After the Tide:
Commodity Currencies and Global Trade

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Abstract

The decade prior to the Great Recession saw a boom in global trade, including a rapid rise in commodity prices, trade volumes, and, consequently, in the cost of transporting goods around the world. At the same time currencies of commodity-exporting currencies appreciated, boosting the carry trade profits in foreign exchange markets. The onset of the Global Recession led to a sharp reversal in all of these trends, with only a weak recovery subsequently. We account for these facts using a two-country general equilibrium model of commodity trade and currency pricing. Our model features specialization in trade and a time-varying trade friction that arises from low frequency movements in shipping capacity, and therefore endogenously generates time-varying dynamics in global market segmentation. Slow adjustment in the shipping sector implies that following a boom and then a bust both trade costs and the carry trade risk premium remain low, even as output and trade volumes recover. We verify this prediction of the model using measures of global shipping costs.

Keywords: shipping, trade costs, carry trade, currency risk premia, exchange rates, international risk sharing, commodity trade

JEL codes: G15, G12, F31
1 Introduction

The decade prior to the Great Recession saw a boom in global trade, including a rapid rise in commodity prices, trade volumes, and, consequently, in the cost of transporting goods around the world. At the same time currencies of commodity-exporting currencies appreciated, boosting the carry trade profits in foreign exchange markets (commodity currencies typically earn higher interest rates, making them attractive to investors).\(^1\) The onset of the Global Recession led to a sharp reversal in all of these trends, with only a weak recovery subsequently. We interpret these facts through the lens of an international asset pricing model, focusing on two groups of countries whose currencies represent the two sides of a typical carry trade strategy. The first group consists of developed countries that are major exporters of basic commodities (Australia, Canada, New Zealand, and Norway) - the typical “investment” currencies.\(^2\) The second group consists of developed economies that primarily export complex manufactured goods (the Euro zone, Japan, Sweden, and Switzerland), typically seen as providing “funding” currencies for exchange rate speculation due to their historically low interest rates.

Building on Ready, Roussanov, and Ward (2016), we develop a two-country general equilibrium model that features complete financial markets and specialization in trade. The key friction that we emphasize in the model is the slow adjustment of capacity in the shipping sector, which results in highly variable costs of international trade (or at least their component that is attributed to shipping). The model can jointly account for the dynamic behavior of real exchange rates and interest rates (and therefore carry trade returns), commodity prices, and shipping costs. All of these series exhibit a sharp drop during the crisis, followed by very slow recovery, with shipping costs being the most sluggish.

The model is designed to capture the carry trade in currency markets. In the model, differences in average interest rates and risk exposures between countries that are net importers

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\(^1\) Early literature on the uncovered interest rate parity puzzle focused on the fact that movements in bilateral exchange rates over time do not offset differences in interest rates - e.g. Hansen and Hodrick (1980), Fama (1984). In contrast, recent literature shows that the bulk of carry trade profitability stems from persistent differences in real interest rates across countries - Lustig, Roussanov, and Verdelhan (2011), Ready, Roussanov, and Ward (2016), Hassan and Mano (2014).

\(^2\) These currencies have been of particular interest in the international economics literature that studies the connection of exchange rates with underlying macroeconomic fundamentals - e.g. Chen and Rogoff (2003), Chen, Rogoff, and Rossi (2010), and Ferraro, Rossi, and Rogoff (2011)
of basic commodities (“producer countries”) and commodity-exporting countries (“commodity countries”) are rationalized by appealing to a natural economic mechanism: trade costs.\footnote{Trade costs have a long tradition in international finance: e.g., Dumas (1992), Hollifield and Uppal (1997). Obstfeld and Rogoff (2001) argue that trade costs hold the key to resolving several major puzzles in international economics.} We model trade costs by considering a simple model of the shipping industry. At any time the cost of transporting a unit of good from one country to the other depends on the aggregate shipping capacity available. While the capacity of the shipping sector adjusts over time to match the demand for transporting goods between countries, it does so slowly, due to gestation lags in the shipbuilding industry. In order to capture this intuition we assume that the marginal costs of shipping an extra unit of good is increasing - i.e., trade costs in our model are convex. Convex shipping costs imply that the sensitivity of the commodity country to world productivity shocks is lower than that of the country that specializes in producing the final consumption good, simply because it is costlier to deliver an extra unit of the consumption good to the commodity country in good times, but cheaper in bad times. Therefore, under complete financial markets, the commodity country’s consumption is smoother than it would be in the absence of trade frictions, and, conversely, the producer country’s consumption is riskier. Since the commodity country faces less consumption risk, it has a lower precautionary saving demand and, consequently, a higher interest rate on average, compared to the country producing manufactured goods. Since the commodity currency is risky - it depreciates in bad times from the perspective of the producer country’s consumer - it commands a risk premium. Therefore, the interest rate differential is not offset on average by exchange rate movements, giving rise to a carry trade.

In order to evaluate the model’s ability to generate quantitatively reasonable magnitudes of currency risk premia and interest rates we calibrate it by allowing for the possibility of very large jumps in productivity - i.e., rare disasters, as in the literature on the equity premium puzzle (e.g., Longstaff and Piazzesi (2004), Barro (2006), Gabaix (2012), Wachter (2013)). The calibrated model is able to account for the observed interest rate differentials and average returns on the commodity currency carry trade strategies without overstating consumption growth volatility, even in samples that contain disasters, or implying an unreasonably high probability of a major disaster.
We use our model as a laboratory for understanding the behavior of commodity currencies around the Great Recession. We simulate a series of positive productivity shocks in the producer country, which generate a boom in commodity prices (as commodity supply struggles to catch up quickly) and a rise in global shipping costs (as shipping capacity also lags behind). The commodity country exchange rate appreciates as well, yielding high carry trade profits. The latter result is not mechanical, as in the presence of complete financial markets terms of trade do not drive exchange rates; rather, this is due to the fact that increasing trade costs make markets more segmented, with marginal utility of producer country (“Japan”) consumers fall faster than that of commodity country (“Australia”). The ensuing global crisis is represented in our model by a large negative productivity shock in the producer country (we abstract from demand shocks or financial frictions for simplicity). As a result, output and trade in the final good collapse, as do commodity prices and shipping costs. The commodity currency depreciates due to a sharp spike in the marginal utility in “Japan” relative to that of “Australia”, generating large losses for the currency carry trade. Since shipping capacity is very slow to adjust, trade costs remain depressed even as output and trade recover. Interest rate differentials and expected carry trade returns also decrease as low trade costs imply a greater degree of risk sharing (i.e., a closer alignment of marginal utilities, and consequently less scope for a currency risk premium). We show that all of these predictions are consistent with the empirical evidence for the countries that we consider.

Our analysis sheds a new light on the role of time-varying transport costs in international trade. In our model trade costs increase in (global) good times endogenously, since that is when exports tend to rise. This is consistent with arguments in Hummels (2007) and papers cited therein, emphasizing the effects of port congestion and delays in shipping and the role of fuel costs, which in the recent decades have behaved procyclically, as well as evidence on the value of speed of shipment analyzed in Hummels and Schaur (2013). It is also corroborated by the empirically observed behavior of the shipping cost indices that we consider as the most explicit (albeit narrow) measures of transportation costs. Other models in international finance instead assume that trade costs increase (exogenously) in bad times, potentially due to a tightening of trade credit - e.g. Maggiore (2012). We do not need to take a stand on the role of trade finance in the trade collapse during the Great Recession, which
is a subject of an empirical debate. Indeed, while some studies, such as Amiti and Weinstein (2011) and Chor and Manova (2012) argue that there is some empirical support for the role of trade credit and financial frictions, Levchenko, Lewis, and Tesar (2011) find very little evidence, at least for the U.S. In addition, Eaton, Kortum, Neiman, and Romalis (2011) and Gopinath, Itskhoki, and Neiman (2012) find little evidence in prices of traded goods that would be consistent with a large role of trade costs in reducing trade volumes during the crisis. While our model of international trade is relatively stylized compared to the recent models that focus on understanding trade frictions at the micro level (e.g., Arkolakis (2010), Alessandria, Kaboski, and Midrigan (2013)), our contribution is to highlight the importance of shipping capital for the dynamics of global trade, international risk sharing, and real exchange rates.

