cost push shocks the central bank is able to simultaneously maintain price stability and close the output gap. Otherwise, a cost push shock generates a tradeoff between the output gap and inflation of the same form that applies for the closed economy. Interestingly, openness does not affect the optimality condition that dictates how aggressively the central bank

price dispersion, where the latter is approximately proportionate to inflation. Note that if there were pricing market, with sticky prices in the final traded goods sector, then a measure of cpi inflation would instead enter the objective function. The same would be true if trade were in intermediate goods, with final goods prices sticky, as in McCallum and Nelson (1999).

should adjust the output gap in response to deviations of inflation from target.[9] Further, openness affects the reduced form elasticity of π_t and x_t with respect to the cost push shock only the the extent it affects the slope of the phillips curve λ. Finally, we observe that while π_t and x_t may be insulated from foreign disturbances, the overall level domestic output will depend on foreign shocks, since the domestic natural level of output depends on foreign output (via the terms of trade), as equation (45) makes clear.

We may combine the IS curve (46) with the solutions for x_t and π_t to obtain an expression for an interest rate rule that implements this policy:

$$r_t = \overline{rr}_t^d + \vartheta E_t \pi_{t+1} \tag{59}$$

with

$$\vartheta = 1 + \xi \sigma_0 \frac{1-\rho}{\rho} > 1$$

As in the case of the closed economy, the optimal rule may be expressed as the sum of two components: the domestic equilibrium real interest rate and a term that has the central bank adjust the nominal rate more than one for one with respect to domestic inflation. Openness affects the slope coefficient only to the extent it affects the interest elasticity of domestic spending, given by σ_0^{-1}. Note also that the terms of trade does not enter the rule directly, but rather does so indirectly via its impact on the domestic equilibrium real rate.

To summarize, we have:

Proposition 1 *In the Nash equilibrium, the policy problem a country faces is isomorphic to the one it would face if it were a closed economy. As in the case of a closed economy, the optimal policy rule under discretion may be expressed as a Taylor rule that it linear in the domestic natural real interest rate and expected domestic inflation. International considerations affect the slope coefficient on domestic inflation in the rule, as well as the behavior of the domestic natural real interest rate.*

[9] This result arises from our explicit derivation of the weight α, which yields a restriction on the ratio λ/α which governs the optimal adjustment of the output gap to inflation. In particular, $\lambda/\alpha = \xi$ which does not depend on openness. The rough intuition for why the optimal policy becomes more aggressive in combating inflation as ξ rises is that the costs of resource misallocation from relative price dispersion depends on the elasticity of demand

Finally, there are some interesting implications for the equilibrium terms of trade and the nominal exchange rate. To gain some intuition we restrict attention to the symmetric case of $\gamma = 1 - \gamma$ (implying $\sigma_o = \sigma_o^*$). In this instance:

$$\begin{aligned} s_t &= \chi[(x_t - x_t^*) + \frac{1+\phi}{\sigma_0 + \phi}(a_t - a_t^*)] \\ &= -\chi\xi\psi(u_t - u_t^*) + \chi\frac{1+\phi}{\sigma_0 + \phi}(a_t - a_t^*)] \end{aligned} \tag{60}$$

with $\chi = (1 - \frac{\widehat{\sigma}}{\sigma_0 + \phi})^{-1}$.[10]The terms of trade depends not only on the relative productivity differentials, but also on the relative cost push shocks. A positive cost push shock in the home country induces an appreciation in the terms of trade

Given that the cpi real exchange rate is unity, we may write:

$$\begin{aligned} e_t &= s_t - s_{t-1} + \pi_t - \pi_t^* + e_{t-1} \\ &= (1 - \chi\xi)\psi\ (u_t -\ u_t^*) + \chi\frac{1+\phi}{\sigma_0 + \phi}(a_t - a_t^*) + e_{t-1} \end{aligned} \tag{61}$$

In general, the nominal exchange rate should respond to relative differences in cost push shocks as well as the terms of trade. The presence of the short run tradeoff motivates the response the relative cost push shocks. In the empirically reasonable case where $(1 - \chi\xi) < 1$, a country that experiences a relative cost push shock should engineer an appreciation of its currency. It is also not optimal to peg in response to terms of trade shocks because the resulting impact on domestic inflation is costly from a welfare standpoint.

