Capital Flows Under Moral Hazard

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Abstract

This work analyzes a model with moral hazard and limited enforcement in a small open exchange economy. I find that when state contingent contracting is allowed adding the moral hazard friction improves the model’s predictions along several dimensions. First, it justifies why non-contingent debt is an optimal way to finance an emerging economy. Second, it explains the limited consumption risk-sharing and high, volatile and counter-cyclical interest rates. Third, it generates realistic crisis like dynamics in which capital inflows are brought to a halt and interest rates sky-rocket. The model also has a strong internal propagation mechanism.

Limited enforcement friction, alone or together with moral hazard has nearly no effect on the model’s performance.

Keywords: moral hazard, limited enforcement, optimal contract, emerging economy

1 Introduction

With access to complete international credit markets, a typical economy is able to diversify all of its country-specific risk. In states in which resources

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are scarce, the economy is able to borrow to finance a stable level of consumption and investment. However, emerging economies seem to be unable to share idiosyncratic risk. First, this is reflected in the almost perfect correlation between consumption and output and consumption being at least as volatile as output. Second, emerging economies face high, volatile and counter-cyclical interest rates. Third, emerging economies often experience sudden stops: a decline in output is accompanied by a sudden and sharp reduction of capital inflows. These three facts put emerging markets in stark contrast with small open developed economies like e.g. Canada. In this work I show how moral hazard that stems from lender’s inability to monitor use of funds could rationalize the above facts.

When markets are assumed to be exogenously incomplete, e.g. only non-contingent debt is traded, limited enforcement improves model’s performance along many dimensions. This line of research dates back to Eaton & Gersowitz (1981) that studies a deterministic economy subject to debt repudiation. Recently Arellano (2008) extends the analysis in Eaton & Gersowitz (1981) to a stochastic environment. Yue (2006) adds a possibility of renegotiation.¹

This work starts with complete markets, that trade fully state-contingent debt, and asks what friction could simultaneously justify observed market incompleteness and conform with the three facts defining an emerging economy. I compare the limited enforcement and moral hazard frictions but, unlike, in the previous studies without restricting the type of contracts that can be written between a creditor and a borrower. Limited enforcement performs poorly when state contingent contracting is allowed. It also does not change model predictions much once moral hazard friction is present. Moral hazard, on the other hand, brings the model in line with the three empirical facts.

Surprisingly little attention has been devoted to the problem of moral hazard in international credit markets.² Gertler & Rogoff (1990) study a two-period model of international lending with moral hazard. In their model

¹For a list of alternative modeling approaches see Arellano & Mendoza (2003).
²Researchers at the IMF studied the question whether its lending generates moral hazard. Rogoff (2002) gives a short overview of the arguments advanced and concludes that evidence on the presence of the moral hazard is mixed. The concept of moral hazard
a risk-neutral agent borrows to finance a risky investment project. Because investment is not observable, the creditor must provide incentives for investing by offering a spread (across states) in the borrower’s net worth next period. As the authors point out

Relative to the perfect-information benchmark, ... capital flows are dampened (and possibly reversed).

Investment is also lower relative to the frictionless economy and increases with the borrower’s net worth, as observed in the data.

Atkeson (1991) studies a model similar to Gertler & Rogoff (1990) that is a fully dynamic, infinite horizon model with a risk-averse borrower. Contracting between the borrower and the lender is restricted both because the lender cannot observe the use of funds by the borrower (moral hazard) and the borrower can renge on the promised repayment (limited enforcement). Atkeson (1991) shows that a binding participation constraint may trigger capital outflows in the lowest output state. In economies that are subject to the risk of repudiation, participation constraints usually bind in high output states. The presence of the moral hazard could reverse this result. Whether this can occur under a reasonable model parametrization is an open question.

I calibrate the model with moral hazard and limited enforcement to Argentina’s business cycle data. I find that the moral hazard friction severely limits the degree of state contingency built into the optimal contract. This result provides a justification for why non-contingent debt is an optimal way of financing an emerging economy. The limited enforcement friction, on the other hand, adds little to the model performance. The moral hazard friction also generates high, volatile and counter-cyclical interest rates.