2 Motivating Evidence

Our approach builds on the evidence in Ready, Roussanov, and Ward (2016) that countries whose primary exports are basic commodities exhibit qualitatively different behavior of macroeconomic aggregates than countries that concentrate in exporting complex manufactured goods. As argued by Lustig, Roussanov, and Verdelhan (2011), persistent differences in countries’ exposures to global shocks drive the bulk of currency risk premia earned in foreign exchange markets. These differences lead to persistent interest rate differentials (or, equivalently, forward discounts), which, in turn, translate into expected excess returns earned on currency positions, as spot exchange rates on average systematically deviate from the forward rates.\footnote{Denoting log forward exchange rate one month ahead $f_t = \log(F_t)$ and log spot exchange rate $s_t = \log(S_t)$, both expressed in units of foreign currency per one U.S. dollar, the forward discount is equal to the interest rate differential: $f_t - s_t \approx i^*_t - i_t$, where $i^*$ and $i$ denote the foreign and domestic nominal one month risk-free rates, by covered interest rate parity. The log excess return $r_x$ on buying a foreign currency in the forward market and then selling it in the spot market after one month is then given by

$$r_{x_{t+1}} = f_t - s_{t+1},$$

while the arithmetic excess return is given by

$$R_{x_{t+1}} = \frac{F_t}{S_{t+1}} - 1.$$}
Table 1: Average Currency Returns and Forward Discounts

<table>
<thead>
<tr>
<th>Country</th>
<th>Currency Excess Return</th>
<th>Forward Discount</th>
<th>Import Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>-2.25</td>
<td>-2.28</td>
<td>-0.54</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-1.54</td>
<td>0.40</td>
<td>-0.21</td>
</tr>
<tr>
<td>Germany (Euro)</td>
<td>-0.28</td>
<td>-0.58</td>
<td>-0.18</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.02</td>
<td>-0.19</td>
<td>-0.11</td>
</tr>
<tr>
<td>Canada</td>
<td>0.25</td>
<td>1.13</td>
<td>0.21</td>
</tr>
<tr>
<td>Norway</td>
<td>1.29</td>
<td>0.69</td>
<td>0.51</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2.81</td>
<td>1.70</td>
<td>0.53</td>
</tr>
<tr>
<td>Australia</td>
<td>2.46</td>
<td>1.47</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table plots average annualized currency returns, forward discounts, and import ratios for the post-euro sample (1999 - 2015). Currency returns are the excess return to rolling over 1-month currency forwards. Forward discounts are the average annualized discount on 1-month forward contracts. Import ratio is calculated as in Ready, Roussanov, and Ward (2016), and reflects the extent to which a country is a net exporter of commodities. This ratio is calculated as:

\[
\text{Import Ratio} = \frac{\text{Net Imports of Complex Goods} + \text{Net Exports of Basic Commodities}}{\text{Total Trade in All Goods}}
\]

Currency data is from Bloomberg and trade data is from the U.N. Comtrade Database. Goods are classified as complex or basic as in Ready, Roussanov, and Ward (2016).
Table 2 displays the average interest rate differentials (proxied by one-month forward discounts) and the corresponding average one-month excess returns on eight currencies vis-a-vis the U.S. dollar, over the period 2000-2015 (since the introduction of the Euro). These currencies are among the 10 most actively traded currencies in the foreign exchange market (known as the G10 currencies). The economies representing the top four currencies - the Euro zone, the Japanese yen, the Swedish Krona, and the Swiss franc - all feature advanced manufacturing sectors and a high share of complex manufactured goods in their exports. The bottom four currencies are known as “commodity currencies”: the Australian, Canadian, and New Zealand dollars, and the Norwegian Krone.\(^5\) Alongside the exchange rate variables the table displays the average Import Ratio introduced by Ready, Roussanov, and Ward (2016), which reflects the extent to which a country is a net exporter of commodities vs. manufactured goods. This ratio introduced in is calculated as

\[
\text{Import Ratio} = \frac{\text{Net Imports of Complex Goods} + \text{Net Exports of Basic Commodities}}{\text{Total Trade in All Goods}},
\]

using U.N. COMTRADE data to construct good-specific trade series at annual frequency. All of the countries in the latter group primarily export basic commodities, as indicated by the positive Import ratios (the other countries’ Import ratios are negative). The table reveals that the top four currencies have persistently low interest rates relative to the U.S., which translate on average into negative excess returns. In contrast, the bottom four currencies exhibit higher interest rates (positive forward discounts for the U.S. dollar), which translate into positive average excess returns. Note that across countries both average forward discounts and average excess returns are roughly monotonic in the Import ratio. As argued by Ready, Roussanov, and Ward (2016), this systematic relationship captures essentially all of the cross-sectional variation in unconditional average currency returns, i.e. the currency carry trade.

While the currency carry trade is a highly profitable trading strategy on average, it performs poorly during global market downturns (see Lustig, Roussanov, and Verdelhan (2011) for detailed analysis). The carry trade suffered particularly large losses during the onset of the global financial crisis and the Great Recession. Figure 2 shows that two groups

\(^5\)The other two countries making up the G10 group - the U.K. and the U.S. - have a relatively equal shares of complex and basic goods in their net exports, as analyzed in Ready, Roussanov, and Ward (2016).
of countries indeed display very different behavior of macroeconomic aggregates during the crisis. In the “producer” countries (complex manufactured goods exporters) both GDP and labor productivity grow rapidly between 2003 and 2007 and then plunge during the Great Recession, largely recovering afterwards, but still remaining below trend until at least 2014. In contrast, for the “commodity countries” we see on average a somewhat slower rate of productivity growth but only a slight downturn in GDP during the crisis and essentially no decline in productivity on average (equally weighing all four countries).

Figure 1: Commodity and Producer Country Outputs and Productivity

Panel A: Real GDP

Panel B: Labor Productivity

Panel C: Real GDP - Disaggregated

Panel D: Labor Productivity - Disaggregated

Panel A plots an equal weighted index of log real GDP for the four commodity countries (Australia, Canada, New Zealand, and Norway) and the four producer countries (Germany, Japan, Switzerland, and Sweden). Panel B repeats this plot using Labor Productivity in place of GDP. Panels C and D show data for the individual countries. Data are from the OECD. Switzerland’s productivity data is omitted due to lack of availability.
The apparent resilience of the commodity countries during the Great Recession is particularly surprising given the dramatic decline in their terms of trade due to the collapse of commodity prices (e.g., see discussion in Eaton, Kortum, Neiman, and Romalis (2011)). Figure 2 Panel A plots the aggregate Commodity Price Index compiled by the Commodity Research Bureau (CRB) along the with the two measures of global shipping costs. The first measure is the widely used Baltic Dry Index (BDI), an exchange-traded composite of dry bulk freight rates, which captures the cost of shipping dry commodities (e.g., iron ore). The second measure (Harpex) is a composite index of freight rates for container shipping along several common routes (e.g., Hong Kong to Rotterdam), and therefore captures the cost of shipping more complex/finished goods. While container shipping rates are typically contracted forward and are therefore relatively smooth, bulk shipping rates are determined largely in a spot market and therefore fluctuate at fairly high frequencies. The figure shows that commodity prices indeed fell by about 20% on average during the global financial crisis, although they recovered between 2009 and 2010, reaching new highs in 2011. Interestingly, global shipping costs, which grew rapidly in the early 2000-s (especially the BDI) fell much more dramatically, both BDI and Harpex losing over 90% of their values between 2007 and 2009. Both indices recover roughly half-way by 2010 but remain depressed throughout the remainder of our sample.

What explains this discrepancy between the behavior of commodity prices and the price of shipping (both commodities and finished goods)? Panel B plots the plots the annual growth rate of freight shipping capacity. Both dry bulk and container ships are large and essentially irreversible investments: they take between 2 to 3 years to build but can be operated for decades. During the trade boom of the early 2000-s as the demand for shipping services drove up shipping costs, used ship prices increased rapidly, the largest of them (CapeSize) selling for over $100 million each (see Greenwood and Hanson (2015) for details). This increased demand drove shippers to increase investment in new ships, resulting in annual increases in shipping capacity between between 7 and 9% in 2007-2009. With the collapse of global trade during the Great Recession shipping costs (and prices of ships collapsed by between 70 and 80%), yet new ships ordered during the boom continued coming on-line, increasing capacity and driving down shipping costs even as the global economy began to
recover.

Our model aims to account for several features of the data relating to the period around the global financial crisis and the Great Recession that are summarized here. The first is the average interest rate differential between the “commodity” and the “producer” countries, and the associated carry trade risk premium. The second is the relative resilience of the commodity countries during the global downturn, despite a substantial decline in their terms of trade. The third key component is the extremely volatile behavior of global shipping costs. In what follows we show that these features are inherently connected via a simple risk sharing mechanism that relies complete financial markets and costly trade in goods.

3 Model

3.1 Setup

There are two countries, each populated by a representative consumer endowed with CRRA preferences over the same consumption good, with identical coefficients of relative risk aversion $\gamma$ and rates of time preference $\rho$. The countries differ in their production technologies, each specializing in the production of a single good. The “commodity” country produces a basic input good using a simple production technology

$$y_c = z_c l_c^\alpha;$$

assuming one unit of commodity country’s non-traded input $l_c$ (e.g., labor, land, etc.) is supplied inelastically, so that this is equivalent to an exogenous endowment of basic commodity equal to the productivity shock $z_c$ ($y_c = z_c$).

The “producer” country only produces a final consumption good using basic commodity input $b$ and labor:

$$y_p = z_p b^{1-\beta}l_p^\beta,$$

which is subject to a productivity shock $z_p$, with one unit of producer country’s non-traded input also supplied inelastically.
Figure 2: Shipping Costs and Commodity Prices

Panel A plots the log of the CRB Spot Commodity index, as well as the logs of the Harpex container ship index and the Baltic Dry Index. Panel B plots the annual growth of the global merchant shipping fleet. Price index data are from Datastream. Shipping fleet data are from United Nations Conference on Trade and Development.
The countries are spatially separated so that transporting goods from one country to the other incurs shipping costs. Our model of shipping costs extends the variable iceberg cost of Backus, Kehoe, and Kydland (1992), where each unit of good shipped in either direction loses a fraction

$$\tau_i(x, z_k) = \kappa_0^i + \kappa_1^i \frac{x}{z_k},$$

which depends on the total amount of goods shipped in the same direction, $x$, and the shipping capacity available at time $t$, $z_k$. For simplicity we assume that this shipping capacity (or, equivalently, shipping sector productivity) is exogenous (although a model with investment in shipping capacity yields similar implications). Since the costs of shipping raw commodities and manufactured goods are likely to be different, we allow two sets of parameters ($i \in c, f$).

