Growth Uncertainty, Generalized Disappointment Aversion and Production-based Asset Pricing*

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Abstract

We study a production economy with regime switching in the conditional mean and volatility of productivity growth. The representative agent has generalized disappointment aversion preferences. We show that volatility risk in productivity growth carries a positive and sizable risk premium. Our model can endogenously reproduce long-run risks in the volatility of consumption growth and return predictability by the conditional volatility of productivity growth observed in the data. Our model matches well key moments in the data on quantities and asset prices and cyclical variations in asset prices.

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1 Introduction

Financial data have provided many stylized facts that are challenging for standard economic models to explain. For example, the mean and volatility of the equity premium are high, the mean and volatility of the risk-free rate are low, and the conditional equity premium is time varying and moves with business cycles countercyclically. Using the consumption-based asset pricing approach in the finance literature, one needs to introduce either various sources of risk into the exogenously given aggregate consumption process or nonstandard preferences into the representative agent model. An important source of risk is the long-run risk in the mean and volatility of consumption growth (Bansal and Yaron (2004), Hansen et al. (2008), and Bansal et al. (2013)). Nonstandard preferences are crucial for how risks are perceived and priced in the market.¹

The importance of volatility risk or time-varying macroeconomic uncertainty has also been noticed in the recent macroeconomics literature.² This literature typically studies dynamic stochastic general equilibrium (DSGE) models and focuses on the implications for quantities rather than asset prices. While DSGE models are more coherent in that consumption and dividends are endogenous, they typically fail to explain many asset pricing puzzles (e.g., Rouwenhorst (1995)).

The goal of this paper is to provide a production-based asset pricing model with time-varying macroeconomic uncertainty by linking the preceding two strands of literature. In addition to capital adjustment costs and financial leverage, our model has two key features. First, we assume that aggregate productivity growth follows a Markov-switching process (Hamilton (1989)). In particular, the conditional mean and volatility of aggregate productivity growth follow two independent two-state Markov chains. The persistent changes in the mean and volatility capture long-run risks in the expected productivity growth and the time-varying macroeconomic uncertainty, respectively. We estimate the Markov-switching model for the historical productivity growth data from 1956:Q1 to 2012:Q4. Our empirical estimates strongly support the presence of shifts in mean and volatility regimes, with volatility regimes being more persistent. We show that an increase in the volatility of productivity growth reduces consumption as in the data and raises marginal utility. Thus the model implied market price of productivity volatility risk is negative. Since aggregate stock returns are negatively exposed to this risk both in the data and in the model, the volatility risk carries a positive risk premium.

Second, we assume that the representative agent has generalized disappointment aversion (GDA) preferences that are recently introduced by Routledge and Zin (2010). As in Routledge and Zin

¹Important nonstandard preferences include Epstein-Zin preferences (Epstein and Zin (1989)), habit formation preferences (Campbell and Cochrane (1999)), disappointment-aversion preferences (Gul (1991) and Routledge and Zin (2010)), and ambiguity-sensitive preferences (Hansen and Sargent (2010) and Ju and Miao (2012)).

we embed GDA in the Epstein and Zin (1989) recursive utility framework to further disentangle risk aversion from the elasticity of intertemporal substitution (EIS). Compared with an agent with Epstein-Zin preferences, the agent with GDA preferences puts more weight on disappointing outcomes below a threshold set at a proportion of the certainty equivalent of the continuation value. This causes the pricing kernel to be more countercyclical than that for the Epstein-Zin utility, thereby helping raise the mean and volatility of the equity premium. Unlike the Epstein-Zin utility, GDA preferences imply first-order risk aversion, which has a large quantitative impact on the risk premium. Moreover, GDA preferences generate stronger precautionary saving motives and hence help lower the risk-free rate.

Our calibrated model can match well many salient features of business cycles and asset returns in the data, including the first and second moments of the equity premium and the risk-free rate and the volatilities of consumption growth, investment growth, and output growth as well as their correlations. Moreover, our model can also successfully replicate the volatility of volatility in consumption growth, output growth, and investment growth in the data.

Notably our model can generate endogenous long-run risks in the mean and volatility of consumption growth observed in the data. We estimate a Markov-switching model of consumption growth using the US data from 1956:Q1 to 2012:Q4. This model is a nonlinear version of the long-run risks model studied in Bansal and Yaron (2004). Our estimates are broadly consistent with Lettau et al. (2008) and Boguth and Kuehn (2013). Using model simulated consumption data, we can closely replicate persistent volatility regimes estimated using the US data. Thus our production-based model provides a foundation for the consumption process adopted in the long-run risk literature.

Our model can also generate asset return dynamics observed in the data. In particular, our model generates countercyclical variations in the equity premium and equity volatility and procyclical variations in the price-dividend ratio. Moreover, our model can produce the predictability of excess stock returns by the investment rate, the dividend yield, and the consumption-wealth ratio documented in the literature.

Importantly we provide new empirical evidence that the conditional volatility of productivity growth, estimated from an AR(1)-GARCH(1, 1) model, can forecast future stock returns significantly at 4- and 5-year horizons. Our model reproduces this predictability pattern quite closely. The intuition is that an increase in the volatility of productivity growth reduces dividends due to the precautionary saving motive. The decrease in dividends reduces the current stock price, leading to a rise in the conditional expected stock return.

In our benchmark calibration with GDA, capital adjustment costs are small and the EIS is greater than 1. These parameter choices enable our model to produce smooth consumption growth
and volatile investment growth observed in the data. Small adjustment costs imply a small real friction in capital accumulation. As a result, investment reacts strongly to productivity fluctuations. A high EIS implies a large substitution effect and a low desire to smooth consumption. This causes consumption to rise less in response to a permanent shock to productivity growth. Thus, a high EIS reduces consumption growth volatility in our model.

The long-run risks literature typically assumes that the EIS is greater than 1. But we recognize that the estimate of the EIS is under debate in the literature (e.g., Hall (1988), Campbell and Mankiw (1989), among others). Bonomo et al. (2011) show in an endowment economy that a model with GDA and an EIS less than 1 can deliver asset pricing results that are almost fully consistent with the well-known stylized facts. They then conclude that the EIS is not important for asset pricing once GDA is incorporated in the model. However, in a production economy, the value of the EIS is crucial for matching moments of macroeconomic quantities. We show that an EIS lower than 1 implies that investment growth becomes too smooth while consumption growth becomes excessively volatile. Consequently, this model is less successful in reproducing low-frequency movement in the volatility of consumption growth, generating a low equity premium and a high risk-free rate.

The combination of the volatility risk in productivity growth and GDA preferences is the key ingredient of our model. To see its importance, we consider three comparison models. First, we remove GDA and keep other parameter values fixed as in the benchmark calibration. We find that shutting down GDA does not affect macroeconomic quantity moments significantly since preference parameters related to risk attitudes are not important for quantity dynamics. However, this model performs much worse in the asset pricing dimension even though the agent has the Epstein-Zin utility and the risk aversion parameter is 10. In particular, we find that the market price of risk, the mean equity premium, and the equity volatility are too low, while the mean risk-free rate is too high, compared to the data. The quantitative impact of GDA is that it raises the mean equity premium by 5.5 percentage points, lowers the mean risk-free rate by 1 percentage point, and raises the equity volatility by 8.5 percentage points.

Second, we ask whether the Epstein-Zin model with high risk aversion can generate similar results to those in the benchmark model. To address this question, we recalibrate the subjective discount factor and the risk aversion parameter to match the mean risk-free rate and the mean equity premium in the data. We find that the recalibrated risk aversion parameter is equal to 31, which is too high according to the discussion in Mehra and Prescott (1985). This recalibrated Epstein-Zin model generates results similar to our benchmark model with GDA at the expense of an extremely high degree of risk aversion.

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3Tallarini (2000) makes this point in a production-based model with the Epstein-Zin utility.
Third, we shut down the channel of economic uncertainty by fixing the productivity growth volatility at its mean value. We find that this comparison model cannot generate endogenous long-run risks in the volatility of consumption growth or strong predictability by the volatility of productivity growth observed in the data. This model also generates much lower mean equity premium and equity volatility. This result suggests that the volatility risk in productivity growth carries a positive and economically significant risk premium. We show that the volatility risk in productivity growth raises the mean equity premium and the equity volatility by about 46 and 42 percent, respectively.

This paper is related to two strands of literature cited earlier, namely, the recent macroeconomics literature focusing on the effect of time-varying uncertainty on business cycles and the long-run risks literature exploring the impact of economic uncertainty on asset prices. Our result that macroeconomic volatility risk carries a positive and economically significant risk premium is consistent with the finding of Bansal and Yaron (2004) and Bansal et al. (2013). Our economic mechanism is also similar to theirs: Volatility risk raises marginal utility of consumption and hence the market price of volatility risk is negative. But the stock return has a negative exposure to the volatility risk. The key difference is that consumption growth is endogenously determined in our production-based model rather than exogenously assumed.

We now discuss some closely related recent studies on production-based asset pricing models. Croce (2010) studies a DSGE model with Epstein-Zin preferences and long-run risks in the conditional mean of productivity growth. Kaltenbrunner and Lochstoer (2010) show that long-run risks in expected consumption growth can endogenously arise in an otherwise standard DSGE model with Epstein-Zin preferences. However, their model cannot account for long-run risks in the volatility of consumption growth. Our analysis suggests that the time-varying volatility in productivity growth is important to generate this result.

Campanale et al. (2010) study GDA preferences, but do not take into account long-run productivity risks, which are, however, the focus of our paper. They consider both transitory and permanent productivity shocks. But their main analysis focuses on transitory shocks by calibrating a small EIS less than 1 and a large capital adjustment cost. Their model implies an excessively high volatility of the risk-free rate as in models with habit formation preferences (Jermann (1998) and Boldrin et al. (2001)). In addition, their model does not generate significant time variations in equity premium and thus does not produce significant predictability of stock returns.

Jahan-Parvar and Liu (2013) investigate a model with regime switching in the mean of productivity growth, learning, and ambiguity aversion by adapting the model of Ju and Miao (2012) to a production economy. The model of Jahan-Parvar and Liu (2013) can reconcile a number of salient features of business cycles and asset prices. As Hansen and Sargent (2010) and Ju and Miao
show, ambiguity aversion raises the countercyclicality of the pricing kernel because the agent pessimistically attaches more weight to bad states, just like disappointment aversion.

The rest of the paper is organized as follows. Section 2 estimates a regime-switching model of productivity growth and presents empirical evidence of the impact of productivity growth uncertainty on macroeconomic quantities and asset prices. Section 3 presents a benchmark production-based asset pricing model with GDA preferences and a Markov-switching productivity growth process. Section 4 calibrates the model and presents quantitative results. Section 5 discusses the robustness of our model. Section 6 concludes. Technical details are relegated to appendices.

2 Regime Shifts in Productivity Growth

In standard real business cycle (RBC) models, the aggregate productivity shock is typically assumed to follow a homoscedastic AR(1) process. To capture time-varying macroeconomic uncertainty, the recent macroeconomics literature introduces stochastic volatility in the productivity process. In the finance literature on long-run risks, economic uncertainty is typically modeled as stochastic volatility in consumption growth in endowment economies. While most papers in these two strands of literature adopt linear processes, we will use a nonlinear Markov-switching process to model productivity growth because this process is easy to estimate. We will estimate this process in Section 2.1 and provide empirical evidence of the impact of volatility risks on macroeconomic quantities and asset returns in Section 2.2.

2.1 Estimation of the Markov-Switching Model

Let \( A_t \) denote the level of aggregate productivity and \( \Delta a_t \equiv \ln (A_t/A_{t-1}) \) denote the growth rate. The empirical specification for productivity growth takes the form

\[
\Delta a_t = \mu(z_t) + \sigma(s_t) \epsilon_t, \quad \epsilon_t \sim N(0, 1),
\]

where \( z_t \) and \( s_t \) determine the regimes of the conditional mean and volatility of productivity growth, respectively. The conditional mean has two states, \( \mu(z_t) \in \{\mu_l, \mu_h\} \) with \( \mu_l < \mu_h \). The conditional volatility also has two states, \( \sigma(s_t) \in \{\sigma_l, \sigma_h\} \) with \( \sigma_h > \sigma_l \). To keep the model parsimonious, we assume that \( z_t \) and \( s_t \) follow two independent Markov chains. This assumption allows us to characterize the joint transition matrix by only four parameters. The transition probabilities of the
two Markov chains are given by

\[ P(\mu_t = \mu_l | \mu_{t-1} = \mu_l) = p_{ll}^\mu, \quad P(\mu_t = \mu_h | \mu_{t-1} = \mu_h) = p_{hh}^\mu, \]
\[ P(\sigma_t = \sigma_l | \sigma_{t-1} = \sigma_l) = p_{ll}^\sigma, \quad P(\sigma_t = \sigma_h | \sigma_{t-1} = \sigma_h) = p_{hh}^\sigma. \]

Combining together the two Markov chains, we obtain a four-state Markov chain. The four states are characterized by (1) low mean and low volatility, (2) low mean and high volatility, (3) high mean and low volatility, and (4) high mean and high volatility.

We use quarterly data on total factor productivity (TFP) growth from 1956:Q1 to 2012:Q4 constructed by Fernald (2012) to estimate the model. We apply the expectation maximization (EM) algorithm developed by Hamilton (1990) to obtain the maximum likelihood estimates of the parameters.\(^4\) The estimation results are reported in Table 1. The average growth rate is 0.48% per quarter in the high expected growth regime and -0.23% per quarter in the low expected growth regime. The estimated transition probabilities, \(p_{ll}^\mu = 0.78\) and \(p_{hh}^\mu = 0.92\), suggest that the expansion state is more persistent than the contraction state. The implied average duration of the expansion state is about 13 quarters and that of the contraction state is about 4 quarters. These results are in line with Hamilton (1989) and Cagetti et al. (2002). The filtered probabilities of the low mean growth states (\(\{\mu_l, \sigma_l\}\) and \(\{\mu_l, \sigma_l\}\)) are plotted in Figure 1. The plot reveals that the filtered probability spikes during recessions, suggesting that expected growth regime switching has a strong linkage with the business cycle. The contemporaneous correlation between the plotted filtered probability and the dummy variable of the NBER recessions is about 0.63.

[Insert Table 1 here.]