The moral hazard problem generates a strong internal propagation mechanism. The shocks that the economy faces are i.i.d. However, due to moral hazard investment often deviates and significantly so from the unconstrained optimal level. Large swings in investment lead to large and persistent deviations in output. A scenario that is supposed to mimic a crisis is 5.17 times more likely under the moral hazard friction than in the frictionless economy. In such a scenario the borrower gradually accumulates debt. Current account in this paper differs from that of the IMF.
and debt initially increase gradually and then jump suddenly. Capital inflows stop and interest rates sky-rocket. Yet, interest rates are slow to react to a worsening financial state.

Notably, the model parametrization does not rely heavily on a differences in time preference between a creditor and a borrower. This implies that the model gives an economy a good chance to climb out of the debt. In this way this model could explain both less and more successful emerging economies.

The model is however unable to replicate current account data. In the data current account is strongly countercyclical and very volatile. In the model it is neither. I explain that no model for an exchange economy cannot simultaneously fit the observed consumption and trade balance data. The model in ? is much richer, features default and generates an endogenous drop in output at the time of default. My work on the other hand emphasizes the friction that was not investigated systematically before. Albeit this is done in a very stylized model. As such I view the mechanism explained here as complementary rather than competing with other models of emerging economies.

2 Empirical Facts

In this work I analyze Argentina over the period from 1993:Q1 to 2005:Q4 as a representative emerging market economy. For comparison I choose Canada as a quintessential developed small economy.3

Figure 2 shows selected macroeconomic series for Argentina and Canada over the chosen sample period. Vertical dashed lines on each plot in the left column mark Argentina’s default in December 2001. Around this period, output and private consumption plummeted (see panel a) – just in one year, from 2001:Q1 to 2001:Q4, real output per capita decreased by 11.6%. During the entire period, consumption closely followed output and declined by 13.3%, exemplifying Argentina’s inability to diversify country risk.

Fact 1. Emerging economies are unable to diversify idiosyncratic risk.

At the same time the EMBI\textsuperscript{4} spread over the USD LIBOR, the premium

\textsuperscript{4}This is the emerging markets’ bond index computed by JP Morgan. It is considered
at which Argentina could borrow, surged from 18 to 29 percent at the time of default and subsequently reached levels above 60%. These extraordinary spread levels effectively cut off Argentina from world credit market. Interest rates inside of Argentina were also high. The spread between rates on USD (fixed rate) loans to private non-financial institutions, as a measure of the cost of funds in Argentina, and USD LIBOR also increased significantly at the time of default and reached 33%. These loan spreads are also highly correlated with the EMBI spreads, as has been noted by Perri & Neumeyer (2005). For the period 1993:Q1-2001:Q4 correlation between these series is 0.96. In view of such a strong correlation I will later use bond spreads constructed Perri & Neumeyer (2005) instead of unavailable loan spreads before 1993.

Fact 2. Interest rates are strongly counter-cyclical.

Up to the forth quarter of 2001 Argentina had also been accumulating debt by running trade balance deficits. However, in 2001:Q4, the trade balance reversed to positive values. For the pre-default period, the correlation between detrended log real GDP/capita and the trade balance to GDP ratio is -0.81 (see Table 1). This means that Argentina on average experienced capital outflows (negative trade balance) when its output was high, and capital inflows when output was low.

Fact 3. After a sequence of negative output realizations capital inflows either stop or reverse.

The above stated facts are in stark contrast with observations on small open developed economies. Table 1 summarizes important data moments for Argentina and compares them with the corresponding moments for Canada. In the table, $r$ denotes the spread between the domestic USD loan rate and the USD LIBOR.