Since the commodity country has no alternative use for the basic good it produces, in equilibrium all of its supply is shipped to the producer country. Total output of the final consumption good is therefore

$$y_p = z_p[z_c(1 - \tau_c(z_c, z_k))]^{1-\beta}l_p^\beta.$$

In the producer country, the representative competitive firm solves

$$\max_{l_p \in [0,1]} \pi_p = z_p(z_c(1 - \tau_c(z_c, z_k)))^{1-\beta}l_p^\beta - w_pl_p - Pz_c(1 - \tau_c(z_c, z_k)),$$

where $w_p$ is the wage paid to labor and $P$ is the price of one unit of basic commodity. From the first-order conditions and zero profits, the price of the basic commodity is given by

$$P = \frac{(1 - \beta)y_p}{(1 - \tau_c(z_c, z_k))y_c} = (1 - \beta)z_p[z_c(1 - \tau_c(z_c, z_k))]^{-\beta}.$$

The production economy outlined here is very simple (e.g., it is essentially static, as there are no capital or other inter-temporal investment margins), intended to highlight the main mechanism based on the interplay of specialization and trade costs. Gourio, Siemer, and Verdelhan (2013) and Colacito, Croce, Ho, and Howard (2013) study currency risk premia in fully dynamic production economies that could potentially be generalized to incorporate
the type of heterogeneity we consider.

Consumption allocations for the commodity country and the producer country, $c_c$ and $c_p$, are determined by the output of the producer country $y_p$ and the amount $X$ of final consumption good exported to the commodity country. We will consider complete financial markets as our benchmark case, so that equilibrium consumption allocations to the two countries over time and across states of nature will be determined as a result of a risk-sharing arrangement, and the real exchange rate is pinned down by the absence of arbitrage in the financial markets (as well as the markets for the consumption good). In contrast, in (financial) autarky, whereby trade is balanced in every period since trade in financial claims is impossible, the producer country consumption equals to its share of output $\beta z_p[z_c(1 - \tau_c(z_c, z_k))]^{1-\beta}$ (if labor is the only non-traded factor, this quantity represents the total wage bill in the competitive equilibrium), while the remainder of the output is exported to the commodity country in the form of payment for the basic commodity $(X_{aut} = (1 - \beta)z_p[z_c(1 - \tau_c(z_c, z_k))]^{1-\beta})$, which implies that after trade cost the commodity country income/consumption would equal $X_{aut}(1 - \tau_f(X_{aut}, z_k))$. The real exchange rate in this case is determined by the terms of trade (i.e., the relative price of the basic commodity). As long as commodity demand shocks (i.e., shocks to the final good productivity $z_p$) are large relative to supply shocks $z_c$, the commodity prices are procyclical, in the sense that $P$ comoves positively with output of the final good $y_p$ and therefore with the world consumption. In financial autarky this would imply that the commodity currency is “risky” since it comoves positively with consumption. This may explain why currencies of some commodity-producing emerging countries that are not well-insured via the world financial markets may be more volatile and tend to depreciate during global economic downturns. However, even for such countries (e.g., Brazil, Russia) a total absence of trade in financial assets is a rather extreme assumption. Moreover, it would not explain why currencies of developed countries that are well-integrated into the global financial system, such as Australia, Canada, or Norway, may also be highly procyclical. In our main analysis we therefore focus on the polar case of perfect international financial markets, where physical trade frictions are the only impediment to full risk sharing.
3.2 Dynamics

We assume that the shocks to productivity experienced by the final good producer are permanent, so that its evolution (in logs) follows a jump-diffusion process:

\[ d \log z_{pt} = (\mu - \mu Z \eta) \, dt + \sigma_p dB_{pt} + dQ_t. \]

Let \( N(t) \) be a Poisson process with intensity \( \eta \), and let \(-Z_1, -Z_2, \ldots\) be a sequence of identically distributed random variables drawn from a truncated Pareto distribution with minimum jump \( Z_{\text{min}} \), maximum jump \( Z_{\text{max}} \), and shape parameter \( \alpha \). Denote this distribution’s mean as \( \mu_Z \). Define the compound Poisson process:

\[ Q(t) = \sum_{j=1}^{N(t)} Z_j = \int_0^t Z_s dN_s, \quad t \geq 0. \]

\[ \Rightarrow dQ(t) = Z_{N(t)} dN_t, \]

so that \( \mu \) is the uncompensated drift of the jump-diffusion, and the growth rate of the productivity shock process can be written as

\[ \frac{dz_{pt}}{z_{pt^-}} = \left( \mu - \mu Z \eta + \frac{1}{2} \sigma_p^2 \right) dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN_t \]

\[ \Rightarrow \mu_p dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN_t, \]

where \( z_{pt^-} = \lim_{s \uparrow t} z_{ps} \) is the process’s left-limit, a convention used throughout.

In order to ensure stationarity of the model economy, we further assume that commodity country productivity shock are cointegrated with the producer country shocks. Specifically, we assume that their cointegrating residual

\[ q_t = \log z_{pt} - \beta \log z_{ct} \]

is stationary, following a mean-reverting jump-diffusion process

\[ dq_t = [(1 - \beta)(\mu - \mu Z \eta) - \beta \psi q_t] \, dt + \sigma_p dB_{pt} - \beta \sigma_c dB_{ct} + dQ_t, \]
so that the commodity country productivity shock process (in logs) follows

\[ d \log z_{ct} = (\mu + \psi q_t) dt + \sigma_c dB_{ct}, \]

and therefore we can write

\[ \frac{dz_{ct}}{z_{ct}} = \left( \mu + \psi q_t + \frac{1}{2} \sigma_c^2 \right) dt + \sigma_c dB_{ct} \]

\[ \Rightarrow \mu_{ct} dt + \sigma_c dB_{ct}. \]

This cointegrated relationship can be interpreted as a reduced form representation of an economy where supply of the commodity is inelastic in the short run (based on the currently explored oil fields, say) but adjusts in the long run to meet the demand by the final good producers (e.g., as new fields are explored more aggressively when oil prices are high).

Similarly, we assume that shipping sector productivity is cointegrated with the commodity supply, with the cointegrating residual defined

\[ q_{kt} = \log z_{ct} - \log z_{kt}, \]

which follows a mean-reverting process

\[ dq_{kt} = (\psi q_t - \psi_k q_{kt}) dt + \sigma_c dB_{ct} - \sigma_k dB_{kt} \]

so that the shipping shock process follows

\[ d \log z_{kt} = (\mu + \psi_k q_{kt}) dt + \sigma_k dB_{kt} \]

\[ \Rightarrow \frac{dz_{kt}}{z_{kt}} = \left( \mu + \psi q_{kt} + \frac{1}{2} \sigma_k^2 \right) dt + \sigma_k dB_{kt} \]

\[ \Rightarrow \mu_{kt} dt + \sigma_k dB_{kt}, \]

where the Brownian motions \( B_{pt}, B_{ct}, \) and \( B_{kt} \) are independent. The latter assumption captures the idea that shipping capacity cannot be adjusted quickly in response to shocks, which can lead to substantial volatility in costs of shipping over time, and therefore shipping
costs that are very sensitive to demand shocks in the short run (e.g., Kalouptsidi (2014), Greenwood and Hanson (2015)). Our modeling of cointegrated jump-diffusion processes is similar to the model of cointegrated consumption and dividend dynamics in Longstaff and Piazzesi (2004). We can solve for output and commodity price dynamics by application of Ito’s lemma (see Appendix).

3.3 Complete markets and consumption risk sharing

In order to emphasize that our mechanism does not rely on any financial market imperfections, we consider consumption allocations under complete markets. This is a standard benchmark in international finance, and is reasonable at least when applied to developed countries. Under complete markets, the equilibrium allocation is identical to that chosen by a central planner for a suitable choice of a (relative) Pareto weight \( \lambda \).

The planner’s problem is therefore

\[
V(z_{ct}, z_{pt}, z_{kt}) = \max_{\{X_t\}} \mathbb{E} \left[ \int_t^\infty e^{-\rho(s-t)} \left( \frac{c_{cs}^{1-\gamma} - 1}{1 - \gamma} + \frac{c_{ps}^{1-\gamma} - 1}{1 - \gamma} \right) ds \bigg| \mathcal{F}_t \right],
\]

where \( X_s \) is exports of final good to the commodity country, the commodity country consumption is \( c_{cs} = X_s(1 - \tau_f(X_s, z_k)) \), and the producer country consumption is \( c_{ps} = y_{ps} - X_s \). The first-order condition implies that

\[
g(X_t, z_{ct}, z_{pt}, z_{kt}) \equiv \left[ X_t(1 - \kappa_0^f - \kappa_1^f \frac{X_t}{z_{kt}}) \right]^{-\gamma} \left( 1 - \kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}} \right) - \lambda(y_{pt} - X_t)^{-\gamma} = 0
\]

must hold state by state for all \( t \). In general, this nonlinear equation must be solved numerically, except for the special case of log utility (\( \gamma = 1 \)).