The estimated transition probabilities for volatility regimes are \(p_{ll}^\sigma = 0.98\) and \(p_{hh}^\sigma = 0.96\). These estimates imply that average duration of the low volatility states is about 50 quarters, and that of the high volatility states is about 26 quarters. Compared to expected growth regimes, volatility regimes are far more persistent, suggesting that long-run volatility risk in productivity growth is pronounced in the data. Figure 1 plots the filtered probabilities of the high conditional volatility states. The figure shows that (1) the TFP growth of the U.S. economy features a significant decline in volatility since the 1980s, and this pattern is persistent in decades, and (2) the variation in the filtered probability of the high volatility states is mostly detached from the business cycle. In fact, the contemporaneous correlation between the plotted filtered probability and the dummy variable of the NBER recessions is only about 0.12. In the next section, we present a production-based general equilibrium model to show that the low-frequency movement in the conditional volatility

\(^4\)As noted by Hamilton (1990), the EM algorithm permits analytical derivatives that are easy to obtain from the smoothed inferences about the unobserved regimes, and is quite robust to the initial guess of the parameters.
of productivity growth can induce time-varying risk premium, and this variation does not appear to be strongly connected with the business cycle.

[Insert Figure 1 here.]

2.2 Impact of Volatility Risks

What is the impact of volatility risks in productivity growth on the macroeconomic quantities and asset prices in the data? To answer this question, we first estimate an AR(1)-GARCH(1,1) model of productivity growth by the maximum likelihood method,

\[
\Delta a_t = \mu + \vartheta \Delta a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, h_t^2),
\]

\[
h_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \theta h_{t-1}^2,
\]

where \( h_t^2 \) is the variance of \( \epsilon_t \). Panel A of Table 2 presents the estimates for this model. It shows that the estimates of \( \theta \) and \( \vartheta \) are both significant, indicating strong evidence of time-varying volatility of productivity growth. We then extract data for \( h_t \) from the preceding estimation and use \( \ln(h_t) \) to proxy volatility risk.

[Insert Table 2 here.]

Next we run predictive regressions of stock returns and excess stock returns using \( \ln(h_t) \) as a predictor,\(^5\)

\[
r_{t \rightarrow t+j} = a + b \ln(h_t) + \varepsilon_{t,j},
\]

\[
r_{t \rightarrow t+j}^e - r_{t \rightarrow t+j}^f = a_{ex} + b_{ex} \ln(h_t) + \varepsilon_{ex,t},
\]

where \( j \) is the horizon of returns, \( r_{t \rightarrow t+j}^e \) and \( r_{t \rightarrow t+j}^f \) are cumulative log stock returns and the log risk-free rate from periods \( t \) to \( t+j \), respectively. We take stock returns data on the NYSE/AMEX/NASDAQ value-weighted index and 3-month T-bill rates (risk-free rates) from the Center for Research in Security Prices (CRSP). We use the CPI data from FRED at St. Louis to deflate all nominal variables. The sample period is from 1956:Q1 to 2012:Q4. We consider horizons \( j = 4, 8, 12, 16, \) and 20 quarters. Since overlapping observations in the long-horizon predictive regressions causes serial correlation in the error terms, we use Hansen and Hodrick (1980) standard errors to account for this issue.

\(^5\)Taking logs makes the volatility measure less sensitive to outliers and does not qualitatively affect our empirical results.
Panel B of Table 2 shows the estimation results. The \( t \)-statistics adjusted using Hansen and Hodrick (1980) standard errors are reported in parentheses. Remarkably, the volatility of productivity growth has predictive power for future excess returns at long horizons. The results in Panel B show that the slope estimates are significant at the 4- and 5-year horizons. Both the slope estimates and the \( R^2 \)'s are increasing in the horizon. The \( R^2 \) is about 9 percent at the 5-year horizon. Moreover, the slope estimates are all positive, implying that a high current volatility of productivity growth forecasts high future excess returns. Using equity returns as the predicted variable gives similar results. Thus the strong predictability of excess returns is not driven by the variation of the risk-free rate.

Finally, we estimate a VAR model by OLS, where the state vector includes seven variables in the estimation order: \( \ln (h_t) \) and the logs of dividends, investment, consumption, output, price-dividend ratios, and stock returns. Dividends, investment, consumption, and output are detrended. We also use the maximum likelihood method and the BVAR method (Sims and Zha (1998)) to estimate this model and find similar results.

We take the data on real fixed investment \((I)\), nondurable consumption goods and services \((C)\), and aggregate output (defined as \( I + C \)) from the national income product accounts, Bureau of Economic Analysis. We follow Bansal et al. (2005) and use monthly returns data including and excluding dividends to generate quarterly dividend growth data and price-dividend ratio data for the period 1956:Q1–2012:Q4.

Figure 2 presents the impulse responses to a positive one-standard-deviation shock to the volatility of productivity growth from the VAR estimation. This figure shows that investment, consumption, and output fall, consistent with the finding in the literature.\(^6\) Thus an increase in the volatility risk in productivity growth reduces consumption and raises marginal utility. This implies that the market price of the volatility risk is negative. Figure 2 also shows that dividends, the price-dividend ratio, and stock returns all fall in response to a positive volatility shock. This result is in line with the finding of Bansal and Yaron (2004), Lettau et al. (2008), and Bansal et al. (2013) that high macroeconomic uncertainty is associated with low equity valuation. Our result indicates that stock returns are negatively exposed to the volatility risk in productivity growth. Consequently, the volatility risk in productivity growth carries a positive risk premium. In the next section we build a model to generate this result and use the calibrated model to quantify the size of the volatility risk premium.

\[^6\]There are several different measures of uncertainty shock in the literature. For example, Bloom (2009) uses the VIX index, Bloom et al. (2013) use firm-specific uncertainty, and Leduc and Liu (2013) use survey data. They all find similar results.
3 A Benchmark Model

In this section we present a benchmark production-based asset pricing model. As is standard in the literature (e.g., Jermann (1998), Campanale et al. (2010), Croce (2010), and Kaltenbrunner and Lochstoer (2010), among others), we assume that labor is exogenously given. We first describe the utility function and then discuss the social planner’s problem. Finally we study decentralization and present asset pricing formulas.

3.1 Generalized Disappointment Aversion

We follow Routledge and Zin (2010) to model the representative agent’s utility function. Routledge and Zin (2010) generalize Gul (1991)’s disappointment aversion utility and embed this static utility in the recursive utility framework of Epstein and Zin (1989). Formally the agent derives utility from a consumption stream \( C_t \) only. Suppose that the labor supply is exogenously given and normalized to 1. The agent’s continuation utility at date \( t \) is given by

\[
V_t = \left[ (1 - \beta)C_t^{1-\psi} + \beta R_t (V_{t+1})^{1-\psi} \right]^{1-\psi},
\]

where \( \psi > 0 \) represents the elasticity of intertemporal substitution (EIS), and \( R_t (V_{t+1}) \) represents the (conditional) certainty equivalent of future utility \( V_{t+1} \). The conditional certainty equivalent is defined as follows

\[
\frac{R_t (V_{t+1})^{1-\gamma}}{1 - \gamma} = \mathbb{E}_t \left[ \frac{V_{t+1}^{1-\gamma}}{1 - \gamma} \right] - \eta \mathbb{E}_t \left\{ \mathcal{I} \left( \frac{V_{t+1}}{R_t (V_{t+1})} < \kappa \right) \left[ \frac{(\kappa R_t (V_{t+1}))^{1-\gamma}}{1 - \gamma} - \frac{V_{t+1}^{1-\gamma}}{1 - \gamma} \right] \right\},
\]

where \( \mathbb{E}_t \) represents the conditional expectation operator given information at time \( t \), \( \gamma > 0 \) represents the risk aversion parameter, and the parameters \( \eta > 0 \) and \( 0 < \kappa \leq 1 \) capture disappointment aversion.\(^7\) We use \( \mathcal{I} (A) \) to denote an indicator function that is equal to 1 if the event \( A \) is true, and zero otherwise. When \( \psi = 1 \), the utility function (2) reduces to

\[
V_t = C_t^{1-\beta} [R_t (V_{t+1})]^{\beta}.
\]

When \( \eta = 0 \), \( R_t (V_{t+1}) \) is equal to the certainty equivalent for expected utility and the model in (2) reduces to the familiar Epstein-Zin recursive utility. If \( \eta > 0 \), there is a utility cost for outcomes below the scaled certainty equivalent \( \kappa R_t (V_{t+1}) \). This represents the agent’s aversion to

\(^7\)Routledge and Zin (2010) also consider the case \( \kappa > 1 \), in which states with continuation value greater than the conditional certainty equivalent are also deemed as disappointing.
disappointing outcomes. If $\kappa = 1$, the model reduces to the pure disappointment aversion model of Gul (1991). Routledge and Zin (2010) generalize Gul’s model of disappointment aversion by moving the disappointment reference point with $\kappa \in (0, 1)$. The idea is that utility outcomes are disappointing only if they lie sufficiently far below the certainty equivalent. This allows for first-order-risk-aversion effects away from certainty and is useful for asset pricing. More importantly, GDA allows the disappointing outcomes correspond to tail events and hence generate countercyclical risk aversion (or pricing kernel). This is useful to generate time-varying equity premium.

It is straightforward to show that the pricing kernel for the utility model in (2) is given by (see Bonomo et al. (2011))

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{V_{t+1}}{R_t (V_{t+1})} \right)^{\frac{1}{\psi}-\gamma} \left( \frac{1 + \eta I \left( \frac{V_{t+1}}{R_t (V_{t+1})} < \kappa \right)}{1 + \kappa^{1-\gamma} \eta E_t \left[ I \left( \frac{V_{t+1}}{R_t (V_{t+1})} < \kappa \right) \right]} \right). \quad (4)$$

There are three components in the pricing kernel. The first component is the standard power utility pricing kernel. The second component reflects the adjustment due to the separation between risk aversion and the EIS as in the Kreps-Porteus and Epstein-Zin utility. When $\gamma > 1/\psi$, the agent prefers earlier resolution of uncertainty and the ratio of continuation utility $V_{t+1}$ to the certainty equivalent of this continuation utility $R_t (V_{t+1})$ adds a premium for long-run risk. If consumption growth is persistent, a bad shock to productivity growth today will reduce $V_{t+1}/R_t (V_{t+1})$, which will raise the pricing kernel when $\gamma > 1/\psi$. The third component reflects the impact of disappointment aversion with $\eta > 0$. Whenever the ratio of future utility to its certainty equivalent is less than the threshold $\kappa$, the agent attaches a larger weight to the pricing kernel. Thus the last two components make the pricing kernel more countercyclical and help raise the equity premium. We will discuss this point further in Section 4.

What is the impact of the time-varying volatility of productivity growth on the pricing kernel? In Section 4.5 we will show that the time-varying volatility of productivity growth is the key to generating long-run risks in the volatility of consumption growth observed in the data. An increase in the volatility of consumption growth will have persistent effects and make future utility more volatile because disappointing outcomes will be more likely. As a result, the increased consumption growth volatility will raise the volatility of the third component in the pricing kernel. Persistent changes in expected productivity growth can also increase the volatility of future utility and of the GDA pricing kernel, but the effect is indirect. The comparative statics analysis in Section 5.1 shows that this effect is much smaller in magnitude compared to that of the time-varying volatility of productivity growth.
3.2 Social Planner’s Problem

Suppose that aggregate output is produced according to the standard Cobb-Douglas production function

\[ Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \quad \alpha \in (0,1), \]

where \( K_t \) and \( N_t \) represent aggregate capital and labor inputs, respectively, and \( N_t \) is normalized to 1. The aggregate productivity shock \( A_t \) follows the process in Section 2.

Capital adjustment is costly and the law of motion of capital is given by

\[ K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t, \tag{5} \]

where the adjustment cost function is given by

\[ \phi \left( \frac{I_t}{K_t} \right) = a_1 + \frac{a_2}{1 - 1/\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi}, \quad a_2 > 0, \quad \xi > 0. \]

Here \( \xi \) represents the elasticity of the investment rate to Tobin’s Q and the parameters \( a_1 \) and \( a_2 \) are chosen such that there is no adjustment cost in the steady state.\(^8\)

The social planner’s problem is to choose \( \{C_t, I_t\} \) so as to maximize (2) subject to (5) and the resource constraint

\[ Y_t = C_t + I_t. \]

3.3 Asset Prices

The decentralization of the planner’s solution is standard (e.g., Jermann (1998)). The representative household works for the firm and trades firm shares and risk-free bonds. The representative firm chooses labor and investment demand to maximize its firm value. Firm value is equal to the discounted present value of its future cash flows. The cash flows in period \( t \) are given by

\[ F_t = Y_t - w_t N_t - I_t = \alpha Y_t - I_t, \tag{6} \]

where \( w_t \) represents the wage rate and the second equality follows from the fact that \( w_t = \partial Y_t / \partial N_t = (1 - \alpha) A_t^{1-\alpha} K_t^\alpha N_t^{-\alpha} \). Optimal investment is characterized by the \( Q \) theory (Hayashi

\[^{8}\text{In particular, } \phi (I/K) = I/K \text{ and } \phi' (I/K) = 1. \text{ This means that} \]

\[ a_1 = \frac{\exp (\bar{\mu}) - 1 + \delta}{1 - \xi}, \quad a_2 = (\exp (\bar{\mu}) - 1 + \delta)^{1/\xi}, \]

where \( \bar{\mu} \) denotes the unconditional mean of \( \mu (z_t) \).
(1982)). In particular, Tobin’s marginal $Q$ satisfies

$$Q_t = \frac{1}{\phi'(\frac{I_t}{K_t})} = a_2 \left( \frac{I_t}{K_t} \right)^{1/\xi}.$$  

This implies that the investment rate increases with Tobin’s $Q$. As is well known, unlevered firm value $FV_t$ is given by $FV_t = Q_tK_{t+1}$.

In equilibrium, the following asset pricing equation must hold for any asset $j$

$$\mathbb{E}_t \left[ M_{t+1} R^j_{t+1} \right] = 1,$$

where $R^j_{t+1}$ denotes asset $j$’s return between $t$ and $t + 1$. In particular, it holds for the investment return $R^I_{t+1}$, defined by

$$R^I_{t+1} = \frac{1}{Q_t} \left\{ Q_{t+1} \left[ 1 - \delta + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) \right] + \frac{\alpha Y_{t+1} - I_{t+1}}{K_{t+1}} \right\}.$$  

This return is also equal to the return on the unlevered firm with cash flows given by (6).