Technology Shocks? Aguiar & Gopinath (2007) argue that volatile business cycles in many emerging economies are caused by a volatile TFP trend. Garcia-Cicco, Pancrazi & Uribe (2010) estimate a real business cycle (RBC) model allowing for both permanent and transitory productivity shocks using one of the major indicators of a country’s riskiness.
Table 1: Data moments

<table>
<thead>
<tr>
<th>Country</th>
<th>$\frac{\sigma(c)}{\sigma(y)}$</th>
<th>$\rho(c,y)$</th>
<th>$\mu(r)$</th>
<th>$\sigma(r)$</th>
<th>$\rho(r,y)$</th>
<th>$\rho(tb,y)$</th>
<th>$\rho(tb,r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina 93:1–01:4</td>
<td>1.11</td>
<td>0.97</td>
<td>8.18</td>
<td>4.73</td>
<td>-0.58</td>
<td>-0.81</td>
<td>0.65</td>
</tr>
<tr>
<td>Argentina 93:1–05:4</td>
<td>1.15</td>
<td>0.99</td>
<td>7.86</td>
<td>4.78</td>
<td>-0.68</td>
<td>-0.82</td>
<td>0.30</td>
</tr>
<tr>
<td>Canada 93:1–01:4</td>
<td>0.55</td>
<td>0.62</td>
<td>1.51</td>
<td>0.33</td>
<td>0.23</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>Frictionless credit mkt</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\approx$ 1</td>
<td>0</td>
</tr>
<tr>
<td>Argentina 1980–2005</td>
<td>n.a.</td>
<td>0.95</td>
<td>11.00</td>
<td>4.89</td>
<td>-0.80</td>
<td>-0.77</td>
<td>0.89</td>
</tr>
</tbody>
</table>

1 Output and consumption are filtered with Hodrick-Prescott filter.
2 Output, consumption and trade balance-to-GDP ratio are seasonally adjusted.

a long data sample for Argentina and Mexico. They find that an RBC model driven by technology shocks alone cannot explain business cycle statistics in the two emerging economies.

It the beginning of the 2001 crisis in Argentina capital utilization was 48.2%. Assuming that the pre-crises utilization level at 65% (pre-crisis level observed in mid-2004), fluctuations in capital utilization alone can justify large swings in output.\(^5\) This indicates at capital market imperfections rather than productivity as a cause of volatile business cycles in emerging economies.

3 Economic Environment

Time is discrete and the planning horizon is infinite, $t \geq 0$. There is one perishable good at each date. There is one infinitely lived borrower. The borrower at each date receives a stochastic endowment and decides how to split available resources between consumption and investment. The borrower ranks alternative consumption streams according to

$$U^b = E_0 \sum_{t=1}^{\infty} \beta^t u(c_t), \quad \beta \in (0,1), u'(c) > 0, u''(c) < 0.$$  \hspace{1cm} (1)

The borrower starts period 0 with a given net worth $n_0$ which is a claim to $n_0$ units of date-0 good. Endowment at all consecutive dates depends

\(^5\)Gertler, Gilchrist & Natalucci (2003) also find that capacity utilization, as measured by electricity use, can explain large movements in the measured productivity during the Korean crisis.
on the previous period’s investment, $I$. Having invested $I$ units of date-$t$ good, the borrower’s endowment at date $t+1$ is drawn from a distribution $g(\cdot|I)$. It is assumed that, for all investment levels, the distribution of the next period’s endowment has a fixed and finite support $Y = \{Y_1, \ldots, Y_s\}$ with $Y_1 < \ldots < Y_s$. Following Atkeson (1991), $g$ is specified in the following way:

$$
\begin{align*}
g(Y_j|I) &= (1 - \lambda(I))g_{0j} + \lambda(I)g_{1j},
\end{align*}
$$

where $\lambda : R_+ \longrightarrow [0, 1]$ is an increasing and strictly concave function. Densities $g_0 > 0$ and $g_1 > 0$ are exogenously given and satisfy:

\begin{itemize}
  \item Assumption A1. $g_0$ and $g_1$ satisfy monotone likelihood ratio condition.
  \item Assumption A2. $\lim_{I \to 0+} \lambda'(I) = \infty$.
\end{itemize}

Assumption A2 implies that production technology satisfies the Inada condition at zero and that optimal investment is always strictly positive. This enable me to use the relaxed first-order approach to the incentives problem in section 4.