Since the trade costs are increasing in the amount of goods shipped (holding shipping capacity fixed), the cost of transporting an extra unit of the final consumption good is increasing in total output \( y_{pt} \). When output is high, the social planner allocates greater amounts of the good to the commodity country while shipping becomes increasingly costly.\(^7\)

\(^6\)For example, Fitzgerald (2012) estimates that risk-sharing via financial markets among developed countries is nearly optimal, while goods markets trade frictions are sizeable.

\(^7\)The share of final good output that is exported can be increasing or decreasing in output, depending on the curvature of the utility function and the steepness of the trade cost profile: if the utility function
The effects of individual state variables on the final good trade cost $\tau_f$ are displayed in Figure 3 as functions of one shock while holding all other shocks constant at a value of 1.3. These effects are intuitive: greater shipping capacity decreases the cost of shipping, while higher productivity of the final goods producer increases trade costs by raising output and, consequently, the amount of goods shipped to the commodity country (higher productivity in the commodity country has a similar effect, as it feeds into final good output).

### 3.4 Exchange rates

The spot exchange rate in the absence of arbitrage is proportional to the ratio of the marginal utilities of the two representative agents,

\[
S_t = \frac{\pi_{pt}}{\pi_{ct}} = \frac{\lambda c_{ct}}{c_{pt}} = \lambda \left( \frac{X_t(1 - \tau_f(X_t, z_{kt}))}{y_{pt} - X_t} \right)^\gamma
\]

(2)

\[
= \lambda \left( \frac{1 - \tau_f(X_t, z_{kt})}{1 - x_t} \right)^\gamma = \left( 1 - \kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}} \right)
\]

(3)

is sufficiently concave, the planner compensates the increasing losses due to rising trade costs by increasing export share in good times (the empirically relevant case); otherwise, the share declines to reduce the dead-weight loss.
where the last equality follows from (1), implying that the real exchange rate is proportional
to the marginal value to the commodity country consumer of a unit of the consumption good
shipped from the country where it is produced (e.g., see Dumas (1992), Hollifield and Uppal
(1997), Verdelhan (2010)).

The real exchange rate is monotonic in the ratio of the two countries’ consumption levels,
is linear in the quantity of final good output exported to the commodity country, $X_t$, and
is therefore closely related to the trade costs. Following good productivity shocks in either
final good or commodity producing countries, total output $y_p$ and exports $X$ both increase,
and therefore the producer country exchange rate depreciates. This is due to the fact that
shipping costs lower the value of a marginal unit of the consumption good exported by
its producer to the commodity country consumer, and more so when more of the good is
shipped. Consequently, as (2) shows, both consumption and its marginal utility declines
more slowly for the commodity country consumer than for the producer country consumer in
good times, and also rises more slowly in bad times. Positive shocks to the shipping capacity
$z_k$ reduce the cost of shipping and therefore act in the opposite direction, increasing the value
of the unit of $X$ to the commodity country and therefore lowering its exchange rate ($X$ will
increase endogenously in response to higher shipping capacity, however, partially offsetting
the influence of shipping cost shocks on the exchange rate.). These effects are displayed in
Figure 4, which plots the exchange rate $S$ (in units of commodity currency per one unit of
final good producer currency), as a function of the three shocks, holding the other shock
constant at a value of 1.3.

3.5 Asset pricing

Stochastic discount factors for the two countries are given by

\footnote{In autarky, the commodity currency appreciates following good shocks to the final good production
technology as its good becomes more highly demanded - this is the terms-of-trade effect, which is present
even in the absence of complete financial markets, as emphasized by Cole and Obstfeld (1991). The effects of
the commodity country productivity differ, however: terms of trade logic implies that commodity currency
appreciates when the commodity becomes scarce following a bad supply shock. This is not generally true in
our complete markets setup, as a decline in commodity supply leads to lower output of the final good, and
higher value for the producer country currency.}
Figure 4: Shocks and Exchange Rates

\[
\pi_{pt} = e^{-\rho t} e^{-\gamma}
\]

\[
\Rightarrow \frac{d\pi_{pt}}{\pi_{pt}} = - \left\{ \rho + \gamma \mu_{cpt} - \frac{1}{2} \gamma (1 + \gamma) \sigma_{cpt}^T \sigma_{cpt} \right\} dt - \gamma \sigma_{cpt}^T dB_t + \left( e^{-\gamma J_p} - 1 \right) dN_t
\]

for the final good producer and

\[
\pi_{ct} = e^{-\rho t} e^{-\gamma}
\]

\[
\Rightarrow \frac{d\pi_{ct}}{\pi_{ct}} = - \left\{ \rho + \gamma \mu_{cct} - \frac{1}{2} \gamma (1 + \gamma) \sigma_{cct}^T \sigma_{cct} \right\} dt - \gamma \sigma_{cct}^T dB_t + \left( e^{-\gamma J_c} - 1 \right) dN_t
\]

for the commodity producer, where \( J_p \) and \( J_c \) are log changes in the marginal utilities induced by jumps.

Risk-free rates are the (negative) drifts of the stochastic discount factors:

\[
r^f_{pt} = \rho + \gamma \mu_{cpt} - \frac{1}{2} \gamma (1 + \gamma) \sigma_{cpt}^T \sigma_{cpt} - \eta E[Z] \left( e^{-\gamma J_p} - 1 \right)
\]
and

\[ r_{ct}^f = \rho + \gamma \mu_{ct} - \frac{1}{2} \gamma (1 + \gamma) \sigma^2_{ct} \sigma_{ct} - \eta \mathbb{E}_Z \left[ e^{-\gamma J_c} - 1 \right], \]

for the final goods and commodity producer, respectively. The terms \( \mathbb{E}_Z \) denote expectations taken over the distribution of jump sizes conditional on a jump occurring. The first two terms of the interest rate expressions above are equal between the two countries on average, as long-run consumption growth rates are equalized by the social planner. However, the last terms – the precautionary saving demands – differ. Since the final goods producer absorbs the bulk of productivity shocks to output, consuming a greater share in good times and a lower share in bad times, it experiences greater consumption volatility. Consequently, it has a greater precautionary demand and a lower interest rate on average. Similarly, the conditional expectation of marginal utility growth upon a jump is greater for the producer country consumer due to the same effect.

Since trade costs are persistent as long as shipping capacity adjusts slowly in response to demand, the interest rate variation is driven in part by the expected convergence in consumption due to cointegration (captured by the drift terms) and by the dispersion in conditional risk exposures of the pricing kernels (captured by the precautionary and jump terms). In particular, when output outstrips shipping capacity, the dispersion between the risk terms in the two countries is high, whereas when shipping capacity is abundant relative to output this dispersion is lower. Figure 5 illustrates this effect for the case of logarithmic utility and no jumps: the difference between conditional consumption volatilities increases following good productivity shocks (or bad shipping sector shocks), which increase trade costs and consequently the degree of goods markets segmentation, reducing risk-sharing opportunities.

### 3.6 Expected excess returns: the carry trade

We can define the instantaneous excess return process for the currency trading strategy that is long the commodity currency (and short the producer currency) as

\[ dRet_t = (r_{ct}^f - r_{pt}^f)dt - \frac{dS_t}{S_t}. \]
This return can be earned by a final-good producing country investor directly, by shipping a unit of consumption good (borrowed at rate $r_{ft}^{f}$) and purchasing $S_t$ units of the commodity-country risk free bonds, earning interest $r_{ct}^{f}$ on these bonds, and converting it back into its own consumption good by shipping fewer units of the consumption good to the commodity country. It can also be obtained indirectly, by trading a state-contingent claim that replicates the payoff on this strategy, given complete financial markets. A commodity country investor can obtain a similar return, adjusted for the exchange rate.

The conditional expected excess return on this strategy (i.e., the currency risk premium) is given by the covariance of the exchange rate with the producer country pricing kernel (e.g., Bakshi and Chen (1997)):

$$E [dRet_t | F_t] = E \left[ \frac{dS_t}{S_t} \frac{d\pi_{pt}}{\pi_{pt}} | F_t \right],$$

since the returns are expressed in the producer country numeraire (an equivalent statement holds for the commodity country pricing kernel if the returns are expressed in the commodity currency units). In general, this risk premium is not equal to zero, so that the uncovered interest parity relation $E \left[ \frac{dS_t}{S_t} | F_t \right] = (r_{ct}^{f} - r_{pt}^{f}) dt$ need not hold.
In fact, this commodity currency trading strategy is profitable, on average, since the commodity currency is risky: it tends to appreciate in good times (when final good output is high) and depreciate in bad times, so that \( E[dRet_t] = E\left[\frac{dS_t}{S_t} - \frac{d\pi_{pt}}{\pi_{pt}}\right] > 0 \). As long as exchange rates are persistent and close to random walks, the bulk of average carry excess return comes from the interest rate differentials.