It is also interesting to observe that the nominal exchange rate is nonstationary in this instance. This result obtains because each central bank is operating under discretion, which implies that inflation targeting is optimal within this kind of framework (see, Clarida, Gali and Gertler, 1999.) Under commitment, price level targeting is optimal for this type of environment. In

[10] In the asymmetric case:

$$s_t = [1 - \frac{\widehat{\sigma}_0}{\sigma_0 + \phi}\frac{\widehat{\sigma}_0^*}{\sigma_0^* + \phi}]^{-1}\{(1 - \frac{\widehat{\sigma}_0^*}{\sigma_0^* + \phi})(x_t + \frac{1+\phi}{\sigma_0 + \phi}a_t) - (1 - \frac{\widehat{\sigma}_0}{\sigma_0 + \phi})(x_t^* + \frac{1+\phi}{\sigma_0^* + \phi}a_t^*)$$

this instance, so long as both central banks commit, the nominal exchange rate will be stationary.

5 The Cooperative Equilibrium

We now consider optimal policy under cooperation. First, as we show in the appendix, both economies will set a common subsidy rate independent of their relative, in order to offset the presence of market power in goods and labor markets. This subsidy satisfies the following condition:

$$(1-\tau)(1+\mu^w)(1+\mu^p) = 1 \tag{62}$$

Under cooperation the two central banks agree to pursue a policy that maximizes a weighted average of the utilities of home and foreign households, with weights determined by the relative size of the two economies. As shown in the appendix, a second order approximation to that objective function around the cooperative steady state takes the form:

$$\begin{aligned} W^C &= (1-\gamma)W^H + \gamma W^F \qquad (63) \\ &= -\frac{\Lambda}{2}E_0\sum_{t=0}^{\infty}\beta^t \ \left[(1-\gamma)\ (\alpha\ \widetilde{x}_t^2 + \pi_t^2) + \gamma\ (\alpha^*\widetilde{x}_t^{*2} + \pi_t^{*2}) + (1-\sigma)\gamma(1-\gamma)\ \ \widetilde{x}_t\widetilde{x}_t^*\right] \end{aligned}$$

The key way that cooperation differs from Nash is that the centrals banks explicitly take into account the spillovers of foreign output on the domestic marginal cost of production. As we show, in the absence of these spillovers, there are no gains from cooperation.

We assume that the central banks can commit to cooperating with each other, in the respect that they abide by a policy that jointly maximizes the objective given by (63) on period by period basis. But, as in the Nash case we studied, they cannot commit to a policy rule they will stick to in the future. (Thus we are considering cooperation under discretion.) In this instance they maximize equation (63) subject to the IS curve (46) and the condition linking the domestic and overall output gaps (54), as well as the counterparts for the foreign country, taking expectations of the future as given.

The solution to this joint maximization problem will satisfy the following optimality condition for the home country:

$$x_t = -\xi\ \pi_t + \frac{\widehat{\sigma}\xi}{\sigma_o + \phi}\pi_t^* \tag{64}$$
$$= -\xi^c \pi_t + (\xi^c - \xi)(\pi_t^* - \pi_t)$$

where

$$\xi^c = (1 + \frac{\widehat{\sigma}}{\sigma_o + \phi})\xi$$

With cooperation, the central bank now pays attention to foreign inflation as well as domestic inflation.

We note immediately the following important special case:

Proposition 2 *There will be a gain to monetary policy cooperation unless* $\sigma = 1$.

Corsetti and Pesenti (2001), who assume $\sigma = 1$, also find no gain to monetary policy cooperation for the special case of their model that features complete pass-through. It will be noted that this is a global result that does not depend on the quadratic approximation. When $\sigma = 1$ the objective function itself is strongly separable in home and foreign output. However, in general, there will be a gain to monetary policy cooperation even with complete pass-through if $\sigma \neq 1$. This is because of the spillovers from one country's output gap to the other country's marginal cost and inflation that are ignored in the Nash game. Unless $\sigma = 1$, the optimal choice of x_t will be a function of u_t and u_t^*, as will π_t and the foreign counterparts x_t^* and π_t^*.

To gain intuition for the nature and implications of these spillover effects under cooperation, we study in detail the case in which $\gamma = \frac{1}{2}$ (and, hence, $\sigma_0 = \sigma_o^*$, $\widehat{\sigma} = \widehat{\sigma}^*$, and $\lambda = \lambda^*$). In this instance, combining the optimality conditions for the home and foreign countries as well as the IS curves yields:

$$x_t - x_t^* = -\xi^c(\pi_t - \pi_t^*) \tag{65}$$

$$\pi_t - \pi_t^* = \lambda x_t + \beta(E_t \pi_{t+1} - \pi_{t+1}^*) + u_t - u_t^* \tag{66}$$