The international credit market is represented by a sequence of overlapping generations of lenders each living for two periods. A lender living in periods $t$ and $t+1$ ranks alternative consumption streams according to:

$$
U_t^c = c_t + \beta_c E_t c_{t+1}, \quad \beta_c \in (0, 1].
$$

Lenders receive endowment, $M \gg 0$, in each period that they live and can borrow and lend at the risk-free rate of $1/\beta_c$. Presence of $M$ emphasizes that

\begin{itemize}
  \item Distributions $g_0$ and $g_1$ are said to satisfy monotone likelihood ratio property if $g_{0j}/g_{1j}$ is increasing in $j$.
  \item Because $\lambda$ is a concave function, production technology is also concave: expected output tomorrow is a concave function of the working capital $I$.
\end{itemize}
the international credit market in not unlimited.\footnote{If the borrower were allowed to default, it would be possible to endogenize \( M \). I do not take this route in order to keep the model simple. In what follows I will make sure that these bounds are sufficiently large.} Thus, the amount lent cannot exceed the lender’s endowment:

\[ b \leq M. \]  

The borrower with net worth \( n \) faces the following budget constraint:

\[ n + b = c + \theta I, \]

where \( b \) is the loan amount provided by a creditor, \( i \) is investment and \( \theta > 0 \) represents cost of investment.

**Assumption A3.** \( \beta \leq \beta_c \).

This assumption reflects the fact that the government of an emerging economy may have a shorter planning horizon than a typical international lender.\footnote{It also serves a technical purpose by reducing the maximal net worth that can be attained by the borrower.}

**Remark about production.** The production process in this model is represented in a non-standard way. This is needed to facilitate the analysis. Yet, this specification includes the standard production function as a special case.\footnote{First, note that it is a concave production function in the sense that expected output is a concave function of investment:}

\[
\frac{d^2}{dt^2} \left( \sum_{j=1}^{s} g_j(I)Y_j \right) = \lambda''(I) \sum_{j=1}^{s} \Delta g_j Y_j < 0.
\]

Next, consider the example in which log of output \( y_j = \log(Y_j) \) is distributed according to \( (1 - \lambda(I))\Phi(y_j|0, 1) + \lambda(I)\Phi(y_j|1, 1) = \Phi(y_j|\lambda(I), 1) \).\footnote{Assuming \( \lambda(I) = \alpha \cdot \log(I) \), the output process can be also represented as \( Y_j = z_j I^\alpha \) where \( \log(z_j) \) is a standard normal random variable. In view of the above current specification includes as a special case the model in Clementi & Hopenhayn (2006) albeit with one-period capital.} Assuming \( \lambda(I) = \alpha \cdot \log(I) \), the output process can be also represented as \( Y_j = z_j I^\alpha \) where \( \log(z_j) \) is a standard normal random variable. In view of the above current specification includes as a special case the model in Clementi & Hopenhayn (2006) albeit with one-period capital.
could be the effort by the Argentina’s government to reform its public sector. Reforms, as represented by higher $I$, could have a beneficial effect on the economy in the future but the cost, in the form of reduced consumption, has to be paid now.

Had I assumed the one period utility of the form $u(c_t) - \theta I_t$, we would obtain a model in Clementi, Cooley & Di Giannatale (2010) albeit with one-period capital capital. This specification while simplifying the analysis considerably, however, is not suitable here because it predicts that wealthier economies invest less.\footnote{This becomes apparent from the borrower’s individual rationality condition (12) introduced later.}

\section*{3.1 Contracts and frictions}

Each period, the borrower signs a contract $C$ with the creditor. In a frictionless economy (in which the borrower’s investment is observed and contracts are enforceable) $C$ specifies 1) the loan amount $b$ provided by the creditor, 2) the repayment schedule $(d_1, ..., d_n)$, where $d_j$ is the amount to be repaid by the borrower in state $i$, and 3) the investment, $I$, made by borrower. I analyze the unconstrained efficient contract in section (3.2.1).