To highlight the connection between trade costs, commodity prices, and exchange rates we plot the sample paths of a simulation that realizes an instance of a jump, a “global recession”, in 6. Consistent with intuition, commodity currency exchange rate comoves with the commodity price \( P \) as well as realized shipping costs measured by \( \tau_f(X, z_k) \) (for \( S \) the relationship is inverse). Interestingly, while carry trade returns are positively correlated with these variables, so are expected returns on the carry trade. This is due to the fact that the degree of dispersion between the conditional expected marginal utilities (and therefore the risk premium) is pro-cyclical, as trade costs are high in good times (especially if shipping capacity is lagging behind).

We explore this mechanism quantitatively using the fully-specified model in Section 4 below when we calibrate our model to match the dynamics of the Great Recession.
4 Quantitative analysis

So far we have only explored the qualitative implications of our model. We now turn to quantitative analysis. Ideally, we would like to calibrate the model parameters to closely match empirical moments. The fact that the model features only two countries (each completely specialized in producing one kind of good) makes such a moment-matching exercise challenging. In order to circumvent this challenge we make an assumptions that countries that are ranked at the top of the final good exporter measure as a group are representative of a final-good producer country in the model, while countries that rank at the bottom (i.e., the final good importers) are representative of the commodity country. Our empirical results above appear to corroborate this distinction, even though the difference between the two types of countries is much less stark in reality than our model assumes. We form two baskets using the set of G10 countries: one of the countries with the four highest import ratios (commodity countries) and the other of the four lowest (final good producer countries). We average macroeconomic and financial variables across countries within each basket and compare their properties to those implied by the model. Table 2 summarizes these moments while Table 3 lists the parameter values used in the calibration.

We present the summary statistics from the model-generated simulated data in three ways: we simulate the model 10,000 times, each time generating sample periods of approximately the same length as those in our data (30 years). Besides reporting both mean and median statistics across the simulations we also report means conditional on no “disasters” occurring in the sample (i.e. jumps that imply an annual consumption drop in the final good producer country that is greater than 5%). This definition is conservative, as Barro (2006) defines disasters as consumption drops of 10% and greater. We calibrate the distribution of jump sizes so that its tail approximately corresponds to the distribution of empirically observed consumption disasters compiled by Barro and Ursua (2008) (the largest disaster in their sample corresponds to a consumption drop of 70%, which is approximately the same as the upper bound of our jump distribution $Z_{max} = 1.2$). Disasters - large jumps that cause a 5% or greater drop in consumption - occur at least once over a 30-year period with probability of 16% in the simulated samples given that the jump intensity $\eta$ is such that a jump occurs
Table 2: Calibration moments

This table reports summary statistics generated by the model and compares them to data analogues from the G10 country set. All variables are annualized. The macroeconomic variables (consumption, output, exports) are time-aggregated quarterly. All of the financial variables (real interest rates, commodity prices, exchange rates, currency returns) are sampled monthly (monthly carry trade returns are based on continuously rolled-over positions in the model and one-month forward contract returns in the data). Real interest rates are calculated using 1 year lags of realized inflation to proxy for expected inflation. “AC” is the sample autocorrelation. The commodity country set includes Australia, Canada, New Zealand and Norway. The producer country set consists of Germany/Euro, Japan, Sweden, and Switzerland. All means and standard deviations are annualized, in percentage points. The model moments are averages across 10,000 simulated paths of 30 year length, reported as unconditional means and medians, as well as means conditional on “no disasters” - i.e., no jumps generating producer-country annual consumption declines greater than 5% over the 30-year period (disasters of such magnitude occur at least once in approximately 16% of simulated paths).

<table>
<thead>
<tr>
<th></th>
<th>Means</th>
<th>Means, no disasters</th>
<th>Data</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std</td>
<td>AC</td>
</tr>
<tr>
<td><strong>Quarterly growth rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{pt}$</td>
<td>1.69</td>
<td>2.55</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta y_{ct}$</td>
<td>1.69</td>
<td>1.63</td>
<td>0.71</td>
</tr>
<tr>
<td>$\Delta c_{pt}$</td>
<td>1.70</td>
<td>2.61</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta c_{ct}$</td>
<td>1.72</td>
<td>1.10</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta X_t$</td>
<td>1.68</td>
<td>2.51</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Carry trade return components</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{pf}$</td>
<td>4.51</td>
<td>2.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$r_{cf}$</td>
<td>7.61</td>
<td>1.42</td>
<td>0.73</td>
</tr>
<tr>
<td>$dRet_t$</td>
<td>2.98</td>
<td>11.6</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Monthly growth rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dS_t$</td>
<td>0.036</td>
<td>9.25</td>
<td>-0.01</td>
</tr>
<tr>
<td>$dP_t$</td>
<td>0.09</td>
<td>2.19</td>
<td>0.05</td>
</tr>
<tr>
<td>$dP_t \times \partial (z_{ct} \tau_c(z_{ct}, z_kt)) / \partial z_{ct}$</td>
<td>-0.07</td>
<td>3.68</td>
<td>0.28</td>
</tr>
</tbody>
</table>
on average every 25 years, the smallest jump size is 2%, and the power law distribution of jump sizes has a tail exponent of 1.1.\textsuperscript{9} Since the probability of such jumps is sufficiently small, these conditional statistics capture the sense in which rare disasters contribute to the observed risk premia. There is some debate in the literature about the extent to which rare disasters and peso problems contribute to currency risk premia\textsuperscript{10}. While the economic mechanism of our model does not rely on rare disasters, the simulation results reveal that the possibility of such disasters that may occur but are not observed in sample substantially improves the model’s ability to quantitatively account for the carry trade risk premium that is generated by the spread between the higher-order moments of the marginal utilities in the two countries.

The modest degree of relative risk aversion $\gamma = 5$ ensures that the model does not overshoot the exchange rate volatility observed in the data too much in the absence of disasters, with the levels of the risk-free rates matching closely to the interest rates in the data (with the caveat that the empirical interest rates are nominal rather than real), and matching the spread between the rates closely at about 3.1% per annum. Consequently, the Sharpe ratio is roughly as high as in the data on average (around 0.5 on average in no disaster samples and just under 0.26 overall). However, the model does not completely rely on the peso-problem explanation of the carry trade profitability, as even in the samples including disasters the average carry trade return is essentially of the same magnitude. The volatility of exchange rates (and therefore currency carry strategy returns) in the model averaged over no disaster samples matches closely to the empirical volatility of the IMX returns for G10 currencies, at just over 6.25% per annum. This is below the unconditional mean over the simulated samples of 9.25%. Similarly, volatilities of consumption and output growth in the no disaster samples on average match those in the data, and are roughly between the means and the medians of the unconditional distributions. Thus, the model’s ability to match unconditional currency risk premia does not rely on an unreasonably large magnitude (and

\textsuperscript{9}Backus, Chernov, and Martin (2011) argue that equity option prices imply lower probabilities of consumption disasters than the magnitude required to match the equity premium.

\textsuperscript{10}Models such as Farhi and Gabaix (2008) and Gourio, Siemer, and Verdelhan (2013) rely on rare disasters for explanations of the forward premium puzzle. Empirical evidence in Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009), Jurek (2009), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2008), and Chernov, Graveline, and Zviadadze (2012) points to the importance of crash risk in explaining jointly the carry trade risk premia and prices of currency options.
probability) of a rare disaster.

The trade cost coefficients combined with the shipping sector dynamics imply that the fraction of total exports of the final good that is lost to transportation frictions is substantial, at close to 40% (but much smaller, around 11%, for commodities). These costs appear large but are in fact well within the range of values estimated by Anderson and van Wincoop (2004). The dynamics of the trade costs produced by the model are much less volatile than those observed in the data (we use the Baltic Dry Shipping index as our empirical proxy)\(^\text{11}\).

\(^{11}\)The parameters governing mean reversion of the commodity production and shipping prices are chosen so that the commodity production reverts more quickly than the shipping capital. This is broadly consistent with the behavior of commodity prices and shipping costs after the crisis, and also consistent with Bessember, Coughenor, Seguin, and Smoller (1995) who document relatively rapid mean reversion in commodity prices, and Kalouptsidi (2014) who emphasizes the long production lags in the shipping industry.

---

**Table 3: Parameter values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda)</td>
<td>1</td>
<td>Relative Pareto weight</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.7</td>
<td>Cobb-Douglas producer-country labor share</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>5</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.001</td>
<td>Rate of time preference (annualized)</td>
</tr>
<tr>
<td>(\kappa_c^0)</td>
<td>0.01</td>
<td>Fixed commodity trade cost</td>
</tr>
<tr>
<td>(\kappa_c^1)</td>
<td>0.5</td>
<td>Variable commodity trade cost</td>
</tr>
<tr>
<td>(\kappa_f^0)</td>
<td>0.01</td>
<td>Fixed final trade cost</td>
</tr>
<tr>
<td>(\kappa_f^1)</td>
<td>0.9</td>
<td>Variable final trade cost</td>
</tr>
<tr>
<td>(\sigma_p)</td>
<td>0.0025</td>
<td>Productivity shock volatility (annualized)</td>
</tr>
<tr>
<td>(\sigma_k)</td>
<td>0.0001</td>
<td>Shipping shock volatility (annualized)</td>
</tr>
<tr>
<td>(\sigma_c)</td>
<td>0.0015</td>
<td>Commodity shock volatility (annualized)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.018</td>
<td>Uncompensated TFP growth rate (annualized)</td>
</tr>
<tr>
<td>(\psi)</td>
<td>0.005</td>
<td>Mean reversion of commodity supply ((z_c) to (z_p))</td>
</tr>
<tr>
<td>(\psi_k)</td>
<td>0.00001</td>
<td>Mean reversion of shipping capacity ((z_k) to (z_c))</td>
</tr>
<tr>
<td>(\eta)</td>
<td>1 per 25 years</td>
<td>Jump frequency</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.1</td>
<td>Power tail of jump</td>
</tr>
<tr>
<td>(Z_{\text{min}})</td>
<td>2%</td>
<td>Minimum jump size</td>
</tr>
<tr>
<td>(Z_{\text{max}})</td>
<td>120%</td>
<td>Maximum jump size</td>
</tr>
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</table>
5 Great Recession and Currency Returns: Model and Data

In order to gauge the severity of the Great Recession on shipping capacity and globalization post-recession, we simulate a Great Recession in our model and compare its predictions regarding exchange rates and currency returns to the observed patterns data.