As is well known, unlevered equity premium is typically low compared to the data because cash flows in (6) are relatively smooth (e.g., Jermann (1998)). Financial leverage can raise dividends volatility and hence raise the levered equity premium. In addition, the observed equity prices and returns in the data correspond to leveraged corporations. The price of unlevered cash flows $FV_t$ and the investment return $R^I_{t+1}$ are not directly observable. Thus it is important to study levered equity value and returns in order for the model to be confronted with the data.

We now follow Jermann (1998) to introduce financial leverage. Suppose that the Modigliani and Miller Theorem holds so that capital structure does not matter for a firm’s investment decision and firm value. There are no corporate taxes or default. Suppose that in each period $t$, a fraction $\omega$ of the firm’s capital stock $K_t$ is financed by long-term discount bonds. We use $B_{t,n}$ to denote the price of the $n$-period discount bonds.\(^9\) We can then write dividends as

$$D_t = Y_t - w_tN_t - I_t + \omega K_t - \omega K_{t-n}/B_{t-n,n},$$  

where $\omega K_t$ represents the bond proceeds and $\omega K_{t-n}/B_{t-n,n}$ represents the repayment to the bond holders purchasing the $n$-period discount bonds in period $t - n$ at the price $B_{t-n,n}$. According to this formulation, leverage increases dividends volatility for two reasons: (1) A shock that raises capital will raise debt. This will dampen the decline in dividends caused by higher investment,

\(^9\)The payoff of one unit $n$-period discount bond is equal to one at all states $n$ periods later in the future.
and raise dividends later on. (2) If interest rates decrease after a shock, the firm repays less debt interests. This will raise dividends.

The $n$-period bond price satisfies the no-arbitrage equation

$$ B_{t,n} = \mathbb{E}_t \left[ M_{t+1} B_{t+1,n-1} \right], $$

with the boundary condition $B_{t,0} = 1$ for any $t$. The one-period risk-free rate is given by

$$ R_{t+1}^f = \frac{1}{B_{t,1}} = \frac{1}{\mathbb{E}_t [M_{t+1}]]. $$

Let $P^e_t$ denote the (levered) equity price. The (levered) equity return is given by

$$ R_{t+1}^e = \frac{D_{t+1} + P^e_{t+1}}{P^e_t}. $$

It follows from (7) that equity value satisfies

$$ P^e_t = \mathbb{E}_t \left[ M_{t+1} (D_{t+1} + P^e_{t+1}) \right] = \sum_{j=0}^{\infty} \mathbb{E}_t \left[ M_{t+1} \cdots M_{t+1+j} D_{t+1+j} \right], $$

where the second equality follows from repeated substitution and the no-bubble (or transversality) condition. By (9), we can compute that equity value is equal to firm value minus debt value,

$$ P^e_t = FV_t - DV_t, $$

where debt value is equal to the total market value of all outstanding bonds from period $t - n + 1$ to period $t$,

$$ DV_t = \sum_{j=1}^{n} \frac{B_{t,j} \omega K_{t-n+j}}{B_{t-n+j,n}}. $$

Using (7), we can show that the conditional equity premium satisfies

$$ \mathbb{E}_t \left[ R_{t+1}^e - R_{t+1}^f \right] = \frac{\sigma_t(M_{t+1}) - \text{Cov}_t(M_{t+1}, R_{t+1}^e)}{\mathbb{E}_t(M_{t+1})} \frac{\sigma_t(M_{t+1})}{\sigma_t(M_{t+1})}, $$

where $\sigma_t(M_{t+1})/\mathbb{E}_t(M_{t+1})$ represents the conditional market price of risk and the last term represents the conditional quantity of risk. Here, $\sigma_t(\cdot)$ and $\text{Cov}_t(\cdot, \cdot)$ denote the conditional standard deviation and covariance operators given date $t$ information, respectively. The above equation shows that in order for the conditional equity premium to be positive, the pricing kernel and stock returns must be negatively correlated. The intuition is that the pricing kernel reflects the marginal utility of consumption and is high in bad times. The agent must be compensated for holding stocks when they do poorly in bad times. When the pricing kernel and stock returns are more negatively
correlated, the equity premium will be larger. Thus raising the countercyclicality of the pricing kernel is important for explaining asset pricing puzzles.

We can also derive an unconditional version of equation (10). Moreover, we can derive the Hansen-Jagannathan bound (Hansen and Jagannathan (1991)) as follows

\[
\frac{\sigma(M_{t+1})}{E(M_{t+1})} \geq \frac{E(R_{t+1}^e - R_{t+1}^f)}{\sigma(R_{t+1}^e - R_{t+1}^f)},
\]

where \(\sigma(M_{t+1})/E(M_{t+1})\) is the unconditional market price of risk and the expression on the right-hand side represents the Sharpe ratio. The above inequality shows that the Sharpe ratio provides a lower bound for the market price of risk. Another version of the equity premium puzzle is that the market price of risk implied by many asset pricing models is smaller than the Sharpe ratio observed in the data, violating the above inequality.

4 Quantitative Results

Since the model does not admit a closed-form solution, we use numerical methods to solve the model. We solve the social planner’s problem numerically using the standard value function iteration method with Chebyshev interpolation and then derive asset pricing results using formulas in Section 3.3. Appendix B describes the numerical procedure. We calibrate the model at a quarterly frequency and obtain unconditional moments of macroeconomic quantities and financial variables from 20,000 Monte Carlo simulations, where each simulation contains 228 periods of data.

4.1 Calibration

We choose the estimates presented in Table 1 as the parameter values in the productivity growth process. We fix the capital share at \(\alpha = 0.36\) and the depreciation rate at \(\delta = 0.02\) as in the RBC literature. We then choose other parameter values in the model to match moments in the data described in Section 2.

The quantity dynamics are insensitive to preference parameters related to risk attitudes such as \(\gamma, \eta,\) and \(\kappa\). But the consumption dynamics are sensitive to the EIS parameter \(\psi\), a finding similar to Croce (2010). When the EIS parameter \(\psi\) increases, the agent is more willing to substitute consumption intertemporally by adjusting the investment level. Thus, investment absorbs most of the (permanent) shock to productivity growth such that consumption growth becomes less volatile.\(^{10}\)

\(^{10}\)Campanale et al. (2010) and Kaltenbrunner and Lochstoer (2010) show that the case of transitory shocks will generate a different result.
We choose $\psi = 2.0$ and $\xi = 11$ to match the volatilities of consumption growth and investment growth in the data, where $a_1$ and $a_2$ are normalized such that there are no adjustment costs in the deterministic steady state. The value of $\psi$ is consistent with the long-run risk literature (Bansal and Yaron (2004) and Bansal et al. (2013)). This value is still under debate in the literature. Some researchers (e.g. Hall (1988), Campbell and Mankiw (1989), and Ludvigson (1999)) find that the estimated EIS is relatively small and close to zero using aggregate consumption data. Other researchers find higher values using cohort level data (Attanasio and Weber (1993) and Beaudry and van Wincoop (1996)), or when the analysis is restricted to asset market participants using household-level data (Vissing-Jørgensen (2002)). More recently, Attanasio and Vissing-Jørgensen (2003) find that the estimated EIS for stockholders is typically above 1. Bansal and Yaron (2004) argue that estimates of the EIS based on aggregate data will be biased down if the usual assumption that consumption growth and asset returns are homoskedastic is relaxed. In Section 5.1 we will conduct a sensitivity analysis for various values of the EIS parameter.

We fix the risk aversion parameter $\gamma$ at 10 in the benchmark calibration as in Bansal and Yaron (2004). Mehra and Prescott (1985) argue that a plausible risk aversion should be between 1 and 10. Our chosen value for $\gamma$ is moderate compared with existing production-based asset pricing models such as Tallarini (2000), Croce (2010) and Li and Palomino (2013), given the enormous difficulty to match the size of equity premium in production-based models. We set the GDA parameter $\eta$ at 2.44. Epstein and Zin (1991) find that the estimate of $\eta$ is 1.63 for nondurable consumption and 2.45 for nondurables and services. We then choose $\kappa = 0.989$ to match the mean equity premium in the data. The calibrated values for $\eta$ and $\kappa$ are also close to those chosen by Bonomo et al. (2011). Since the risk-free rate is sensitive to the subjective discount factor $\beta$, we choose $\beta$ to match the mean risk-free rate in the data.

For financial leverage, we consider 15-year discount bonds, i.e., $n = 60$ quarters. We set $\omega = 0.0138$ in the benchmark calibration to keep the average debt-to-equity ratio to be about 1:1. The leverage ratio is held constant across models in the calibration and sensitivity analysis. Using a comprehensive dataset on debt structure, Rauh and Sufi (2013) document that the leverage ratio is about 50% in their dataset. Jermann and Quadrini (2012) report a similar level of leverage, using the US Flow of Funds data. Table 3 presents the calibrated benchmark parameter values.

4.2 Impulse Responses and Price of Risk

Our model features three sources of independent aggregate shocks: an innovation shock to productivity growth and two regime switching shocks to the mean and volatility of productivity growth.
We now examine the responses of quantities and asset prices to each of these shocks. We assume that the economy initially stays in the high mean and low volatility state \((\mu_h, \sigma_l)\) and the capital stock is at the stochastic steady-state level.\(^{11}\) We then study the impact of a shock that occurs in the third period and only lasts for that period. Appendix C describes the details of the implementation.

Figure 3 presents the impulse responses of several key variables to a positive innovation shock \(\varepsilon_t\) with the size \(2\sigma_l\). Because the impact of the innovation shock on the productivity level is permanent, both consumption and investment rise on impact and then grow at the original rate \(\mu_h\). This effect decreases the representative agent’s marginal utility and the pricing kernel. Thus the price of the innovation risk in productivity growth is positive. The immediate increase in investment leads to a temporary decline in dividends. But dividends rise eventually and grow at the rate \(\mu_h\). Since the equity price grows at the rate of productivity and productivity growth rises on impact, the equity price also rises on impact and then grows at the rate \(\mu_h\). This leads to an increase in the realized equity return on impact. Thus the pricing kernel and the realized equity return react in opposite directions, generating a positive risk premium. The conditional market price of risk and conditional equity premium are, however, not quite responsive to the innovation shock, because the innovation shock does not change economic regimes.

[Insert Figure 3 here.]

Figure 4 displays the impulse responses to a shock that makes the economy switch from state \((\mu_h, \sigma_l)\) to state \((\mu_l, \sigma_l)\). This shock can be interpreted as a negative long-run risk shock to the expected productivity growth, in the spirit of Bansal and Yaron (2004). As expected, investment and consumption fall and then move to the new long-run trend because aggregate productivity decreases to a permanently lower level. Since the pricing kernel rises when the mean productivity growth decreases, the price of the regime shift in mean is positive. Dividends rise on impact and then grow at the rate \(\mu_h\). Equity prices decrease on impact and then rise and grow at the rate \(\mu_h\). Stock returns also fall on impact. This implies that stock returns are positively exposed to the risk of regime switching in mean. Thus this risk carries a positive risk premium.

[Insert Figure 4 here.]

We now study the impulse responses to an uncertainty shock that makes the economy switch from state \((\mu_h, \sigma_l)\) to state \((\mu_h, \sigma_h)\). Figure 5 presents the results. Due to increased uncertainty, the precautionary saving motive induces the representative agent to reduce consumption. Note that output \(Y_t = K_t^\alpha A_t^{1-\alpha}\) does not respond to the uncertainty shock on impact because (1) capital \(K_t\)

\(^{11}\)Starting at any other state will give a similar result and will not change our insights.
is predetermined, (2) labor is exogenously fixed at 1, and (3) the uncertainty shock does not change the productivity level \( A_t \). It follows that investment must rise on impact. Thus our benchmark model cannot explain the comovement among consumption, investment, and output in response to uncertainty shocks observed in the data presented in Section 2.2. However, the consumption response in our model is consistent with the data. The decreased consumption causes the marginal utility and the pricing kernel to rise. Thus the market price of the volatility risk in productivity growth is negative.

Figure 5 also shows that dividends, equity prices, and stock returns all fall on impact, a result consistent with the data. This implies that stock returns are negatively exposed to the volatility risk in productivity growth. Thus this risk carries a positive risk premium.

Unlike the first two shocks discussed earlier, the responses of the pricing kernel, the conditional market price of risk, and the conditional equity premium to the uncertainty shock are large, suggesting that volatility risks should play a quantitatively important role in explaining asset pricing phenomena. The intuition is as follows. When the economy switches to the high uncertainty state, it is more likely for disappointing outcomes to occur. Thus the third component of the pricing kernel in (4) rises substantially. In addition, because the agent prefers early resolution of uncertainty \((\gamma > 1/\psi)\), the second component of the pricing kernel also rises. These two effects together cause the responses of the pricing kernel and the conditional market price of risk to the uncertainty shock to be significant.

4.3 Unconditional Moments

Table 4 presents results related to unconditional moments of quantity and financial variables. It reveals that the benchmark model matches closely the targeted moments in the data. To better evaluate the model performance, we focus on the moments that are not used as targets in calibration. In terms of quantities, the benchmark model matches the following moments reasonably well: the output growth volatility, the correlation between consumption growth and output growth, and the correlation between investment growth and output growth. It is challenging for production-based models to simultaneously match investment volatility and equity premium. Previous research such as Jermann (1998) and Campanale et al. (2010) relies on high adjustment costs to increase the equity claim risk and equity premium, but high adjustment costs will counterfactually imply a low investment volatility. In the benchmark model, we assume low adjustment costs to raise investment volatility, while we rely on GDA to match equity premium in the data.

Recent research in macroeconomics has focused on time variations in the volatility of macroe-
economic quantities (e.g., Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2011), and Bloom et al. (2013)). It is important for DSGE models to match the variability of the volatility in quantities observed in the data. Table 4 shows that the vol-of-vol in investment growth is five times as high as the vol-of-vol in consumption growth in the data. Standard DSGE models with linear homoscedastic productivity shocks cannot explain this salient feature, due to the lack of variations in the second moment of productivity growth. The benchmark model matches well the vol-of-vol in consumption, investment, and output growth in the data. However, the benchmark model cannot match the autocorrelations of consumption, investment, and output growth. This is a common problem in the RBC literature because most RBC models lack a propagation mechanism.