The first friction that I study is moral hazard. Output is observable which means that repayment can be made contingent on output realization. However, the lender is able to observe neither investment nor consumption. This induces moral hazard and the lender can no longer fully insure the borrower as the latter would then optimally invest nothing. Formally, private information restriction implies that investment, rather than part of the explicit terms of the contract, is part of the borrower’s optimal response to the creditor’s contract $(b, d)$.

The second friction that I study is limited enforcement. It is assumed that contracts with a sovereign nation cannot be enforced. If a borrower decides to renege on a contract, he is forever excluded from international credit market. Thus a contract has to be designed so that the borrower always (weakly) prefers participation in the international credit market to financial autarky. Formally, a limited enforcement restriction imposes an
upper bound on repayments that the creditor can obtain from the borrower.

In what follows I study recursive economies. For the economy with moral hazard and enforcement constraints the existence of a recursive representation is shown by Atkeson (1991) in proposition 5 (page 1083). The existence of a recursive representation for the economy with moral hazard only follows directly.

3.2 Two benchmarks
3.2.1 Frictionless Credit Market

In this section I analyze the case when a borrower has access to a frictionless credit market. The borrower’s actions are observable and can be contracted upon. There is no enforcement problem on either side of the market. The value function defined in this section is largest value that can be achieved in this environment.\(^\text{13}\)

Let \(d_j\) be the quantity of one period Arrow securities that pay out in state \(i\) issued by the borrower and let \(q(Y_j|I)\) be the price of such Arrow securities given last period’s investment \(I\). Because the lender is risk-neutral, Arrow securities are traded at

\[ q(Y_j|I) = \beta g(Y_j|I). \tag{4} \]

Let \(V_{ad}(n)\) be the optimal value to a borrower with net worth \(n > -M\).

The value function \(V_{ad}(n)\) satisfies the following Bellman equation:

\[ V_{ad}(n) = \max_{I, b, d} \left\{ u(n + b - \theta I) + \beta \sum_{j=1}^{s} g(Y_j|I)V_{ad}(Y_j - d_j) \right\} \tag{5} \]

subject to lender’s individual rationality condition\(^\text{14}\)

\[ b \leq \beta c \sum_{j=1}^{s} g(Y_j|I)d_j, \tag{6} \]

\(^{13}\)For a numerical solution it is important to start iterations from the largest possible value function. otherwise iterations may converge to the value of financial autarky defined in the next section.

\(^{14}\)According to this condition the lender’s gross rate of return is no less than \(1/\beta\).
where \( b \) is the amount of good that the borrower receives in exchange for his promise, \( \{d_j\} \), to deliver goods tomorrow. Loan amount \( b \) must also satisfy equation (3): \( b \leq M \). If this constraint were absent then a borrower would always invest the optimal amount \( I^* \): \(^{15}\)

\[
\theta = \beta \lambda^*(I^*) \sum_{j=1}^{s} \Delta g_j Y_j. 
\]  

Let \( E^* \) denote the expectation with respect to the unconstrained optimal distribution of output \( g(Y_j|I^*) \). Denote the lowest sustainable net worth level by \( n_c \equiv E^*(Y_j) - M/\beta_c \). Then the optimal contract can be summarized as follows.

1. If \( n \in [-M, n_c) \) then the agent is borrowing constrained. The borrower receives maximal loan amount \( M \) and agrees to a repayment schedule that delivers state non-contingent net worth \( n' \in [n_0, n_1] \) next period: \( d_j = Y_j - n' \). Investment is increasing in the borrower’s net worth in this region.

2. If \( n \in [n_c, \infty) \) then the borrower receives a contract under which the borrower’s net worth is constant across states and time: \( d_j = Y_j - n' \), \( b = \beta(E^*(Y_j) - n') \) for some \( n' \in [n_1, \infty) \). Investment is equal to the unconditionally optimal level \( I^* \).