5.1 Simulating a Great Recession in the Model

To simulate a Great Recession in the model, we simulate many realizations of the model, but extract only those that experienced a jump in sample. We define a jump as a drop of at least 5% in producer-country consumption. We center the time series of each realization around the date of its jump. These centered dates are set to coincide with the timing of the Great Recession in the data, year-end 2008. The size of each jump is stochastic and therefore the model generates a distribution of jumps. We then see how well the distribution of jumps in the calibrated model, specifically the mean and the 5-95% confidence bounds, can match the time series realization in the data. Both the model and the data are aggregated to a quarterly frequency.

Figure 7 plots the result of the exercise. The data is captured relatively well by the model within its confidence bounds. While the model does not perfectly match asset prices, it does generate a large drop in trade costs, which persists due to the slow adjustment of shipping capital. This is largely consistent with the behavior of shipping costs in the recession which remain persistently low as shown in Figure 2.

This slow adjustment in the shipping sector leading to not only a bust in trade costs, but also the carry trade risk premium. Figure 8 plots time series of the carry trade risk premium. Before the recession, global trade booms and marginal trade costs are high. Upon the impact of the recession, global trade drops, leaving a glut of shipping capacity and a low marginal trade cost. These low costs allow risks to be better shared across economies and therefore, in equilibrium, the risk premium demanded by global investors of the carry trade declines. The persistence in the trade cost ($q_k$ slowly mean-reverts to producer-country productivity $z_p$) leads to a long-lived increase in risk sharing and a commensurate decline in expected returns.
The calibration of our model to the global financial crisis. In every panel the top blue line is the 95th percentile across all simulations, the middle line is the mean, and the bottom line is the 5th percentile. The top-left panel shows the GDP-weighted GDP index of producer countries (Germany, Japan, and Sweden) in the data; and the model shows the output of the producer country $y_p$. The top-right panel shows the realized return to a carry trade which goes long the commodity country currencies and short the producer country currency. In the model this is $dRet$. The bottom-left panel shows the commodity price: the CRB Spot Commodity Index and the commodity price $P$ in the model. The bottom-right panel shows the trade cost: the Baltic Dry Index and the marginal “dollar” commodity trade cost, $P \times \frac{\partial (\tau_c(z_c, z_k) z_c)}{\partial z_c}$. All exchange rate, commodity price, trade cost, and output variables are normalized to one in December 2007. Data from Datastream and the OECD.
Figure 8: Carry Trade Expected Return over Great Recession

Figure plots of the average expected return to the carry trade which goes long the commodity country currency and short the producer country over the model's simulated recession period.

Having established the behavior in the model, we now examine empirically the behavior of exchange rate

5.2 Exchange Rates, Interest Rates, and Currency Returns around the Global Recession

As shown in Section 2 the behavior of commodity prices and trade costs around the great recession are largely consistent with the model. In this section we examine the behavior of exchange rates, relative interest rates, and currency returns over this same time period. The primary prediction of the model is that a large recession leads to a drop in spot exchange rates, along with a drop in expected carry trade returns coinciding with the persistently low shipping costs following the drop in output.

Figure 9 plots data relevant for currency returns along with the BDI and Harpex to provide evidence of these predictions in the data. Panel A first plots the relative spot exchange rate between equal weighted averages of commodity and producer countries. Consistent with the model, the drop in shipping costs coincides with a large depreciation of the commodity currencies relative to the producer countries, and that this difference in exchange rate persists following the initial drop.

Panel B then plots the time series of the differential between the average of commodity
country and producer country interest rates. Here we see that this depreciation in currency also is accompanied by a persistent decrease in this differential. As this differential is the primary source of carry trade profitability, this is consistent with the reduction in expected returns predicted by the model.

While all variables in the model are real variables, it is instructive to examine whether this decreased differential reflected changes in inflation expectations. As a proxy for these expectations, in Panel C we plot the 1-year rolling difference in realized inflation among the commodity and producer countries. While commodity countries generally have higher inflation, there is no evidence that inflation behavior experienced a large change after the crisis.

Finally, in Panel D we plot cumulative realized returns on a strategy which goes long an equal weighted portfolio of the four commodity countries and short an equal weighted portfolio of the four producer countries. As the plot shows, this return was consistently positive prior to the crisis, but after a quick recover immediately following the recession, as remained largely flat over the subsequent 4-year period. While the time-series is too short to assign statistical significance to this result, it is nevertheless consistent with the decrease in expected returns predicted by the model, and suggested by the reduction in interest rate differentials in the data.

To assign significance to the relations in Panel’s A and B. Table 4 shows results of regressions of quarterly changes in spot exchange rate and interest rate differentials on quarterly changes to our measures of shipping costs and the CRB commodity index. As columns (1) - (3) show, all three of the variables are positively related with exchange rates, with the BDI and commodity price indices being significantly related. As columns (4) - (6) show, both commodity prices and the measures of shipping costs have positive coefficients positive even at this short term quarterly horizon. The relation between the Harpex and interest rate differentials is the only one which is significant at conventional levels, but here the relation between quarterly changes is strikingly strong (T-stat > 4), coinciding with the pattern in Panel B of Figure 9. Since the Harpex measures trade in finished goods it may be a more appropriate measure of barriers to risk-sharing, and hence expected carry trade returns, as predicted by the model.
Panel A plots the difference between the equal weighted average of commodity country and producer country short-term (1-month) interest rates calculated form currency forwards. Panel B plots the cumulative change the relative spot exchange rates between an equal weighted portfolio of the four commodity countries (Australia, Canada, New Zealand, and Norway) and the four producer countries (Germany, Japan, Sweden, and Switzerland). Panel C plots the rolling 1-year difference in commodity country and producer country realized inflations calculated as the relative change in equal weighted averages of the CPIs of the two groups. Panel D plots the cumulative return to a currency strategy which goes long an equal weighted position in the 4 commodity countries and short an equal weighted position in the 4 producer countries. Panels A-C also include plots of the log of the Harpex Container Ship Price Index and the Log of the Baltic Dry Index. Currency data are from Bloomberg. CPI data is from the OECD.
Table 4: Relative Interest and Exchange Rates vs. Shipping Costs and Commodity Prices

<table>
<thead>
<tr>
<th>Dep. Var:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative Spot Change</td>
<td>Relative Interest Rate Change</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\Delta BDI$</td>
<td>0.034**</td>
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<td></td>
<td>(0.013)</td>
<td></td>
<td></td>
<td>(0.001)</td>
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<td>$\Delta Harpex$</td>
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<td>0.011***</td>
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<tr>
<td></td>
<td>(0.055)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\Delta CRB$</td>
<td></td>
<td>0.262***</td>
<td></td>
<td>0.008</td>
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<td>(0.078)</td>
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<td>Constant</td>
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<td>-0.005</td>
<td>-0.000</td>
<td>0.000</td>
<td>-0.000</td>
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<td></td>
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<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.228</td>
<td>0.034</td>
<td>0.288</td>
<td>0.026</td>
<td>0.255</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

This table regresses changes in commodity and producer country relative interest rates and exchange rates against changes in shipping costs and commodity prices. Columns (1) - (3) are regressions where the dependent variable is the relative spot exchange rate, which is calculated as an equal weighted average of the changes in the four commodity countries’ exchange rates less an equal weighted average of the producer countries exchange rates. The independent variables are the change in the two shipping costs indices (Baltic Dry Index and Harpex), and the change in the CRB spot commodity index. Columns (4) - (6) repeat the regressions using the change in the relative interest rate as the dependent variable, calculated as the average 1-month interest rate of the four commodity countries (Australia, Canada, New Zealand, and Norway) against less the average 1-month interest rate of the four producer countries (Japan, Germany, Sweden, and Switzerland). Data are Quarterly from 2000 to 2015.
6 Conclusion

We introduce a simple two-country model featuring specialization in trade and a time-varying trade friction to study the behavior of exchange rates and currency carry trade returns around the Great Recession. In the model, persistent changes in trade costs arise from low frequency movements in shipping capacity, leading to endogenous time-varying dynamics in global market segmentation.