Turning to financial moments, we find that equity volatility implied by the benchmark model is 10.45 percent per year. This level is lower than the empirical estimate in the data (15.03 percent). It is worth mentioning that explaining the equity volatility puzzle (Shiller (1981) and Campbell (1999)) in the production-based framework is much more challenging than in endowment economies because production-based models generally imply countercyclical dividends growth as discussed in Section 4.2 (see Favilukis and Lin (2013)). However, dividends growth is procyclical in the data. Thus excessively volatile dividend growth is needed to generate a high equity volatility in the model. Financial leverage helps generate high dividend volatility. Without financial leverage, the model implied mean equity premium and equity volatility are equal to 0.37 and 0.44 percent, respectively. In Section 5.2 we will directly calibrate the firm’s equity payout to the dividends data and assess the model’s ability in reproducing the key moments of asset returns.

The benchmark model with an EIS greater than 1 generates a smooth risk-free rate, with a standard deviation of 0.23 percent per year. This is in contrast to production-based models with habit formation that generate an excessively volatile risk-free rate. For example, in the habit formation model of Jermann (1998) or Boldrin et al. (2001), the agent displays strong aversion to intertemporal substitution of consumption. Given high capital adjustment costs, the risk-free rate must vary a lot in response to productivity shocks. In a closely related paper Campanale et al. (2010) consider Epstein-Zin recursive utility and calibrate the EIS parameter at a small value substantially less than 1. This inevitably generates a high volatility of the risk-free rate due to the implied low intertemporal substitution.

To see the role of the GDA preference, we compare the benchmark model with the Epstein-Zin model. We first set \( \eta = 0 \) and fix other parameter values as in Table 3. The resulting model is called EZ1. Table 4 reveals that the benchmark model and model EZ1 generate similar values for unconditional moments of quantities. It turns out that the GDA parameters (\( \eta \) and \( \kappa \)) do not matter much for quantity dynamics. This result is generally true for parameters related to risk.

\[ \text{Campbell and Cochrane (1999) overcome this issue in an endowment economy by designing their external habit formation model such that the risk-free rate is held constant.} \]
attitudes, a point first made by Tallarini (2000) using the Epstein-Zin model.

By contrast, the GDA parameters matter a lot for financial moments. Specifically, model EZ1 implies a high risk-free rate (2.42 percent), a low equity premium close to zero (0.02 percent), and a low equity volatility (2.03 percent). We find that GDA preferences greatly reduce the mean risk-free rate and increase the mean equity premium and equity volatility. Since outcomes below the GDA threshold receive relatively more weight, GDA preferences raise effective risk aversion and therefore the price of risk. In particular, the unconditional market price of risk, $\sigma(M)/E(M)$, is 0.82 in the benchmark model but only 0.15 in model EZ1. Model EZ1 also violates the Hansen-Jagannathan bound because the Sharpe ratio is equal to $5.4/15.03 = 0.36$ in the data. Even though model EZ1 entertains time-varying uncertainty in productivity growth, this uncertainty is insufficiently priced due to low risk aversion. The incapability of model EZ1 in replicating the historical mean equity premium is also reflected by the implied unreasonably high average price-dividend ratio. The volatility of the price-dividend ratio is also excessively high in model EZ1.

Can the Epstein-Zin model generate similar results to those for the GDA preferences by raising the relative risk aversion parameter? To answer this question, we consider the case in which we set $\eta = 0$ and take other parameter values as in Table 3 except for $\beta$ and $\gamma$. We recalibrate $\beta$ and $\gamma$ to match the mean risk-free rate and the mean equity premium in the data. We call this calibrated model EZ2. We find that, when $\beta = 0.9943$ and $\gamma = 31$, this model matches financial moments in the data very closely and the results are similar to those for our benchmark model with GDA preferences. However, the required relative risk aversion parameter in model EZ2 is unreasonably high, given the abundant experimental evidence.

To see the role of the volatility risk in productivity growth, we compare our benchmark model with a third model, called GDAC, in which we fix the conditional volatility of productivity growth at its steady-state level implied by the Markov-switching model. When the volatility risk is shut down, the model implied volatilities of macroeconomic quantities are moderately lower. More pronounced is the impact on financial moments. In particular, the volatility risk in productivity growth raises the mean equity premium from 3.71 percent in model GDAC to 5.50 percent in the benchmark model. This means that the volatility risk accounts for a large risk premium (1.8 percent per year). Moreover, it raises the equity volatility from 7.38 percent in model GDAC to 10.45 percent in the benchmark model, a 42 percent increase.

[Insert Table 4 here.]
4.4 Cyclicality of Asset Prices

Asset prices and asset returns move with business cycles as evidenced in the data. Our production-based asset pricing model is suitable to address this issue. Following Gourio (2012), we use cross-correlograms to illustrate cyclicality of several financial variables in Figure 6.

We first consider the cyclicality of equity premium in Panel A of Figure 6. We detrend the log of output using the one-sided version of the Baxter and King (1999) filter and obtain \( \tilde{y}_t \). Gourio (2012) notes that using the one-sided Baxter-King filter can avoid look-ahead bias and better capture the covariation of output with conditional financial moments. We compute the correlation between \( \tilde{y}_t \) and excess stock returns at various leads and lags, i.e., \( \rho \left( \tilde{y}_t, R^e_{t+j} - R^f_{t+j} \right) \), for \( j = -5 \) to 5 quarters. In the data (lines with diamonds), this correlation is positive for \( j < 0 \), suggesting that excess stock returns positively lead output. But this correlation becomes negative for \( j > 0 \), implying that output negatively leads excess stock returns, i.e., future equity premia are lower when current output is high. Panel A shows that the benchmark model, model EZ2, and model GDAC can all match this pattern reasonably well.

Panel B of Figure 6 presents the cyclicality of the price-dividend ratio. In the data, the correlation between detrended output and the log price-dividend ratio is positive at all leads and lags, indicating that the price-dividend ratio is procyclical. Again, all three models can match this pattern, but the magnitude of the correlation is larger than in the data.

Panel C of Figure 6 presents the cyclicality of conditional equity volatility. This panel shows that in the data, equity volatility is countercyclical and negatively related to output at leads and lags up to 5 quarters. All three models can match this pattern. But the negative correlation between output and future equity volatility implied by model GDAC is too large compared to the data.

Panel D of Figure 6 displays the relationship between the stock market and investment. We compute the correlation between the HP filtered log investment \( i_t \) and the log market-to-book ratio, i.e., \( \rho \left( i_t, \ln \left( \frac{P_{t+j}}{K_{t+j}} \right) \right) \), \( j = -5 \) to 5. In the data, this covariance is positive for \( j < 2 \), indicating that the stock market positively leads investment. All three models match well the magnitude of the association between stock prices and investment. But the exact timing of the relation between the stock market and investment cannot be replicated.

In summary, Figure 6 shows that neither the GDA preferences nor the time-varying volatility of productivity growth plays an important role in determining the cyclical patterns of asset prices and returns. This suggests that Epstein-Zin preferences and the regime shifts in the expected growth of aggregate productivity are the two key elements for generating the cyclicality patterns in the data.
To further examine the cyclical properties of asset returns, we present conditional macroeconomic and financial moments in Table 5. Columns 2 through 5 show conditional moments in each of the four states, \((\mu_l, \sigma_l), (\mu_l, \sigma_h), (\mu_h, \sigma_l),\) and \((\mu_h, \sigma_h)\). Columns 6 through 9 further show moments conditional on, respectively, the low and high expected growth regimes and the low and high volatility regimes. For instance, moments in the low expected growth regime are computed as the probability-weighted averages of conditional moments in states \((\mu_l, \sigma_l)\) and \((\mu_l, \sigma_h)\), where the weights are steady-state probabilities of the two volatility regimes. Similarly moments in the low volatility regime are computed as the probability-weighted averages of conditional moments in states \(\{\mu_l, \sigma_l\}\) and \(\{\mu_h, \sigma_l\}\), where the weights are steady-state probabilities of the two expected growth regimes.

Table 5 reveals that conditional volatilities of consumption growth and investment growth are very similar across the expected growth regimes but significantly different across the volatility regimes. The difference is particularly large for investment volatility. This suggests that volatility regime switching is important to explain the variability of investment volatility, i.e., model GDAC without time-varying volatility falls short on this respect.

Regarding conditional financial moments, Table 5 shows that in the benchmark model the conditional equity premium is higher in the low expected growth regime than in the high expected growth regime (8.22 versus 4.70 percent). In model EZ2, the conditional equity premium in the low expected growth regime roughly doubles that in the high expected growth regime. However, this variation of conditional equity premium across the expected growth regimes appears to be far less important in comparison with the variation across the volatility regimes. In the benchmark model, conditional equity premium is 2.88 percent in the low volatility regime, compared to 10.65 percent in the high volatility regime. The difference is more pronounced in model EZ2 (1.85 percent in the low volatility regime versus 12.78 percent in the high volatility regime). These results suggest that time-varying volatility of productivity growth is much more important than time-varying expected growth in accounting for the time variation of equity premium. The conditional equity volatility across different states exhibit similar properties. Model GDAC assumes that the conditional volatility of productivity growth is constant and thus generates relatively small fluctuations in the conditional equity premium and equity volatility. This explains why model GDAC delivers smaller unconditional equity premium and equity volatility shown in Table 4.

[Insert Table 5 here.]
4.5 Consumption Dynamics

Understanding consumption dynamics is fundamental for consumption-based asset pricing models. For example, models with long-run consumption risks including Bansal and Yaron (2004), Lettau et al. (2008) and Bansal et al. (2012) find considerable evidence in the data for time-varying expected consumption growth and consumption volatility. In particular, Bansal and Yaron (2004) and Lettau et al. (2008) both emphasize the importance of time-varying consumption volatility. In this section, we demonstrate that our production economy model can successfully reproduce long-run risks in both expected consumption growth and consumption volatility, and thereby lends support to the long-run consumption risks literature.

We begin by documenting empirical evidence that the historical consumption growth data feature time-varying expected growth and volatility. Following Lettau et al. (2008), we estimate a four-state Markov switching process,

\[ \Delta c_t = \mu^c(z_t^c) + \sigma^c(s_t^c) \varepsilon_{c,t}, \varepsilon_{c,t} \sim N(0, 1), \]

where \( z_t^c \) and \( s_t^c \) follow independent two-state Markov chains and \( \varepsilon_{c,t} \) is a white noise and independent of other shocks. The consumption growth data are quarterly growth rates of consumption of nondurable goods and services from 1956:Q1 to 2012:Q4. We use the EM algorithm to compute the maximum likelihood estimates. The estimation results are reported in the second column of Table 6. The estimation clearly identifies two regimes for expected consumption growth and another two distinct regimes for the conditional volatility of consumption growth. Expected consumption growth is positive in both regimes and the growth rate in the high mean regime is about twice as high as that in the low mean regime. Moreover, the estimated conditional volatility in the high volatility regime is almost three times as high as that in the low volatility regime.

More importantly, it is apparent that both the mean and volatility regimes are persistent, according to the estimated transition probabilities.\(^{13}\) The probabilities of staying in the low mean regime and in the high mean regime are given by 0.92 and 0.97, respectively. The volatility regimes exhibit moderately less persistence. The probabilities of staying in the low volatility regime and in the high volatility regime are given by 0.91 and 0.86, respectively. The estimated transition probabilities imply that the first-order autocorrelation of expected consumption growth is about 0.96 in monthly terms and that of conditional consumption growth volatility is about 0.92, largely in line with the parameter values used by Bansal and Yaron (2004).

Next we estimate the Markov-switching process using simulated consumption growth data from \(^{13}\)Boguth and Kuehn (2013) also estimate the same Markov-switching model and obtain somewhat more persistent volatility regimes. Our estimation results are slightly different because we use a longer sample including the most recent recession period since 2008.
the benchmark model and model EZ2. We simulate 20,000 samples under each model and apply the EM algorithm on each sample. Each sample contains 228 periods of quarterly data. In Table 6, we report average values of the estimates over the simulated samples and 90% confidence intervals for each parameter as well. For both models, although the expected growth estimates in the simulation cannot match the data estimates, the conditional volatility estimates in the simulation are quite close to their data counterparts. Moreover, the estimated transition probabilities in the simulation are very close to the empirical estimates, which also lie in the simulated 90% confidence intervals.

Kaltenbrunner and Lochstoer (2010) show that long-run risks in expected consumption growth can endogenously arise in a DSGE model with Epstein-Zin preferences and i.i.d. productivity growth. But their model lacks a mechanism to reproduce time-varying consumption volatility because they assume homoscedastic productivity growth. Our GDAC model shares the same feature when volatility regime switching is shut down.\textsuperscript{14} Our analysis suggests that it is important to introduce time-varying volatility in productivity growth in order to generate consumption volatility risk.

[Insert Table 6 here.]

4.6 Return Predictability by Volatilities of Productivity Growth

Conditional financial moments in Table 5 and the impulse response in Figure 5 show that an increase in the productivity growth volatility leads to lower equity valuation and a higher equity premium at an aggregate level. This result is consistent with the finding of Bansal and Yaron (2004) and Lettau et al. (2008) that consumption growth volatility is negatively related to the price-dividend ratio and positively related to equity premium in endowment economies. It also suggests that the conditional volatility of productivity growth may forecast future returns. We already presented empirical evidence on this predictability pattern in Section 2.2.

We now investigate whether our production-based model can replicate the empirical finding, we run 20,000 simulations from both the benchmark model and the EZ2 model, with their parameterizations given in Tables 3 and 4. Each simulation contains 228 periods of data, conforming to the number of periods in the actual data. We estimate the preceding AR(1)-GARCH(1,1) specification for each simulation of productivity growth data and extract conditional volatility estimates. We then run predictive regressions for each simulation. Table 7 reports the estimation results averaging over 20,000 simulations. Consistent with the above-documented empirical evidence, two findings are noteworthy. First, the adopted four-state Markov-switching model can reproduce volatility

\textsuperscript{14}When we apply the EM algorithm to the artificial data generated from model GDAC, the algorithm fails to converge for most of the samples. This suggests that the four-state Markov switching specification for consumption growth cannot be supported under the GDAC model.
persistence in productivity growth. The estimate of the GARCH parameter using simulated data
(θ = 0.68) is reasonably close to the empirical estimate (θ = 0.84). Second, both the benchmark
model and model EZ2 replicate the predictability pattern presented in Section 2.2 reasonably well.
The average slope estimates are positive and significant for horizons greater than 1 year, and the R²
is increasing in the horizon. Since the risk-free rate exhibits small variation in the calibration, the
predictability results are similar when excess returns and equity returns are used as the predicted
variable, respectively.