Figure 2 summarizes the above description of the frictionless equilibrium. Note that irrespective of the initial level of net worth the borrower eventually ends up with \( n \in [n_c, \infty) \), which corresponds to the region 2 described above. Thus, net worth, consumption and investment are eventually constant, both across time and states. Current account (capital outflow if positive) each period is \( ca_t \equiv d(Y_t, n_{t-1}) - b(n_t) = Y_t - n_t - b(n_t) \). Since transition from \( n_{t-1} \) to \( n_t \) does not depend on output realization \( Y_t \), the correlation between the ratio of current account balance to output and output equals 1.

\(^{15}\)\( I^* \) is also the level of investment that would be chosen by the lender if he or she were allowed to operate the investment technology directly.
3.3 Financial autarky

Here I assume that a borrower has no access to the international credit market (financial autarky) and the investment technology is the only means available for intertemporal consumption smoothing. Let $V_{aut}(n)$ be the optimal value to a borrower with $n \geq 0$ units of good living in autarky. This value function is the lower bound on the values achieved in other environments. It satisfies the following Bellman equation:

$$V_{aut}(n) = \max_{I \in [0,n]} \left\{ u(n - \theta I) + \beta \sum_{j=1}^{s} g(Y_j | I) V_{aut}(Y_j) \right\}$$

The optimal investment made by the agent is an increasing function of $n$.

Next consider the environment in which the borrower upon entering a financial autarky loses a fraction $(1 - \delta) \in [0, 1)$ of output. Parameter $\delta$ is a “perceived” cost of default. The optimal value to the borrower then is:

$$V_{aut}^\delta(Y_j) = \max_{I \in [0,Y_j]} u(\delta Y_j - \theta I) + \beta \sum_{k=1}^{s} V_{aut}(Y_k - d_k).$$

In what follows I assume that the borrower entering the autarkic regime must suffer an output loss.
4 Moral hazard and limited enforcement

In this section I assume that contracting between the borrower and the lender is restricted both because the lender cannot observe the use of funds by the borrower (moral hazard) and because the borrower can renege on the promised repayment (limited enforcement). Thus, the creditor cannot perfectly insure the borrower as then no investment will be made. In addition, the creditor has a limited choice of repayment schedules as a large repayment may induce a default.

Atkeson (1991) shows that in this environment the optimal allocation can be implemented by a sequence of one-period contracts that depend on the borrower’s net worth. Let \( V(n) \) be the optimal value to the borrower with net worth \( n \) under the optimal contract. The value function \( V(n) \) satisfies the following Bellman equation:

\[
V(n) = \max_{b, d} \left\{ u(n + b - \theta I) + \beta \sum_{j=1}^{s} g(Y_j | I) V(Y_j - d_j) \right\}
\]

subject to

\[
b \leq \beta \sum_{j=1}^{s} g(Y_j | I) d_j \tag{11}
\]

\[
I \in \arg \max_{\hat{I} \in [0, n+b]} \left\{ u(n + b - \theta \hat{I}) + \beta \sum_{j=1}^{s} g(Y_j | \hat{I}) V(Y_j - d_j) \right\} \tag{12}
\]

\[
V_{aut}(Y_j) \leq V(Y_j - d_j), \quad \forall i \in S. \tag{13}
\]

According to the constraint (11) the loan amount cannot exceed the present discounted value of the repayments. The constraint (12) is the borrower’s incentive compatibility constraint. It states that given the contract \((b, d)\) the borrower chooses investment optimally. The constraints (13) are the borrower’s participation constraints. They state that the borrower in every state weakly prefers honoring his debt obligations to exclusion from the credit market (autarky). Note that the participation constraints now do the work of the restriction \( b \leq M \) by setting the lower bound on sustainable levels of net worth.
4.1 First-Order Approach

Constraint (12) is not tractable as it is. Replacing it with the first-order condition is a viable alternative. But, first, it has to be shown that the first-order approach is valid in this setting. Following Rogerson (1985) I replace constraint (12) with the following inequality:

\[-u'(n+b - \theta I) + \beta \lambda'(I) \sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) \geq 0. \tag{14}\]

I need to show that replacing (12) with (14) in the maximization problem (10) does not affect the solution. For this I need to prove that for any feasible contract \((b, d)\) the following two conditions are satisfied:

\[-u'(n+b - \theta I) + \beta \lambda'(I) \sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) = 0, \quad \theta^2 u''(n+b - \theta I) + \beta \lambda''(I) \sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) < 0.\]

The second inequality implies that the borrower’s objective is strictly concave in investment. Given that \(u\) and \(\lambda\) are strictly concave a sufficient condition for the second inequality is \(\sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) > 0\).