We use the calibrated model to account for the behavior of the relevant economic time series before and after the Great Recession. In our simulation, a run-up in global productivity prior to the recession leads to increases in commodity prices (as commodity supply struggles to catch up quickly) and in global shipping costs (as shipping capacity also lags behind). A widening wedge between the marginal utilities in the producer and the commodity countries leads to large carry trade profits. A large negative shock to productivity - such as the Great Recession - causes a sharp decline in commodity prices and trade costs, and a currency appreciation in the country which produces complex manufactured goods relative to the country which produces basic commodities. Subsequently, shipping costs remain depressed, due to overcapacity in the shipping sector.

Low shipping costs lead to low expected returns on the currency carry trade strategy as they imply a high level of global integration. We verify the predictions of the model in the data, and show that the period since the recession has featured persistently low shipping costs, and lower interest rate differentials leading to lower returns to the carry trade strategy.
References


Appendix

6.1 Data Sources

Data on spot exchange rates and on 1-month forward discounts, which are used to calculate relative interest rates and currency returns, are from Bloomberg. Data on shipping and commodity price indices are collected from data stream. Data on worldwide shipping capacity are from the United Nations Conference on Trade and Development. Data on country output, inflation, and labor production are from the OECD. Data on international trade are from the UN’s Comtrade database. The trade classification used to calculate the Import Ratio is from Ready, Roussanov, and Ward (2016).
6.2 Output

Commodity output $y_{ct}$ equals the level of $z_{ct}$, so that the final good output dynamics are given by

$$y_{pt} = z_{pt}[z_{ct}(1 - \tau_c(z_{ct}, z_{kt}))]^{1-\beta}$$

$$= z_{pt} I(z_{ct}, z_{kt})^{1-\beta}$$

$$dy_{pt} = dz_{pt}^{1-\beta}$$

$$+ z_{pt}(1 - \beta) I_t^{-\beta} I_c dz_{ct}^c + \frac{1}{2} z_{pt}(1 - \beta) \left(I_t^{-\beta} I_{cc} - \beta I_t^{-\beta-1} I_t^2\right) dz_{ct}^c dt$$

$$+ z_{pt}(1 - \beta) I_t^{-\beta} I_k dz_{kt}^k + \frac{1}{2} z_{pt}(1 - \beta) \left(I_t^{-\beta} I_{kk} - \beta I_t^{-\beta-1} I_k^2\right) dz_{kt}^k dt$$

$$+ d \left(\sum_{0 < s \leq t} (y_{ps} - y_{ps^-})\right)$$

$$= z_{pt} \mu_p I_t^{1-\beta} dt + z_{pt} \sigma_p I_t^{1-\beta} dB_{pt}$$

$$+ \left(z_{pt}(1 - \beta) I_t^{-\beta} I_c z_{ct} \mu_{ct} + \frac{1}{2} z_{pt}(1 - \beta) \left(I_t^{-\beta} I_{cc} - \beta I_t^{-\beta-1} I_t^2\right) z_{ct}^2 \sigma_c^2\right) dt$$

$$+ \left(z_{pt}(1 - \beta) I_t^{-\beta} I_k z_{kt} \mu_{kt} + \frac{1}{2} z_{pt}(1 - \beta) \left(I_t^{-\beta} I_{kk} - \beta I_t^{-\beta-1} I_k^2\right) z_{kt}^2 \sigma_k^2\right) dt$$

$$+ \left(e^{Z_N(t)} - 1\right) dN_t$$

\[\Rightarrow dy_{pt} = \mu_p dt + \sigma_p dB_{pt}\]

$$+ (1 - \beta) \left[ I_c \frac{z_{ct} \mu_{ct} + \frac{1}{2} \left(I_{cc} - \beta I_t^2 I_t^{1-\beta}\right) z_{ct}^2 \sigma_c^2}{I_t} dt + (1 - \beta) I_c z_{ct} \sigma_c dB_{ct}\right]$$

$$+ (1 - \beta) \left[ I_k \frac{z_{kt} \mu_{kt} + \frac{1}{2} \left(I_{kk} - \beta I_k^2 I_t^{1-\beta}\right) z_{kt}^2 \sigma_k^2}{I_t} dt + (1 - \beta) I_k z_{kt} \sigma_k dB_{kt}\right]$$

$$+ \left(e^{Z_N(t)} - 1\right) dN_t$$

\[\cong \mu_p dt + \sigma_p^T dB_t + (e^{Z_N(t)} - 1) dN_t,\]
where $I(z_{ct}, z_{kt})$ and its derivatives are defined as follows:

$$
I_t = I(z_{ct}, z_{kt}) = z_{ct}(1 - \tau_c(z_{ct}, z_{kt}))
$$

$$
I_c = (1 - \kappa^c_0) - 2\kappa^c_1 \frac{z_{ct}}{z_{kt}}
$$

$$
I_{cc} = -2\kappa^c_1 \frac{z_{ct}}{z_{kt}}
$$

$$
I_k = \kappa^c_1 \frac{z_{ct}^2}{z_{kt}^2}
$$

$$
I_{kk} = -2\kappa^c_1 \frac{z_{ct}^2}{z_{kt}^3}
$$

Commodity price dynamics are given by

$$
P_t = (1 - \beta)z_{pt} [z_{ct}(1 - \tau_c(z_{ct}, z_{kt})))]^{-\beta}
$$

$$
= \frac{(1 - \beta) y_{pt}}{(1 - \tau_c(z_{ct}, z_{kt})) z_{ct}}
$$

### 6.3 Exports of final consumption good

Since in the general case the export function must be found numerically, it is convenient to restate equation (1) as

$$
\left[ \xi_t (1 - \kappa^f_0 - \kappa^f_1 \xi_t) \right]^{-\gamma} \left( 1 - \kappa^f_0 - 2\kappa^f_1 \xi_t \right) - \lambda [\exp (q_t + q_{kt}) (1 - \kappa^c_0 - \kappa^c_1 \exp (q_{kt})])^{1-\beta} - \xi_t \right]^{-\gamma} = 0
$$

where $\xi_t = \frac{x_t}{z_{kt}} \equiv \xi (q_t, q_{kt})$ is exports of final good per unit of shipping capacity as a function of the two stationary state variables. Then the numerical solution for $\xi_t$ can be interpolated for use in simulations.

In the special case of log utility ($\gamma = 1$) equation (1) simplifies to

$$
\kappa^f_1 (2 + \lambda)X_t^2 = [z_{kt} (1 - \kappa^f_0) (1 + \lambda) + 2\kappa^f_1 y_{pt}]X_t + (1 - \kappa^f_0) y_{pt} z_{kt} = 0.
$$
Solving this equation yields
\[
X_t = \frac{z_{kt}(1 - \kappa_0^f)(1 + \lambda) + 2\kappa_1^f y_{pt} - \sqrt{[z_{kt}(1 - \kappa_0^f)(1 + \lambda) + 2\kappa_1^f y_{pt}]^2 - 4(1 - \kappa_0^f) y_{pt} z_{kt} \kappa_1^f (2 + \lambda)}}{2\kappa_1(2 + \lambda)}
\]
which is the only root that allows positive producer-country consumption. We can write
\[
X_t = \frac{h(z_{ct}, z_{pt}, z_{kt}) - \sqrt{g(z_{ct}, z_{pt}, z_{kt})}}{2\kappa_1(2 + \lambda)},
\]
where
\[
\begin{align*}
h(z_{ct}, z_{pt}, z_{kt}) &= z_{kt}(1 - \kappa_0)(1 + \lambda) + 2\kappa_1 z_{pt} I_t^{1 - \beta}, \\
g(z_{ct}, z_{pt}, z_{kt}) &= h(z_{ct}, z_{pt}, z_{kt})^2 - 4(1 - \kappa_0)\kappa_1 (2 + \lambda) z_{pt} I_t^{1 - \beta} z_{kt}.
\end{align*}
\]

The derivatives of the export function and its components follow:
\[
\begin{align*}
X_i &= \frac{h_i - \frac{1}{2} g^{-1/2} g_i}{2\kappa_1(2 + \lambda)}, \quad \forall i \in \{c, p, k\} \\
X_{ii} &= \frac{h_{ii} + \frac{1}{4} g^{-3/2} g_i^2 - \frac{1}{2} g^{-1/2} g_{ii}}{2\kappa_1(2 + \lambda)}.
\end{align*}
\]
In the general CRRA case the derivatives of the export function can be found by implicit differentiation:
\[
\begin{align*}
\frac{dX}{dz_i} &= -\frac{g_{zi}}{g_X} \text{ for } i \in c, p, k \\
\frac{d^2X}{(dz_i)^2} &= -\left(\frac{g_X \left( g_{zi, X} \frac{dX}{dz_i} + g_{zi, z_i} \right) - g_{zi} \left( g_{X, X} \frac{dX}{dz_i} + g_{X, z_i} \right)}{(g_X)^2}\right).
\end{align*}
\]
By normalizing each partial differential by $X_t$ and by Ito’s lemma,

$$dX_t(z_{ct}, z_{pt}, z_{kt}) = X_{ct}X_t dz_{ct}^c + X_{pt}X_t dz_{pt}^c + X_{kt}X_t dz_{kt}^c$$

$$+ \frac{1}{2} X_{cct}X_t dz_{ct}^c dz_{ct}^c + \frac{1}{2} X_{ppt}X_t dz_{pt}^c dz_{pt}^c + \frac{1}{2} X_{kkt}X_t dz_{kt}^c dz_{kt}^c$$