[Insert Table 7 here.]

4.7 Return Predictability by Standard Predictors

Many researchers have documented empirical evidence of return predictability at an aggregate level
by valuation ratios. One of the most widely used predictors is the dividend yield, for example, see
Campbell and Shiller (1988), Fama and French (1988a), and Welch and Goyal (2008). Recently,
Lettau and Ludvigson (2001) show that the consumption-wealth ratio (cay) is another important
predictive variable. In a production-based model, Cochrane (1991) shows that macroeconomic
quantities such as output growth and the investment rate also have predictive power for stock
returns. In particular, empirical evidence suggests that low dividend yields and low consumption-
wealth ratios forecast low future returns, while low investment rates forecast high future returns.

In our production-based model, the dividend yield and investment rate can be endogenously
determined. Moreover, we can follow Epstein and Zin (1989) and Ju and Miao (2012) to show that
the wealth-consumption ratio satisfies

\[ \frac{W_t}{C_t} = \frac{1}{1 - \beta} \left( \frac{V_t}{C_t} \right)^{1 - \frac{1}{\psi}}, \tag{11} \]

where \( W_t \) is the wealth level in period \( t \). Thus we can compute the consumption-wealth ratio in
the model.

Table 8 presents empirical evidence of return predictability and simulation results for the bench-
mark model and models EZ2 and GDAC. We regress the 1-, 2-, 3-, 4-, and 5-year cumulative excess
log returns onto investment rates, log dividend yields, and log consumption-wealth ratios, respec-
tively. We construct investment rates and dividend yields in a standard way. The empirical measure
of the consumption-wealth ratio is the cay variable constructed by Lettau and Ludvigson (2001). The
empirical results in Table 8 are largely in line with the extant literature on return predictability.

We simulate 20,000 samples of data from each of the benchmark model, models EZ2 and GDAC.
Each sample contains 228 periods of data. Table 8 reports the estimation results averaging over
the simulated samples. For the sake of brevity, we do not report the $t$—values adjusted for Hansen and Hodrick (1980) standard errors. All the three models can reproduce the aforementioned predictability patterns in the data, though the magnitude of the simulated slope estimates and $R^2$s cannot match the empirical estimates exactly. Specifically, the investment rate negatively forecasts future returns, while the dividend yield and the consumption-wealth ratio positively forecast future returns. In addition, the $R^2$s are increasing in the horizon and are large for long-horizon returns.

[Insert Table 8 here.]

5 Robustness

5.1 Sensitivity Analysis

In this section we conduct an extensive sensitivity analysis to examine the impact of different assumptions or different parameter values on macroeconomic and financial moments. When changing a particular parameter or assumption, we hold everything else constant as in the benchmark model. We focus only on a few key macroeconomic and financial moments, including the volatilities of consumption growth and investment growth, the mean risk-free rate, the mean equity premium, the equity volatility, and the market price of risk. Table 9 presents the results.

We first examine the role of the specification of the productivity process by studying two alternative models. In model GDAC1, we assume that expected productivity growth is constant at the steady-state level implied by the estimated Markov switching model of productivity growth. The third row of Table 9 reveals that the absence of regime shifts in expected productivity growth does not significantly alter the results relative to the benchmark model. Both the mean equity premium and equity volatility are moderately lower than those in the benchmark model. In model GDAC2, we further assume that the volatility of productivity growth is constant at the steady-state level. That is, model GDAC2 assumes IID productivity growth, a commonly adopted assumption in standard RBC models. The fourth row of Table 9 shows that model GDAC2 performs much worse than the benchmark model, especially in the asset pricing dimension, suggesting that time-varying volatility of productivity growth is an important model element.

Next we study the role of GDA parameters $\eta$ and $\kappa$. Table 9 shows that the magnitude of macroeconomic moments does not change much when we vary the GDA parameter $\eta$. However, the impact on financial moments is significant. When $\eta$ decreases, disappointing outcomes receive less weight. The risk-free rate rises as the agent deems the one-period risk-free bond less valuable. The mean equity premium and the price of risk fall since a less disappointment averse agent demands smaller risk premium. Reducing the relative risk aversion parameter $\gamma$ has similar effects.
Similarly, quantity moments are not very sensitive to changes in the GDA threshold parameter $\kappa$, but financial moments are quite sensitive. In particular, when $\kappa$ increases from 0.972 to 0.992, the mean risk-free rate decreases, the mean equity premium increases, and the equity volatility decreases. This result is consistent with that in Bonomo et al. (2011). The intuition is that a larger value of $\kappa$ implies that it is more likely for continuation values to be below the GDA threshold. The agent must be compensated more for holding stocks because there are more disappointing outcomes.

Routledge and Zin (2010) emphasize the importance of GDA for generating countercyclical price of risk in an endowment economy in which aggregate consumption randomly switches between two states. If $\kappa = 1$, only one state can be disappointing. Thus the conditional equity premium is similar across the two states. But when $\kappa < 1$, it is possible that there is only one disappointing continuation value conditional on the bad state and there is no disappointing continuation value conditional on the good state. This can generate a large variation in the conditional equity premium. Thus GDA is important for generating large equity volatility and significant return predictability.

The EIS parameter $\psi$ is an important parameter for matching macroeconomic and financial moments in the data. The sensitivity analysis with respect to the EIS parameter $\psi$ suggests that when the agent is less willing to substitute consumption intertemporally, the volatility of consumption growth rises whereas the volatility of investment growth falls. The mean risk-free rate is inversely related to the EIS parameter because a high risk-free rate induces more intertemporal substitution. The mean equity premium and equity volatility both decrease. For a low EIS, investment is not responsive to fluctuations in productivity growth, leading to a low volatility of returns on investment and a low equity volatility in consequence. Thus, even though the price of risk does not change much in response to a change in the EIS, the mean equity premium falls with lower EIS values. Bonomo et al. (2011) show that the EIS is not important for understanding financial data in a consumption-based model with GDA preferences. Our analysis suggests that their finding is not robust to the extension to a production-based model.

It is well-known in the literature that the role of capital adjustment costs is two-fold. On one hand, high adjustment costs induce high equity volatility and equity premium. On the other hand, high adjustment costs imply counterfactually low investment volatility. The comparative statics results regarding the parameter $\xi$ show that with a value of $\xi$ lower than that in the benchmark model, the volatility of consumption growth rises while the volatility of investment growth falls substantially. High adjustment costs impede consumption smoothing and increase the variation of the marginal product of capital, causing equity premium and equity volatility to increase. These results are consistent with the findings of Jermann (1998).

[Insert Table 9 here.]
5.2 Pricing a Claim to Stock Market Dividends

In consumption-based asset pricing models, aggregate dividends are usually defined as a levered claim to aggregate consumption. The stock market dividends, which are defined as the sum of cash dividends and net equity repurchases, only represent a small fraction of the payouts of all productive capital including private equity, small businesses, real estate, etc. Kaltenbrunner and Lochstoer (2010) have a related discussion. Thus, in a production economy, it is reasonable not to deem the firm’s payouts as equivalent to the stock market dividends.

In this section we follow Ju and Miao (2012) and directly calibrate dividend growth as containing a component proportional to consumption growth and an idiosyncratic component,

$$\Delta d_{t+1} \equiv \ln \left( \frac{D_{t+1}}{D_t} \right) = \lambda \Delta c_{t+1} + g_d + \sigma_d \varepsilon_{d,t+1}$$

where $\varepsilon_{d,t+1}$ is an IID standard normal random variable and is independent of all other shocks in the model. The parameter $\lambda$ can be interpreted as the leverage ratio on expected consumption growth as in Abel (1999). The specification implies that dividend growth is procyclical as opposed to being countercyclical in standard RBC models. This modeling feature allows us to calibrate dividends growth dynamics to the data and simultaneously match equity premium and equity volatility in the data.

We set the leverage parameter $\lambda = 2.74$ as in Ju and Miao (2012) and Abel (1999). We follow Bansal et al. (2005) and use monthly returns data including and excluding dividends to generate the dividend series and obtain quarterly dividend growth data for the period 1956:Q1–2012:Q4. In the data, the annualized standard deviation of dividend growth is about 4.5 percent. We choose $\sigma_d$ to match the volatility of dividend growth, which implies $\sigma_d = 0.018$, given that the model implied volatility of consumption growth is about 0.5 percent per quarter. We follow Bansal and Yaron (2004) and choose $g_d = -0.005$ such that the average rate of dividend growth is equal to that of consumption growth. Our calibration implies that the correlation between consumption growth and dividend growth is about 0.6, close to the level considered by Kaltenbrunner and Lochstoer (2010).

We set the relative risk aversion parameter at $\gamma = 5$, which is lower than that in the benchmark model in Section 4.3. With procyclical dividends growth, the market price of risk required to match the mean equity premium in the data becomes lower than that in the benchmark model. The GDA parameters $\eta$ and $\kappa$ are chosen to match the mean equity premium and equity volatility. The calibration leads to the parameter choices $\eta = 2.33$ and $\kappa = 0.985$. Other parameter values, if relevant, are kept the same as in Table 3. This model is labelled as GDA*. For comparison, we solve several alternative models including (1) the Epstein-Zin model with a low risk aversion
(model EZ1∗), (2) the Epstein-Zin model with a high risk aversion (model EZ2∗), and (3) the GDA model with a constant volatility of productivity growth (model GDAC∗).

Table 10 shows that when the model is calibrated to fit the observed volatility of dividend growth, both the mean equity premium and equity volatility in model GDA∗ are high and close to the data. Model EZ2∗ assumes a high risk aversion parameter (γ = 27) and generates similar results. Model EZ1∗ implies a very low market price of risk and thus a mean equity premium close to zero. Model GDAC∗ lacking time-varying productivity volatility implies a lower mean equity premium of 4 percent per year, as opposed to 5.49 percent in model GDA∗.

[Insert Table 10 here.]

5.3 Comovement

The analysis of impulse responses to shocks in Section 4.2 indicates that our benchmark model cannot generate comovement among investment, consumption, and output in response to uncertainty shocks as in the data. Endogenizing labor choice cannot solve this issue either. If the representative agent can adjust his labor supply and if consumption and leisure are both normal goods, then an increase in uncertainty induces the agent to supply more labor. As current aggregate productivity and the capital stock remain unchanged, labor demand remains unchanged as well. Thus higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple RBC models with uncertainty shocks.15

Recently Basu and Bundick (2012) propose a solution to this problem using a dynamic new Keynesian model with monopolistic competition and sticky prices. In their model, an increase in uncertainty raises labor supply by the representative agent, which reduces firm marginal costs of production. Falling marginal costs with slowly-adjusting prices imply an increase in firm markups over marginal cost. A higher markup reduces the demand for consumption, and especially, investment goods. Since output is demand-determined in the new Keynesian model, output and hours must fall when consumption and investment both decline. Thus, comovement is restored as in the data.

While generalizing our model to the new Keynesian framework of Basu and Bundick (2012) would be interesting, we argue that this generalization will not change our key insights. Our key model mechanism relies on two results. First, consumption falls in response to an increase in productivity growth uncertainty. This result is consistent with the data and is robust, while the responses of output and investment in our benchmark model are not. This implies that marginal

15The model of Gourio (2012) also has a similar comovement problem in response to a change in the disaster probability.
utility of consumption and the pricing kernel rise in response to an increase in productivity growth risk and hence the market price of this risk is negative. Second, stock returns are negatively exposed to uncertainty shocks. This result is also consistent with the data. These two results together imply that productivity growth risk carries a positive risk premium. This model prediction is robust to the extension that can generate comovement.

6 Conclusion

In this paper we use a production-based model to show that the volatility risk in aggregate productivity growth has important asset pricing implications. Combined with GDA preferences, this risk carries a large and positive risk premium and plays an important role in explaining many asset pricing puzzles. We document evidence that the productivity growth volatility can forecast aggregate excess stock returns. Our model can generate this return predictability pattern and other patterns by standard predictors. Our model can also generate endogenous long-run risks in the mean and volatility of consumption growth often postulated in the asset pricing literature on long-run risks.