**Lemma 1.** Replacing constraint (12) with the relaxed first-order condition (14) does not change the solution to the problem (10).

**Proof.** Fix a contract \((b, d)\). Because \(\lim_{I \downarrow 0} \lambda'(I) = \infty\), the contract for which \(\sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) \leq 0\) holds is not optimal because it implies zero investment. In the relaxed problem such a contract is simply rendered infeasible because it would violate (14).

When \(\sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) > 0\) then the borrower’s objective must be strictly concave because \(u''(n+b - \theta I) < 0\). Hence, the first order condition (14) must hold with equality. \(\Box\)

\(^{16}\)Note that availability of the first-order approach also simplifies computation of the optimal contract.
4.2 Properties of optimal contract

As shown in the appendix A, the optimal contract must satisfy the following optimality condition

\[ V'(n) = \frac{\beta}{\beta_c} V'(Y_j - d_j) \left( 1 + \mu \frac{X(I) \Delta g_j}{g(Y_j|I)} + \gamma_j \right), \]  

(15)

where \( \mu \geq 0 \) is the Lagrange multiplier associated with the borrower’s incentives constraint and \( \gamma_j \geq 0 \) is the scaled Lagrange multiplier associated with limited enforcement constraint for state \( j \).

I now explain how different frictions affect the economy’s dynamics. To start assume that \( b = 0 \) and that \( M = \infty \). Under perfect contract enforcement, \( \gamma_j = 0, \forall j \), the Euler equation (15) implies the following relation:

\[ V'(n) > E[V'(Y_j - d_j)]. \]  

(16)

So, \( V'(n) \) is a super-martingale. Given concavity of the value function, this means that moral hazard problem forces the borrower’s net worth to decline on average. Yet, next period net worth \( n'_j = Y_j - d_j \) increases when \( \Delta g_j > 0 \) and declines otherwise. This is how a creditor provides incentives for investment: by spreading the continuation value of the borrower across future states a creditor unloads part of the risk to the borrower.

However, incentive provision is costly – it reduces expected continuation value. The latter must be compensated by an increase in current consumption. Thus, moral hazard problem leads to excessive consumption and immiseration that is also present in the private information economies of Atkeson & Lucas (1992) and Thomas & Worrall (1990) among others.

Figure 3 plots the Lagrange multipliers for the parameter values assumed in table 2. The Lagrange multiplier \( \mu \) on the borrower’s individual rationality constraint (12), shown in panel A, is decreasing on the set of net worth levels where the endogenous borrowing limits, shown in panel B, do not bind. When the borrower is wealthier and can afford higher investment the cost of incentives provision is lower. This multiplier collapses to zero for low values of net worth. The reason is that limited enforcement alone pressures a creditor to spread the continuation values beyond what is needed to mitigate the moral hazard problem.
Figure 3: Lagrange multiplier on the incentives constraint

Note that the Lagrange multiplier on the endogenous borrowing constraint for state 2 binds on a larger set than for state 1. That is the creditor would rather increase payment in the high output state than reduce it in the low output state. That is like in the model with limited enforcement alone the borrowing limit for the high output state is strongest. Moral hazard, unfortunately, does not reverse this pattern.\footnote{The condition in Atkeson (1991), lemma 1 does not hold and so capital outflows in the low output state is not possible.}

Differential discounting $\beta < \beta_c$ encourages benefit front-loading. That is a creditor rewards the borrower with current consumption rather than by promising high utility/consumption in the future. This effect is similar to that of moral hazard. Even small differences in discounting significantly speed up convergence to a stationary distribution of net worth. Yet, because of the moral hazard problem and equation (16) existence of a stationary distribution of net worth is guaranteed even with equal discounting.