Assuming the cost push shocks have the same degree of persistence in each country, we can combine these equations to obtain solutions for the inflation and output gap differentials in terms of the differences in the costs push shocks:

$$\pi_t - \pi_t^* = \psi^c \; [u_t - u_t^*] \tag{67}$$

$$x_t - x_t^* = -\xi^c \psi^c [u_t - u_t^*] \tag{68}$$

where $\psi^c = [(1 - \beta\rho) + \lambda\xi^c]^{-1}$. Differences in the dispersion of inflation rates and output gaps between Nash and cooperation depend on the difference between ξ^c and ξ, which in turn depends on $\widehat{\sigma}$. Combining with the optimality conditions and the aggregate supply curves and optimality conditions yields:

$$\pi_t = \psi^c u_t + (\xi^c - \xi)\lambda(\psi^c)^2[u_t - u_t^*]\} \tag{69}$$

$$x_t = -\xi^c \psi^c u_t - (\xi^c - \xi)(1 + \lambda\psi^c)\psi^c[u_t - u_t^*] \tag{70}$$

Notice that under log utility, $\widehat{\sigma} = 0$ ($\rightarrow \xi^c - \xi = 0$), the central bank in each country finds it optimal to leave the path of the domestic output gap and inflation unchanged in response to foreign shocks.

On the other hand, whenever $\sigma \neq 1$ the optimal response of the output gap and inflation under cooperation is different from that under Nash. The qualitative pattern of those responses depends to a large extent on whether σ is greater or less than one. To gain some intuition consider the case $\sigma > 1$. Suppose an inflationary cost-push shock hits the foreign economy, inducing its central bank to lower x_t^* in order to dampen the effects on inflation. In that case, as the joint objective function makes clear, there is a marginal welfare gain from lowering domestic output gap as well, thus inducing a deflation at home. In that case, consumers in both countries benefit from the induced positive covariance between output gaps at home and abroad.

Finally, we can derive some implications for the behavior of the nominal interest rate and the real exchange rates under cooperation. Combining the optimality condition with the IS curve yields

$$\begin{aligned} r_t &= \overline{rr}_t^d + \vartheta E_t \pi_{t+1} - (\xi^c - \xi)(\vartheta - 1) E_t \pi_{t+1}^* \\ &= r_t^{nash} - (\xi^c - \xi)(\vartheta - 1) E_t \pi_{t+1}^* \end{aligned} \tag{71}$$

Under cooperation, accordingly, the domestic central bank considers not only home inflation but inflation in foreign domestic prices as well. The sign of the response, naturally depends on the gap, $\xi^c - \xi = \frac{\widehat{\sigma}}{\sigma_o + \phi}$. When $\sigma > 1, \widehat{\sigma} < 0$ and a rise in foreign inflation induces a tightening of home monetary policy. This will tend to induce a positive covariance between home and foreign output gaps, and a negative covariance between home and foreign inflation. When $\sigma < 1, \widehat{\sigma} > 0$ and a rise in foreign inflation induces an easing of home monetary policy. This will tend to induce a negative covariance between home and foreign output gaps, and a positive covariance between home and foreign inflation.

We summarize as follows:

Proposition 3 *Optimal policy in the Cooperative equilibrium can be written as a Taylor rule which is linear in the equilibrium real interest rate, domestic inflation, and foreign inflation.*

Finally, the terms of trade on the nominal exchange rate are given by:

$$s_t = -\chi \xi^c \psi^c (u_t - u_t^*) + \chi(a_t - a_t^*)] \tag{72}$$

$$e_t = (1 - \chi \xi^c) \psi^c \ (u_t - \ u_t^*) + \chi(a_t - a_t^*) + e_{t-1} \tag{73}$$

Note that expressions are the same as in the Nash case, except that ξ^c replaces ξ^c. Under cooperation, accordingly, the nominal exchange rate should be free to vary in response to both relative cost push shocks and relative productivity shocks. Thus we have,

Proposition 4 *Under cooperation, a system of floating exchange rates is optimal. The existence of cost push pressures, further, makes the fixed exchange rate alternative less attractive.*

6 Concluding Remarks

A virtue of our framework is that we are able to derive sharp analytical results, while allowing still allowing each central bank to face a short run tradeoff between output and inflation, in contrast to much of the existing literature. Of course, we obtained these results by making some strong assumptions. It would be desirable to consider the implications of relaxing these assumptions, including: allowing for imperfect consumption risk sharing, pricing-to-market, and trade in intermediate inputs. We find considering the latter particularly interesting, as it would additional effect of the terms of trade on marginal cost, which is the avenue in our model through which there are gains from cooperation.

Finally, while we have considered gains from cooperation, we have not considered the gains from commitment that arise in the closed economy variant of our framework (see, e.g., Clarida, Gali and Gertler, 1999 and Woodford, 1999). It is straightforward to allow for commitment, and we expect that doing so will produce some interesting implications for the behavior of the equilibrium exchange rate.

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