In terms of future research, it would be interesting to study the asset pricing implications of productivity growth uncertainty in the new Keynesian framework of Basu and Bundick (2012) because this framework can overcome the comovement problem. It would be also interesting to introduce sticky wages as in Favilukis and Lin (2013) because this modeling can make profits more volatility and dividends more procyclical. This extension could raise equity volatility and improve model performance. Finally, it would be interesting to examine the cross-sectional asset pricing implications of the volatility risk in productivity growth.
Appendices

A Social Planner’s Problem

We solve the social planner problem by dynamic programming. The state variables are $K_t, A_t, s_t,$ and $z_t$. We simply use $J_t$ to denote the value function $J(K_t, A_t, z_t, s_t)$ and write the Bellman equation as

$$J_t = \max_{C_t, I_t, K_{t+1}} \left[ (1 - \beta)U(C_t) + \beta R_t (J_{t+1})^{1-1/\psi} \right]^{1/1-\psi},$$

subject to

$$K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t,$$

$$K_t^\alpha (A_t N_t)^{1-\alpha} = C_t + I_t.$$

Since labor supply is exogenous and normalized to 1, $N_t = 1$. Set up the Lagrangian as follows:

$$\left[ (1 - \beta)U(C_t) + \beta R_t (J_{t+1})^{1-1/\psi} \right]^{1/1-\psi} - \lambda_t \left( C_t + I_t - K_t^\alpha A_t^{1-\alpha} \right)$$

$$- \lambda_t Q_t \left[ K_{t+1} - (1 - \delta) K_t - \phi \left( \frac{I_t}{K_t} \right) K_t \right].$$

The first-order conditions are given by

$$I_t : Q_t = \frac{1}{\phi' \left( \frac{I_t}{K_t} \right)}, \quad (A.1)$$

$$C_t : J_t^\psi \frac{\partial U(C_t)}{\partial C_t} = \lambda_t, \quad (A.2)$$

$$K_{t+1} : E_t \left\{ J_t^\psi \beta R_t (J_{t+1})^\psi \frac{\partial R_t (J_{t+1})}{\partial J_{t+1}} \frac{\partial J_{t+1}}{\partial K_{t+1}} \right\} = \lambda_t Q_t. \quad (A.3)$$

The envelope condition reads

$$\frac{\partial J_t}{\partial K_t} = \lambda_t \alpha K_t^{\alpha-1} + \lambda_t Q_t \left[ 1 - \delta + \phi \left( \frac{I_t}{K_t} \right) - \frac{I_t}{K_t} \phi' \left( \frac{I_t}{K_t} \right) \right].$$

Lead the above equation by one period and substitute into (A.3) to derive

$$\lambda_t Q_t = E_t J_t^\psi \beta R_t (J_{t+1})^\psi \frac{\partial R_t (J_{t+1})}{\partial J_{t+1}} \cdot \lambda_{t+1} \left\{ R_{k t+1} + Q_{t+1} \left[ 1 - \delta + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) - \frac{I_{t+1}}{K_{t+1}} \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \right] \right\}. \quad 31$$
where $R_{kt+1} = \alpha K_{t+1}^{\alpha - 1} A_{t+1}^{1-\alpha}$ is the marginal product of capital or the rental rate of capital. By the definition of the pricing kernel,

$$M_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} J_t^{\phi} \beta R_t (J_{t+1})^{-\frac{1}{\phi}} \frac{\partial R_t (J_{t+1})}{\partial J_{t+1}}.$$  

We then obtain

$$Q_t = E_t M_{t+1} \left\{ R_{kt+1} + Q_{t+1} \left[ \frac{1 - \delta + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) - \frac{I_{t+1}}{K_{t+1}} \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) }{\beta} \right] \right\}.$$  

By (A.1),

$$E_t M_{t+1} \left\{ Q_{t+1} \left[ \frac{1 - \delta + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) + \frac{\alpha Y_{t+1} - I_{t+1}}{K_{t+1}} }{\beta} \right] \right\} = 1.$$  

By the definition of the investment return $R_{t+1}^I$ in (8), we obtain

$$E_t M_{t+1} R_{t+1}^I = 1.$$

## B Numerical Method

Because there is regime switching in both mean and volatility, the standard local perturbation method does not work. We thus use the value function iteration method to solve the model numerically. Because there is long-run growth in the model, we have to detrend the model first. We can show that $C_t, K_t, I_t$ and $Y_t$ grow at the same rate as $A_t$. We thus normalize these variables by $A_t$ and use a tilde to denote a detrended variable.

Let $\Gamma_{t+1} = A_{t+1}/A_t = \exp (\Delta (a_{t+1}))$. We can show that

$$J \left( K_t, A_t, z_t, s_t \right) = A_t \tilde{J} \left( \tilde{K}_t, z_t, s_t \right),$$  

where $\tilde{J}$ satisfies the Bellman equation

$$\tilde{J} \left( \tilde{K}_t, z_t, s_t \right) = \max_{\tilde{C}_t, \tilde{I}_t} \left\{ (1 - \beta) \tilde{C}_t^{1-1/\psi} + \beta \mathcal{R}_t \left( \Gamma_{t+1} \tilde{J} \left( \tilde{K}_{t+1}, z_{t+1}, s_{t+1} \right) \right)^{1-1/\psi} \right\}^{1-1/\psi}, \quad (B.1)$$  

32
subject to

\[
\begin{align*}
\Gamma_{t+1} \tilde{K}_{t+1} &= (1 - \delta) \tilde{K}_{t} + \phi \left( \frac{\tilde{I}_{t}}{\tilde{K}_{t}} \right) \tilde{K}_{t}, \\
\tilde{K}_{t}^{\alpha} &= \tilde{C}_{t} + \tilde{I}_{t}, \\
\ln (\Gamma_{t}) &= \mu (z_{t}) + \sigma^{2} (s_{t}) \epsilon_{t}, \epsilon_{t} \sim N (0, 1).
\end{align*}
\]

This is a stationary dynamic programming problem, which can be solved by the standard value function iteration method. To apply this method, we need to compute \( R_{t} (\Gamma_{t+1} \tilde{J}_{t+1}) \) using (3), where \( \tilde{J}_{t+1} = J \left( \tilde{K}_{t+1}, z_{t+1}, s_{t+1} \right) \). We substitute out \( \tilde{K}_{t+1} \) and \( \tilde{C}_{t} \) into (B.1) using (B.2) and (B.3), respectively. We can then compute the policy function for \( \tilde{I}_{t} \)

\[ \tilde{I}_{t} = g^{i} \left( \tilde{K}_{t}, z_{t}, s_{t} \right). \]

We then use (B.2) and (B.3) to derive the policy functions for \( \tilde{K}_{t+1} \) and \( \tilde{C}_{t} \)

\[ \tilde{K}_{t+1} = g^{k} \left( \tilde{K}_{t}, z_{t}, s_{t}, \Gamma_{t+1} \right), \tilde{C}_{t} = g^{c} \left( \tilde{K}_{t}, z_{t}, s_{t} \right). \]

We describe our algorithm as follows.

1. We compute the detrended capital \( \tilde{K} \) in the deterministic steady state, assuming that the growth rate of TFP is constant and equal to the steady state level. The state space for \( \tilde{K} \) is set at \( [\tilde{K}_{\min}, \tilde{K}_{\max}] = [0.1 \tilde{K}_{ss}, 1.9 \tilde{K}_{ss}] \). The capital grid has \( n_{k} \) equidistant points on this interval. We set \( n_{k} = 100 \) in the numerical computation. Larger values of \( n_{k} \) do not lead to significantly different results. We use the procedure described in the Appendix of Rouwenhorst (1995) to discretize the innovation of productivity growth. We use 10 grid points for the innovation. The resulting grid and transition probability matrix are used for numerical integration with respect to \( \Delta a \).

2. Our goal is to compute \( \tilde{J} \left( \tilde{K}, z, s \right) \) on the grid \( [\tilde{K}_{\min}, \tilde{K}_{\max}] \times \{ l, h \} \times \{ l, h \} \). For each state \( \{ z, s \} \), we follow Kaltenbrunner and Lochstoer (2010) and approximate the value function \( \tilde{J} \left( \tilde{K}, z, s \right) \) using Chebyshev polynomials in \( \tilde{K} \). We start with an initial guess for \( \tilde{J} \left( \tilde{K}, l, l \right), \tilde{J} \left( \tilde{K}, l, h \right), \tilde{J} \left( \tilde{K}, h, l \right), \text{and } \tilde{J} \left( \tilde{K}, h, h \right) \).

3. We compute the conditional certainty equivalent \( R \left( \Gamma' \tilde{J} \left( \tilde{K}', z', s' \right) \right) \) using equation (3) given that the current state \( \left( \tilde{K}, z, s \right) \) is on each grid by using a nonlinear equation solver. The expectations are approximated by numerical integration given the current state \( \left( \tilde{K}, z, s \right) \).
We interpolate the next period value \( \tilde{J}(\tilde{K}', z', s') \) using Chebyshev approximation.

4. Given the current state \( (\tilde{K}, z, s) \), we use a numerical optimizer to solve the Bellman equation

\[
\tilde{J}(\tilde{K}, z, s) = \max_i \left[ (1 - \beta) \left( \tilde{K}^\alpha - \tilde{I} \right)^{1-1/\psi} + \beta R \left( \Gamma' \tilde{J}(\tilde{K}', z', s') \right)^{1-1/\psi} \right]^{1/1-1/\psi},
\]

subject to

\[
\begin{align*}
\Gamma' \tilde{K}' &= (1 - \delta) \tilde{K} + \phi \left( \frac{i}{\tilde{K}} \right) \tilde{K}, \\
\ln(\Gamma') &= \mu (z') + \sigma^2 (s') \epsilon', \epsilon' \sim N(0, 1).
\end{align*}
\]

We obtain the updated value function \( \tilde{J}^* \) on each grid in the state space after an iteration. The algorithm then returns to the previous step. The stopping rule is that the new value function and the old value function satisfies \( |\tilde{J}^* - \tilde{J}| / |\tilde{J}| < 10^{-12} \).

C  Impulse responses

We compute the impulse response functions in Figure 5 using the following procedure.

Step 1. Compute the steady state when the regime is in \((\mu_h, \sigma_l)\) forever. Formally, we solve the solution of the following equation by iteration:

\[
\tilde{K} = g_k \left( \tilde{K}, \mu_h, \sigma_l, \exp(\mu_h) \right).
\]

Step 2. Suppose that the economy is initially at the steady state, \( \tilde{K}_0 = \tilde{K} \). At time zero, there is an unexpected shift of growth volatility from \( \sigma_l \) to \( \sigma_h \). From time one on, the volatility shifts back from \( \sigma_h \) to \( \sigma_l \). Using the policy function, we compute recursively

\[
\begin{align*}
\tilde{K}_1 &= g_k \left( \tilde{K}_0, \mu_h, \sigma_h, \exp(\mu_h) \right), \\
\tilde{K}_2 &= g_k \left( \tilde{K}_1, \mu_h, \sigma_l, \exp(\mu_h) \right), \\
&\vdots \\
\tilde{K}_{t+1} &= g_k \left( \tilde{K}_t, \mu_h, \sigma_l, \exp(\mu_h) \right).
\end{align*}
\]

Step 3. Compute other equilibrium variables \( \tilde{I}_t \) and \( \tilde{C}_t \) using their policy functions.
Table 1: Parameter estimates of productivity growth

<table>
<thead>
<tr>
<th></th>
<th>$\mu_l$</th>
<th>$\mu_h$</th>
<th>$\sigma_l$</th>
<th>$\sigma_h$</th>
<th>$p_{ll}^l$</th>
<th>$p_{lh}^l$</th>
<th>$p_{ll}^h$</th>
<th>$p_{hh}^h$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>-0.227</td>
<td>0.479</td>
<td>0.548</td>
<td>0.933</td>
<td>0.781</td>
<td>0.919</td>
<td>0.979</td>
<td>0.919</td>
</tr>
<tr>
<td>t-values</td>
<td>(-0.887)</td>
<td>(3.698)</td>
<td>(8.524)</td>
<td>(7.289)</td>
<td>(7.464)</td>
<td>(11.205)</td>
<td>(27.007)</td>
<td>(17.893)</td>
</tr>
</tbody>
</table>

This table reports the maximum likelihood estimates of parameters in the Markov-switching model

$$\Delta a_t = \mu(z_t) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2(z_t)),$$

where $z_t$ and $s_t$ follow two independent Markov chains. $\mu_l$ and $\mu_h$ denote the expected growth rates, respectively, in the low mean and high mean states. $\sigma_l$ and $\sigma_h$ denote the standard deviations of the innovation shock, respectively, in the low volatility and high volatility states. $p_{ij}^l$ and $p_{ij}^h$ are the probabilities of transiting to next period’s state $j$ given the current period’s state $i$, for the two independent Markov chains respectively. The maximum likelihood estimates are obtained using the EM algorithm developed by Hamilton (1990). $t$-values are reported in parentheses. Data are quarterly total factor productivity growth rates, taken from Fernald (2012), and span the period 1956:Q1–2012:Q4.
Table 2: Return predictability by the conditional volatility of productivity growth: empirical results

<table>
<thead>
<tr>
<th>Panel A: AR(1)-GARCH(1,1) estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model: $\Delta a_t = \mu + \vartheta \Delta a_{t-1} + e_t, e_t \sim N(0, h_t^2)$</td>
</tr>
<tr>
<td>$h_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \theta h_{t-1}^2$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>0.003</td>
</tr>
<tr>
<td>(5.246)</td>
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<table>
<thead>
<tr>
<th>Panel B: Predictive regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
</tr>
<tr>
<td>Slope</td>
</tr>
<tr>
<td>Horizon</td>
</tr>
<tr>
<td>1Y</td>
</tr>
<tr>
<td></td>
</tr>
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</tr>
<tr>
<td>5Y</td>
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<td></td>
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</tbody>
</table>

This table reports return predictability results when the conditional volatility of productivity growth is used as a predictor. The conditional volatility of productivity growth is estimated by fitting an AR(1)-GARCH(1,1) model to the TFP growth data from 1956Q1 to 2012Q4. Panel A displays the estimates of the AR(1)-GARCH(1,1) model and $t$-values (in parentheses). Panel B reports the slope estimates and $R^2$s when either excess returns or equity returns are used as a predicted variable. The regression models are

$$r_{t\rightarrow t+j}^e = a + b \ln(h_t) + \varepsilon_{e,t},$$
$$r_{t\rightarrow t+j}^f - r_{t\rightarrow t+j}^f = a_{ex} + b_{ex} \ln(h_t) + \varepsilon_{ex,t},$$

where $j$ is the horizon of returns, $r_{t\rightarrow t+j}^e$ and $r_{t\rightarrow t+j}^f - r_{t\rightarrow t+j}^f$ are cumulative log returns and excess log returns from periods $t$ to $t+j$, respectively. The table presents results for $j = 1$ year, 2, 3, 4, and 5 years. The $t$-values (in parentheses) for the slope estimates are adjusted using Hansen and Hodrick (1980) standard errors.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
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</tr>
<tr>
<td>$\xi$</td>
<td>Adjustment cost parameter</td>
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<td>$a_1$</td>
<td>Normalization</td>
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<tr>
<td>$a_2$</td>
<td>Normalization</td>
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<td>$\beta$</td>
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<td>$\gamma$</td>
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<tr>
<td>$\psi$</td>
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<td>$\kappa$</td>
<td>Disappointment cutoff</td>
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<tr>
<td>$\eta$</td>
<td>Disappointment aversion</td>
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<td>$\omega$</td>
<td>Leverage parameter</td>
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<td>$n$</td>
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### Table 4: Unconditional moments