$$+ d \left( \sum_{0<s \leq t} (X_s - X_{s-}) \right)$$

$$\Rightarrow \frac{dX_t}{X_{t-}} = \left\{ X_{ct} \mu_{ct} z_{ct} + X_{pt} \mu_{pt} z_{pt} + X_{kt} \mu_{kt} z_{kt} + \frac{1}{2} X_{cct} \sigma_c^2 z_{ct}^2 + \frac{1}{2} X_{ppt} \sigma_p^2 z_{pt}^2 + \frac{1}{2} X_{kkt} \sigma_k^2 z_{kt}^2 \right\} dt$$

$$+ X_{ct} \sigma_c z_{ct} dB_{ct} + X_{pt} \sigma_p z_{pt} dB_{pt} + X_{kt} \sigma_k z_{kt} dB_{kt} + d \left( \sum_{0<s \leq t} \frac{X_s - X_{s-}}{X_{t-}} \right)$$

$$\Rightarrow \frac{dc_{pt}}{c_{pt-}} = \frac{1}{c_{pt-}} (\mu_{yt} - \mu_{Xt}) dt + \frac{1}{c_{pt-}} (\sigma_{yt}^T - \sigma_{Xt}^T) dB_t + d \left( \sum_{0<s \leq t} \frac{c_{ps} - c_{ps-}}{c_{pt-}} \right)$$

$$\Rightarrow \frac{dc_{pt}}{c_{pt-}} = \mu_{cpt} dt + \sigma_{cpt}^T dB_t + (e^{J_p} - 1) dN_t$$

where $J_X = \log \left( \xi(q_{t-} + Z_{N(t)}, q_{kt-}) \right) - \log \left( \xi(q_{t-}, q_{kt-}) \right)$, the log change in final goods exported.

### 6.4 Consumption

For the consumption allocations we have

$$c_{pt} = y_{pt} - X_t$$

$$\Rightarrow dc_{pt} = dy_{pt} - dX_t + d \left( \sum_{0<s \leq t} (c_{ps} - c_{ps-}) \right)$$

$$\Rightarrow \frac{dc_{pt}}{c_{pt-}} = \frac{1}{c_{pt-}} \left( \mu_{yt} - \mu_{Xt} \right) dt + \frac{1}{c_{pt-}} \left( \sigma_{yt}^T - \sigma_{Xt}^T \right) dB_t + d \left( \sum_{0<s \leq t} \frac{c_{ps} - c_{ps-}}{c_{pt-}} \right)$$

$$\Rightarrow \frac{dc_{pt}}{c_{pt-}} = \mu_{cpt} dt + \sigma_{cpt}^T dB_t + (e^{J_p} - 1) dN_t$$
for the final good producer, and

\[ c_{ct} = X_t \left( 1 - \kappa_0^f - \kappa_1^f \frac{X_t}{z_{kt}} \right) \]

\[ dc_{ct} = (1 - \kappa_0^f) dX_t^c - \kappa_1^f dz_t + \sum_{0 < s \leq t} (c_{cs} - c_{cs-}) \]

\[ \Rightarrow \frac{dc_{ct}}{c_{ct-}} = \frac{1}{c_{ct-}} \left\{ \mu_{Xt}(1 - \kappa_0^f) - \kappa_1^f \left[ \frac{1}{z_{kt}} (2X_t \mu_{Xt} + \sigma_{Xt}^T \sigma_{Xt}) - \frac{X_t^2}{z_{kt}} (\mu_k - \sigma_k^2) - 2X_t X_{kt} \sigma_k^2 \right] \right\} dt 
\]

\[ + \frac{1}{c_{ct-}} (1 - \kappa_0^f) \sigma_{Xt} T dB_t - \frac{1}{c_{ct-}} \kappa_1^f \frac{2X_t}{z_{kt}} \sigma_{Xt} T dB_t - \frac{1}{c_{ct-}} \kappa_1^f \frac{X_t^2}{z_{kt}} \sigma_k T dB_{kt} \]

\[ + d \left( \sum_{0 < s \leq t} c_{cs} - c_{cs-} \right) \]

\[ \cong \mu_{cct} dt + \sigma_{cct} T dB_t + (e^{J_c} - 1) dN_t \]

for the commodity producer.

### 6.5 Risk-free rates

In order to compute risk-free rates the expected growth rate of marginal utility conditional on a jump occurring must be computed as a function of the state variables. Let

\[ \mathbb{E}_Z \left[ e^{-\gamma J_c} \right] = \mathbb{E}_Z \left( \xi (q_t - Z, q_{kt-}) (1 - \kappa_0^f - \kappa_1^f \xi (q_t - Z, q_{kt-})) \right)^{-\gamma} \]

\[ = \zeta_c (q_t-, q_{kt-}) \]

since the distribution of jump sizes is time invariant. Similarly, let

\[ \mathbb{E}_Z \left[ e^{-\gamma J_p} \right] = \mathbb{E}_Z \left( \frac{\exp (q_t - Z + q_{kt}) (1 - \kappa_0^c - \kappa_1^c \exp (q_{kt-})))^{1-\beta} - \xi (q_t - Z, q_{kt-})}{\exp (q_t - q_{kt-}) (1 - \kappa_0^c - \kappa_1^c \exp (q_{kt-}))^{1-\beta} - \xi (q_t-, q_{kt-})} \right)^{-\gamma} \]

\[ = \zeta_p (q_t-, q_{kt-}) \]

These functions can be evaluated by integrating over the distribution of jump sizes \( Z \) given by the pdf \( \varphi (Z) = \frac{\alpha Z_{\text{max}} x^{-\alpha - 1}}{1 - (\frac{Z_{\text{min}}}{Z_{\text{max}}})^{-\alpha}} \); this is done numerically using Gaussian quadrature.
6.6 Exchange rate

Since the spot exchange rate is defined as

\[ S_t = \lambda \left( \frac{c_{pt}}{c_{ct}} \right)^{-\gamma} = \left( 1 - \kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}} \right), \]

we can derive the dynamic evolution of exchange rate changes as

\[
dS_t = -2\kappa_1^f \left[ \frac{1}{z_{kt}} dX_t - \frac{X_t}{z_{kt}} (\mu_{kt} - \sigma_k^2) dt - \frac{X_t}{z_{kt}} \sigma_k dB_{kt} - X_{kt} \sigma_k^2 dt \right] + \frac{1}{z_{kt}} \left( \sum_{0<s\leq t} (X_s - X_{s^-}) \right)
\]

\[
= -2\kappa_1^f \left[ \frac{X_t}{z_{kt}} \left( X_{ct} \mu_{ct} z_{ct} + X_{pt} \mu_{pt} z_{pt} + X_{kt} \mu_{kt} z_{kt} + \frac{1}{2} X_{ct} \sigma_c^2 z_{ct}^2 + \frac{1}{2} X_{pt} \sigma_p^2 z_{pt}^2 + \frac{1}{2} X_{kt} \sigma_k^2 z_{kt}^2 \right) dt 
- \frac{X_t}{z_{kt}} (\mu_{kt} - \sigma_k^2) dt - \frac{X_t}{z_{kt}} X_{kt} \sigma_k^2 dt + \frac{X_t}{z_{kt}} X_{ct} \sigma_c z_{ct} dB_{ct} + \frac{X_t}{z_{kt}} X_{pt} \sigma_p z_{pt} dB_{pt} + \frac{X_t}{z_{kt}} (X_{kt} - 1) \sigma_k dB_{kt} 
+ (\xi (q_t^- + Z_{N(t)}, q_{kt^-}) - \xi (q_t^-, q_{kt^-})) \right] dN(t)
\]  

\[
\Rightarrow \frac{dS_t}{S_t} = \mu_{St} dt + \sigma_{St} dB_t + \left( e^{J_S} - 1 \right) dN(t),
\]

where \( J_S = \log \left( 1 - \kappa_0^f - 2\kappa_1^f \xi (q_t^- + Z_{N(t)}, q_{kt^-}) \right) - \log \left( 1 - \kappa_0^f - 2\kappa_1^f \xi (q_t^-, q_{kt^-}) \right) \).

6.7 Expected Returns

Let

\[
E \left[ dRet_t | F_t \right] = E \left[ \frac{dS_t}{S_t} \frac{d\pi_{pt}}{\pi_{pt^*}} | F_t \right] = \mu_t^{FX} dt,
\]

where \( \mu_t^{FX} \) is the instantaneous conditional currency risk premium, which can be calculated as

\[
\mu_t^{FX} = -\gamma \sigma_{St}^T \sigma_{c_{pt}} + \eta \mathbb{E}_Z \left[ (e^{J_S} - 1) \left( e^{-\gamma J_p} - 1 \right) \right].
\]

Figure 10 displays the final good trade costs \( \tau_f \) and the conditional currency risk premium \( \mu^{FX} \) as functions of the two cointegrating residuals \( q_t^f \) and \( q_t^k \), evaluated at \( q_t = 0 \), so that a higher \( q_t^k \) due to large output of the final good relative to the available shipping capacity translates into high shipping costs and high expected excess returns.
Figure 10: Trade Costs and Currency Risk Premium

Final Good Trade Cost, $\tau_f$ (holding $q = 0$)

Currency Risk Premium, $\mu_{FX}$ (holding $q = 0$)