<table>
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<tr>
<th>Statistic</th>
<th>Benchmark</th>
<th>EZ1</th>
<th>EZ2</th>
<th>GDAC</th>
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<tr>
<td></td>
<td>β = 0.9945</td>
<td>β = 0.9945</td>
<td>β = 0.9943</td>
<td>σₜ = σₜ₀</td>
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<tr>
<td></td>
<td>γ = 10</td>
<td>γ = 10</td>
<td>γ = 31</td>
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<tr>
<td>U.S. data</td>
<td>κ = 0.989</td>
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<td></td>
</tr>
<tr>
<td>Year</td>
<td>1956–2012</td>
<td>η = 2.45</td>
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<tr>
<td>Panel A: Macroeconomic moments</td>
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<tr>
<td>σₐₜ (%)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>σₐᵢ (%)</td>
<td>4.54</td>
<td>4.20</td>
<td>4.20</td>
<td>4.20</td>
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<tr>
<td>σₐᵧ (%)</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
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</tr>
<tr>
<td>ρ(∆cₜ, ∆cₜ₊₁)</td>
<td>0.45</td>
<td>0.12</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>ρ(∆iₜ, ∆iₜ₊₁)</td>
<td>0.50</td>
<td>0.02</td>
<td>0.02</td>
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<tr>
<td>ρ(∆ytₜ, ∆ytₜ₊₁)</td>
<td>0.58</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>ρ(∆iₜ, ∆yₜ)</td>
<td>0.94</td>
<td>0.98</td>
<td>0.99</td>
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<tr>
<td>ρ(∆cₜ, ∆yₜ)</td>
<td>0.77</td>
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<td>Panel B: Vol of vol (macroeconomic quantities)</td>
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<tr>
<td>σ(σₐₜ) (%)</td>
<td>0.30</td>
<td>0.36</td>
<td>0.33</td>
<td>0.38</td>
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<tr>
<td>σ(σₐᵢ) (%)</td>
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<td>1.58</td>
<td>1.59</td>
<td>1.59</td>
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<tr>
<td>σ(σₐᵧ) (%)</td>
<td>0.62</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
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<tr>
<td>Panel C: Financial moments</td>
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<td></td>
</tr>
<tr>
<td>E[R_{f}^t] - 1 (%)</td>
<td>1.44</td>
<td>1.44</td>
<td>2.42</td>
<td>1.44</td>
</tr>
<tr>
<td>σ(R_{f}^t) (%)</td>
<td>1.07</td>
<td>0.20</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>E(R^t - R_{f}^t) (%)</td>
<td>5.40</td>
<td>5.50</td>
<td>0.02</td>
<td>5.50</td>
</tr>
<tr>
<td>σ(R^t - R_{f}^t) (%)</td>
<td>15.03</td>
<td>10.45</td>
<td>2.03</td>
<td>14.65</td>
</tr>
<tr>
<td>E(p - d)</td>
<td>3.49</td>
<td>2.94</td>
<td>4.53</td>
<td>3.11</td>
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<tr>
<td>σ(p - d)</td>
<td>0.39</td>
<td>0.50</td>
<td>1.36</td>
<td>0.63</td>
</tr>
<tr>
<td>σ(M)/E(M)</td>
<td>n.a.</td>
<td>0.82</td>
<td>0.15</td>
<td>0.62</td>
</tr>
</tbody>
</table>

This table reports unconditional moments on macroeconomic quantities and asset returns for the data and four different models: the benchmark model and models EZ1, EZ2 and GDAC. Other parameters are given in Tables 1 and 3. Empirical moments are computed using US quarterly data from 1956:Q1 to 2012:Q4. All means and standard deviations are expressed in annualized terms, except for moments of the variance risk premium, which are expressed in monthly percentage squared terms. All correlations and autocorrelations are in quarterly terms. A lower case variable x denotes its log value. σₐₓ denotes the standard deviation of growth rates of x (in log units). ρ(∆xₜ, ∆yₜ) denotes the contemporaneous correlation of growth rates of x and y. ρ(∆xₜ, ∆xₜ₊₁) denotes the first-order autocorrelation in the growth rate of x. σ(σₐₓ) measures the volatility of volatility in the growth rate of x. Panel C reports the unconditional means and standard deviations of the risk-free rate (R_{f}^t), excess returns (R^t - R_{f}^t), the log of price-dividend ratio (p - d), and the market price of risk (σ(M)/E(M)). The results are generated from 20,000 simulations under each model. Each simulation contains 328 periods of data (the first 100 data points are dropped).
Table 5: Conditional moments

<table>
<thead>
<tr>
<th></th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>Low mean</th>
<th>High mean</th>
<th>Low vol</th>
<th>High vol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\mu_l, \sigma_l$)</td>
<td>($\mu_l, \sigma_h$)</td>
<td>($\mu_h, \sigma_l$)</td>
<td>($\mu_h, \sigma_h$)</td>
<td>($\mu_l$)</td>
<td>($\mu_h$)</td>
<td>($\sigma_l$)</td>
<td>($\sigma_h$)</td>
</tr>
<tr>
<td>Panel A: Benchmark</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>0.87</td>
<td>1.31</td>
<td>0.84</td>
<td>1.28</td>
<td>1.03</td>
<td>1.00</td>
<td>0.85</td>
<td>1.29</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}$</td>
<td>3.35</td>
<td>5.47</td>
<td>3.28</td>
<td>5.39</td>
<td>4.10</td>
<td>4.03</td>
<td>3.30</td>
<td>5.41</td>
</tr>
<tr>
<td>$\mathbb{E}[R^f] - 1$</td>
<td>1.31</td>
<td>0.72</td>
<td>1.76</td>
<td>1.22</td>
<td>1.10</td>
<td>1.57</td>
<td>1.64</td>
<td>1.09</td>
</tr>
<tr>
<td>$\mathbb{E}(R^e - R^f)$</td>
<td>4.65</td>
<td>14.69</td>
<td>2.23</td>
<td>9.18</td>
<td>8.22</td>
<td>4.70</td>
<td>2.88</td>
<td>10.65</td>
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<tr>
<td>$\sigma(R^e - R^f)$</td>
<td>9.57</td>
<td>15.69</td>
<td>7.47</td>
<td>12.67</td>
<td>11.75</td>
<td>9.32</td>
<td>8.03</td>
<td>13.48</td>
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<tr>
<td>$\sigma(M)/\mathbb{E}(M)$</td>
<td>0.80</td>
<td>0.88</td>
<td>0.78</td>
<td>0.87</td>
<td>0.83</td>
<td>0.81</td>
<td>0.79</td>
<td>0.87</td>
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<tr>
<td>Panel B: EZ2</td>
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<tr>
<td>$\sigma_{\Delta c}$</td>
<td>0.87</td>
<td>1.30</td>
<td>0.85</td>
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<td>1.02</td>
<td>1.01</td>
<td>0.86</td>
<td>1.30</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}$</td>
<td>3.38</td>
<td>5.51</td>
<td>3.31</td>
<td>5.41</td>
<td>4.14</td>
<td>4.06</td>
<td>3.33</td>
<td>5.44</td>
</tr>
<tr>
<td>$\mathbb{E}[R^f] - 1$</td>
<td>1.35</td>
<td>0.84</td>
<td>1.74</td>
<td>1.24</td>
<td>1.17</td>
<td>1.56</td>
<td>1.64</td>
<td>1.13</td>
</tr>
<tr>
<td>$\mathbb{E}(R^e - R^f)$</td>
<td>5.11</td>
<td>17.49</td>
<td>0.67</td>
<td>11.07</td>
<td>9.52</td>
<td>4.37</td>
<td>1.85</td>
<td>12.78</td>
</tr>
<tr>
<td>$\sigma(R^e - R^f)$</td>
<td>13.62</td>
<td>21.92</td>
<td>10.35</td>
<td>17.83</td>
<td>16.57</td>
<td>13.01</td>
<td>11.22</td>
<td>18.92</td>
</tr>
<tr>
<td>$\sigma(M)/\mathbb{E}(M)$</td>
<td>0.60</td>
<td>0.64</td>
<td>0.61</td>
<td>0.64</td>
<td>0.61</td>
<td>0.62</td>
<td>0.61</td>
<td>0.64</td>
</tr>
<tr>
<td>Panel C: GDAC $\sigma_l = \sigma_h$</td>
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<tr>
<td>$\sigma_{\Delta c}$</td>
<td>0.98</td>
<td>0.94</td>
<td>0.98</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
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</tr>
<tr>
<td>$\sigma_{\Delta i}$</td>
<td>3.99</td>
<td>3.94</td>
<td>3.99</td>
<td>3.94</td>
<td>3.95</td>
<td>3.95</td>
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<tr>
<td>$\mathbb{E}[R^f] - 1$</td>
<td>1.04</td>
<td>1.55</td>
<td>1.04</td>
<td>1.55</td>
<td>1.41</td>
<td>1.41</td>
<td></td>
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</tr>
<tr>
<td>$\mathbb{E}(R^e - R^f)$</td>
<td>6.45</td>
<td>2.69</td>
<td>6.45</td>
<td>2.69</td>
<td>3.69</td>
<td>3.69</td>
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<tr>
<td>$\sigma(R^e - R^f)$</td>
<td>9.02</td>
<td>6.61</td>
<td>9.02</td>
<td>6.61</td>
<td>7.25</td>
<td>7.25</td>
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</tr>
<tr>
<td>$\sigma(M)/\mathbb{E}(M)$</td>
<td>0.82</td>
<td>0.81</td>
<td>0.82</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports conditional moments of consumption growth, investment growth, the risk-free rate, the excess return, and the conditional market price of risk for the benchmark model and models EZ2 and GDAC. Columns 2–5 present moments conditional on, respectively, state 1 ($\mu_l, \sigma_l$), state 2 ($\mu_l, \sigma_h$), state 3 ($\mu_h, \sigma_l$) and state 4 ($\mu_h, \sigma_h$) of productivity growth. Columns 6–9 present moments conditional on, respectively, the low mean, high mean, low volatility, and high volatility states of productivity growth. The results are based on 20,000 simulations, where 3579 simulations belong to state 1 ($\mu_l, \sigma_l$), 1807 simulations to state 2 ($\mu_l, \sigma_h$), 9686 simulations to state 3 ($\mu_h, \sigma_l$), and 4928 simulations to state 4 ($\mu_h, \sigma_h$).
This table reports the simulated average of the maximum likelihood estimates of the parameters in the Markov switching model for consumption growth $\Delta c_t = \mu(z_t^c) + \epsilon_{c,t}, \quad \epsilon_{c,t} \sim N(0, \sigma^2(s_t^c))$.

where $z_t^c$ and $s_t^c$ follow two independent Markov chains. The model has two states for the conditional mean and two states for the conditional volatility of consumption growth. $\mu_l^c$ ($\mu_h^c$) denotes the consumption growth rate in the low (high) mean state, and $\sigma_l^c$ ($\sigma_h^c$) denotes the standard deviation of the innovation in the low (high) volatility state. $p_{ij}^{\mu,c}$ and $p_{ij}^{\sigma,c}$ denote the probabilities of transiting to next period’s state $j$ given the current period’s state $i$, for the two independent Markov chains, respectively. The second column presents the estimates based on quarterly growth rates of real consumption (nondurable and services) for the period 1956:Q1–2012:Q4, with t–values reported in the brackets. We then simulate 20,000 samples of consumption growth data from the benchmark and EZ2 models, and apply the EM algorithm on each simulated sample. Each sample contains 228 periods of data. The average values of the estimates and 90% confidence intervals (in parentheses) are reported for the benchmark model and model EZ2, respectively, in the third and fourth columns.
Table 7: Return predictability by conditional volatility of productivity growth: simulation results

Panel A: AR(1)-GARCH(1,1) estimation

Model: \[ \Delta a_t = \mu_a + \vartheta \Delta a_{t-1} + e_t, \quad e_t \sim N(0, h_t^2) \]
\[ h_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \theta h_{t-1}^2 \]

<table>
<thead>
<tr>
<th>( \mu_a )</th>
<th>( \vartheta )</th>
<th>( \alpha_0 )</th>
<th>( \theta )</th>
<th>( \alpha_1 )</th>
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<tr>
<td>0.0027</td>
<td>0.117</td>
<td>0.000</td>
<td>0.668</td>
<td>0.113</td>
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<tr>
<td>5.0888</td>
<td>(1.618)</td>
<td>(1.215)</td>
<td>(7.043)</td>
<td>(1.678)</td>
</tr>
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</table>

Panel B: Predictive regressions: benchmark model

| Horizon | Excess return | | Equity return | |
|---------|---------------|----------|----------------|
|         | Slope | \( R^2 \) | Slope | \( R^2 \) |
| 1Y      | 0.133 | 0.042   | 0.124 | 0.038   |
|         | (1.686)|         | (1.566)|         |
| 2Y      | 0.235 | 0.071   | 0.219 | 0.064   |
|         | (2.001)|         | (1.869)|         |
| 3Y      | 0.311 | 0.090   | 0.289 | 0.082   |
|         | (2.218)|         | (2.081)|         |
| 4Y      | 0.366 | 0.102   | 0.341 | 0.093   |
|         | (2.328)|         | (2.193)|         |
| 5Y      | 0.404 | 0.110   | 0.376 | 0.101   |
|         | (2.366)|         | (2.239)|         |

Panel C: Predictive regressions: EZ2 model

| Horizon | Excess return | | Equity return | |
|---------|---------------|----------|----------------|
|         | Slope | \( R^2 \) | Slope | \( R^2 \) |
| 1Y      | 0.185 | 0.044   | 0.176 | 0.040   |
|         | (1.749)|         | (1.672)|         |
| 2Y      | 0.327 | 0.074   | 0.311 | 0.069   |
|         | (2.082)|         | (1.996)|         |
| 3Y      | 0.431 | 0.094   | 0.411 | 0.088   |
|         | (2.312)|         | (2.224)|         |
| 4Y      | 0.506 | 0.107   | 0.483 | 0.100   |
|         | (2.431)|         | (2.347)|         |
| 5Y      | 0.557 | 0.114   | 0.531 | 0.108   |
|         | (2.478)|         | (2.401)|         |

This table reports simulated return predictability results when the conditional volatility of productivity growth is used as a predictor. The estimates and \( t \)-values (in parentheses) are based on averaging over 20,000 simulated samples. Each sample contains 228 quarters of data. The conditional volatility of productivity growth is estimated by fitting an AR(1)-GARCH(1,1) model to the TFP growth data simulated from the benchmark and EZ2 models. Panel A displays the average estimates of the AR(1)-GARCH(1,1) model and \( t \)-values. Panel B displays the slope estimates and \( R^2 \)’s for excess returns and equity returns respectively used as a predicted variable. The \( t \)-values for the slope estimates are adjusted using Hansen and Hodrick (1980) standard errors.
<table>
<thead>
<tr>
<th>Horizon</th>
<th>Data Slope</th>
<th>Data $R^2$</th>
<th>Benchmark Slope</th>
<th>Benchmark $R^2$</th>
<th>EZ2 Slope</th>
<th>EZ2 $R^2$</th>
<th>GDAC Slope</th>
<th>GDAC $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: $I/K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>-15.490</td>
<td>0.067</td>
<td>-13.389</td>
<td>0.064</td>
<td>-19.437</td>
<td>0.069</td>
<td>-13.194</td>
<td>0.107</td>
</tr>
<tr>
<td>2 years</td>
<td>-31.044</td>
<td>0.137</td>
<td>-24.502</td>
<td>0.116</td>
<td>-35.030</td>
<td>0.123</td>
<td>-24.007</td>
<td>0.198</td>
</tr>
<tr>
<td>3 years</td>
<td>-52.313</td>
<td>0.256</td>
<td>-33.862</td>
<td>0.160</td>
<td>-47.691</td>
<td>0.166</td>
<td>-33.042</td>
<td>0.275</td>
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<tr>
<td>4 years</td>
<td>-69.120</td>
<td>0.340</td>
<td>-41.768</td>
<td>0.197</td>
<td>-57.980</td>
<td>0.200</td>
<td>-40.635</td>
<td>0.338</td>
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<tr>
<td>5 years</td>
<td>-84.158</td>
<td>0.407</td>
<td>-48.351</td>
<td>0.226</td>
<td>-66.191</td>
<td>0.226</td>
<td>-46.906</td>
<td>0.389</td>
</tr>
<tr>
<td>Panel B: $d − p$</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>0.104</td>
<td>0.056</td>
<td>0.026</td>
<td>0.026</td>
<td>0.035</td>
<td>0.034</td>
<td>0.015</td>
<td>0.024</td>
</tr>
<tr>
<td>2 years</td>
<td>0.189</td>
<td>0.105</td>
<td>0.046</td>
<td>0.047</td>
<td>0.060</td>
<td>0.059</td>
<td>0.026</td>
<td>0.045</td>
</tr>
<tr>
<td>3 years</td>
<td>0.239</td>
<td>0.131</td>
<td>0.063</td>
<td>0.065</td>
<td>0.080</td>
<td>0.079</td>
<td>0.035</td>
<td>0.063</td>
</tr>
<tr>
<td>4 years</td>
<td>0.257</td>
<td>0.139</td>
<td>0.077</td>
<td>0.081</td>
<td>0.095</td>
<td>0.093</td>
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<td>0.079</td>
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<tr>
<td>5 years</td>
<td>0.304</td>
<td>0.160</td>
<td>0.088</td>
<td>0.094</td>
<td>0.106</td>
<td>0.104</td>
<td>0.048</td>
<td>0.092</td>
</tr>
<tr>
<td>Panel C: $c − w$</td>
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<td></td>
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</tr>
<tr>
<td>1 year</td>
<td>2.729</td>
<td>0.071</td>
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<td>0.159</td>
<td>2.989</td>
<td>0.160</td>
<td>1.452</td>
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</tr>
<tr>
<td>2 years</td>
<td>4.955</td>
<td>0.125</td>
<td>3.704</td>
<td>0.277</td>
<td>5.260</td>
<td>0.277</td>
<td>2.594</td>
<td>0.221</td>
</tr>
<tr>
<td>3 years</td>
<td>6.931</td>
<td>0.181</td>
<td>4.972</td>
<td>0.365</td>
<td>7.013</td>
<td>0.363</td>
<td>3.537</td>
<td>0.301</td>
</tr>
<tr>
<td>4 years</td>
<td>8.421</td>
<td>0.238</td>
<td>5.976</td>
<td>0.429</td>
<td>8.368</td>
<td>0.425</td>
<td>4.327</td>
<td>0.367</td>
</tr>
<tr>
<td>5 years</td>
<td>9.138</td>
<td>0.227</td>
<td>6.759</td>
<td>0.476</td>
<td>9.395</td>
<td>0.469</td>
<td>4.979</td>
<td>0.420</td>
</tr>
</tbody>
</table>

This table reports predictive regression results when the investment rate ($I/K$), dividend yield ($d − p$), and consumption-wealth ratio ($c − w$) are used as a predictor, respectively. The results are for the data (1956:Q1–2012:Q4) and for the benchmark, EZ2 and GDAC models. We first simulate 20,000 samples and each sample contains 228 periods of data. For each sample we regress excess returns (in log terms) onto a predictor. We obtain the slope estimate and $R^2$ from fitting a univariate linear model for each predictor. The simulation results are computed by taking the average of OLS estimates over 20,000 artificial samples.
Table 9: Sensitivity analysis

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_{\Delta c}$</th>
<th>$\sigma_{\Delta i}$</th>
<th>$E[R_f^d] - 1$</th>
<th>$E(R^e - R_f^f)$</th>
<th>$\sigma(R^e - R_f^f)$</th>
<th>$\sigma(M)/E(M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1.00</td>
<td>4.20</td>
<td>1.44</td>
<td>5.50</td>
<td>10.45</td>
<td>0.82</td>
</tr>
<tr>
<td>GDAC1</td>
<td>1.00</td>
<td>4.10</td>
<td>1.30</td>
<td>5.17</td>
<td>8.98</td>
<td>0.81</td>
</tr>
<tr>
<td>GDAC2</td>
<td>0.90</td>
<td>4.02</td>
<td>1.73</td>
<td>0.27</td>
<td>4.50</td>
<td>0.75</td>
</tr>
<tr>
<td>$\eta = 1.50$</td>
<td>1.08</td>
<td>4.19</td>
<td>1.54</td>
<td>3.09</td>
<td>10.57</td>
<td>0.62</td>
</tr>
<tr>
<td>$\eta = 1.00$</td>
<td>1.08</td>
<td>4.15</td>
<td>1.72</td>
<td>0.82</td>
<td>9.08</td>
<td>0.49</td>
</tr>
<tr>
<td>$\eta = 0.67$</td>
<td>1.09</td>
<td>4.20</td>
<td>1.91</td>
<td>-0.10</td>
<td>9.02</td>
<td>0.39</td>
</tr>
<tr>
<td>$\eta = 0.43$</td>
<td>1.10</td>
<td>4.23</td>
<td>2.07</td>
<td>-0.50</td>
<td>7.90</td>
<td>0.31</td>
</tr>
<tr>
<td>$\kappa = 0.992$</td>
<td>0.98</td>
<td>4.18</td>
<td>1.37</td>
<td>6.05</td>
<td>9.83</td>
<td>0.83</td>
</tr>
<tr>
<td>$\kappa = 0.981$</td>
<td>1.13</td>
<td>4.21</td>
<td>1.37</td>
<td>4.24</td>
<td>10.00</td>
<td>0.75</td>
</tr>
<tr>
<td>$\kappa = 0.972$</td>
<td>1.28</td>
<td>4.32</td>
<td>1.65</td>
<td>1.55</td>
<td>10.50</td>
<td>0.59</td>
</tr>
<tr>
<td>$\gamma = 6$</td>
<td>1.02</td>
<td>4.28</td>
<td>1.47</td>
<td>3.46</td>
<td>8.45</td>
<td>0.74</td>
</tr>
<tr>
<td>$\gamma = 2$</td>
<td>1.04</td>
<td>4.20</td>
<td>1.65</td>
<td>2.24</td>
<td>5.10</td>
<td>0.66</td>
</tr>
<tr>
<td>$\psi = 1.5$</td>
<td>1.23</td>
<td>3.85</td>
<td>1.56</td>
<td>5.47</td>
<td>8.52</td>
<td>0.82</td>
</tr>
<tr>
<td>$\psi = 1.2$</td>
<td>1.39</td>
<td>3.62</td>
<td>1.65</td>
<td>5.29</td>
<td>7.13</td>
<td>0.82</td>
</tr>
<tr>
<td>$\psi = 0.9$</td>
<td>1.59</td>
<td>3.37</td>
<td>1.75</td>
<td>4.52</td>
<td>5.25</td>
<td>0.82</td>
</tr>
<tr>
<td>$\psi = 0.7$</td>
<td>1.75</td>
<td>3.21</td>
<td>1.82</td>
<td>3.06</td>
<td>4.38</td>
<td>0.83</td>
</tr>
<tr>
<td>$\xi = 5$</td>
<td>1.20</td>
<td>3.73</td>
<td>1.42</td>
<td>6.56</td>
<td>10.15</td>
<td>0.82</td>
</tr>
<tr>
<td>$\xi = 1.5$</td>
<td>1.76</td>
<td>2.66</td>
<td>1.26</td>
<td>9.95</td>
<td>13.23</td>
<td>0.83</td>
</tr>
</tbody>
</table>

This table reports results of sensitivity analyses by varying different parameters and model specifications. When varying a particular parameter or specification, we keep everything else fixed as in the benchmark model. Model GDAC1 assumes a constant expected growth rate, set at the steady-state level implied by the estimates of the Markov switching model in Table 3. Model GDAC2 assumes that both the expected growth rate and the volatility of productivity growth are constant at their respective steady-state levels. The rest rows present results of changing a particular model parameter. The moments for comparison include the unconditional standard deviations of consumption growth and investment growth ($\sigma_{\Delta c}$ and $\sigma_{\Delta i}$), average risk-free rate ($E[R_f^d] - 1$), average equity premium ($E(R^e - R_f^f)$) and equity volatility ($\sigma(R^e - R_f^f)$), and market price of risk ($\sigma(M)/E(M)$).
Table 10: Pricing a claim to stock market dividends

<table>
<thead>
<tr>
<th>Statistic</th>
<th>GDA*</th>
<th>EZ1*</th>
<th>EZ2*</th>
<th>GDAC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9948</td>
<td>0.9948</td>
<td>0.9945</td>
<td>$\sigma_h = \sigma_l$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>5</td>
<td>5</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.985</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Macroeconomic moments

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>$\eta = 2.33$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}$ (%)</td>
<td>1.00</td>
<td>1.12</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}$ (%)</td>
<td>4.54</td>
<td>4.16</td>
</tr>
<tr>
<td>$\sigma_{\Delta y}$ (%)</td>
<td>2.00</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Panel B: Financial moments

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[R_f^t] - 1$ (%)</td>
<td>1.44</td>
<td>1.48</td>
</tr>
<tr>
<td>$\sigma(R_f^t)$ (%)</td>
<td>1.07</td>
<td>0.21</td>
</tr>
<tr>
<td>$\mathbb{E}(R^e - R^f)$ (%)</td>
<td>5.40</td>
<td>5.57</td>
</tr>
<tr>
<td>$\sigma(R^e - R^f)$ (%)</td>
<td>15.03</td>
<td>12.18</td>
</tr>
<tr>
<td>$\mathbb{E}(p - d)$</td>
<td>3.49</td>
<td>2.99</td>
</tr>
<tr>
<td>$\sigma(p - d)$</td>
<td>0.39</td>
<td>0.15</td>
</tr>
<tr>
<td>$\sigma(M)/\mathbb{E}(M)$</td>
<td>n.a.</td>
<td>0.66</td>
</tr>
</tbody>
</table>

This table reports results when dividends are modeled as a levered claim of aggregate consumption. Specifically, dividends growth follows the process
\[
\Delta d_{t+1} \equiv \ln \left( \frac{D_{t+1}}{D_t} \right) = \lambda \Delta c_{t+1} + g_d + \sigma_d \varepsilon_{d,t+1}
\]
where $\varepsilon_{d,t+1}$ is IID normal and independent of all other shocks in the model, and $\lambda$ is the leverage ratio parameter and set at 2.75. The process is calibrated to match the volatility of the historical dividends growth data from 1956:Q1 to 2012:Q4. The table presents macroeconomic and financial moments for four models, GDA*, EZ1*, EZ2* and GDAC*, with their respective parameterizations given in the table. Other parameter choices are given in Tables 1 and 3.
Notes: This figure plots the filtered probabilities of the low expected growth states and high volatility states in the next period for the historical total factor productivity growth from 1956:Q1 to 2012:Q4. The filtered probabilities are obtained from the EM estimation of a four-state Markov-switching model.
Figure 2: VAR impulse responses

Notes: This figure plots the impulse response functions and the associated 90% confidence bands of several variables when there is a positive shock to the conditional volatility of productivity growth. The results are obtained from estimating a VAR model, where the state vector includes seven variables in the estimation order: $\ln(h_t)$ and the logs of dividends, investment, consumption, output, price-dividend ratios, and stock returns. Dividends, investment, consumption, and output are detrended.
Figure 3: Impulse responses: growth innovation shock

Notes: This figure plots the impulse response functions for the benchmark model when a positive innovation shock to productivity growth occurs in period 3. Suppose that the economy is initially in regime \((\mu_h, \sigma_l)\). The size of the shock is equal to \(2\sigma_l\).
Figure 4: **Impulse responses: mean regime switching**

Notes: This figure plots the impulse response functions for the benchmark model when the mean of TFP growth switches from $\mu_h$ to $\mu_l$ in period 3. Suppose that the economy is initially in regime $(\mu_h, \sigma_l)$. 

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Figure 5: Impulse responses: volatility regime switching

Notes: This figure plots the impulse response functions for the benchmark model when the conditional volatility of TFP growth switches from $\sigma_l$ to $\sigma_h$ in period 3. Suppose that the economy is initially in regime $(\mu_h, \sigma_l)$. 
Figure 6: Relation between financial variables and macroeconomic quantities

Notes: This figure plots the cross-correlograms between output (in log terms and applying one-sided Baxter and King (1999) filter) and excess returns, ln(P/D), and conditional volatility of excess returns, respectively, and between investment and the market-to-book ratio (both are in log terms and HP-filtered). The figure displays results for the data, the benchmark model, and models EZ2 and GDAC. The results for the models are based on averaging over 20,000 simulated samples, and each sample contains 228 quarters of data.
References