The effect of limited enforcement is to constrain how much net worth can decline. Because $V$ is strictly increasing, equation (13) can be rewritten as

$$d_j \leq \bar{d}_j \equiv Y_j - V^{-1}(V_{\text{aut}}(Y_j)).$$

Because $V(n) \geq V_{\text{aut}}(n), \forall n$ we have $\bar{d}_j \geq 0$. That is there will always be some borrowing and lending. I what follows I will refer to the restriction (17)
5 Quantitative Analysis

5.1 Parameterization

I assume that one period corresponds to a quarter. The one period utility function is \( u(c) = c^{1-\gamma}/(1 - \gamma) \) with \( \gamma = 2 \). The creditor’s discount factor \( \beta_c \) is set to 0.99. I choose the borrower’s discount factor to be 0.985 that is very close to the discount factor of a creditor. Even a small difference in discounting support significantly reduces the equilibrium support of the borrower’s net worth \( n \). The difference that is assumed here is negligible relative to what is assumed in other studies cited in this paper. This is significant because when the creditor’s discount factor is much higher than that of the borrower the latter spends most of the time next to the borrowing limit. That is the model is set so that the borrower can never reduce the debt to “manageable” levels in the unlikely event that the highest output realizes ad infinitum.

Output process is governed by parameters \( \theta, \lambda, g_0, g_1 \). I assume only two states and the following distribution:

\[
\text{prob}(Y_1|I) = 1 - \lambda(I), \quad \text{prob}(Y_2|I) = \lambda(I).
\]

The closer is \( \lambda \) to a linear function the more volatile is investment. So, ideally \( \lambda \) would be a linear function. However, in this case optimal investment can be a corner solution invalidating the first order approach. So, I assume the following functional form:

\[
\lambda(I) = \min(I^{0.5}, 1). \tag{19}
\]

Parameter \( \theta \) is set to match the desired expected level of output that I normalize to 1. States \( Y_1 \) and \( Y_2 \) are chosen to match the desired level of income volatility. Endogeneity of the income process requires using the simulated method of moments to compute model parameters.

\(^{18}\)Alvarez and Jermann(2002) refer to the same restriction as the endogenous solvency constraint.
Parameters $\delta$ and $M$ are set to match the average debt to output ratio respectively in the full model and the model with moral hazard only. The assigned values are 0.812 and 0.468 respectively. The value of $\delta$ is somewhat low but not unreasonable.

All the assumed parameter values are gathered in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Data moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_c$</td>
<td>0.990</td>
<td>World interest rate = 0.040</td>
</tr>
<tr>
<td>$M$</td>
<td>0.468</td>
<td>Average debt/output ratio = 0.410</td>
</tr>
<tr>
<td>$Y_2, Y_1$</td>
<td>1±0.055</td>
<td>Output volatility = 0.054</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.102</td>
<td>Average output level = 1.000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.812</td>
<td>Average debt/output ratio = 0.410</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.000</td>
<td>–</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.985</td>
<td>–</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.900</td>
<td>–</td>
</tr>
</tbody>
</table>

### 5.2 Model solution

I compute the solution to the model by solving the Bellman equation iteratively. Value function is approximated by a cubic spline on the interval $\mathcal{N} = [0.2, 1.2]$ with 100 nodes. Stopping criterion is: $\sup_{n \in \mathcal{N}} |V^{k+1}(n) - V^k(n)| < 10^{-5}$. Further computational details are provided in the appendix B.

Model solution is presented in figure 4. The solid line in panel A represents the optimal value function $V(n)$. The upper dotted line is the value function for the frictionless economy $V_{ad}(n)$ and the lower dotted line is the value function for the financial autarky $V_{aut}(n)$. The fact that the value is closer to $V_{ad}$ suggests a high degree of insurance provided by the international credit market. It could be either intertemporal insurance or interstate insurance implicit in the contract.

---

19In the model with moral hazard only I impose the constant $b \leq M$ while in the full model I drop this constraint.
Panel B plots optimal loan $b(n)$ and repayment schedule $\{d_1(n), d_2(n)\}$. All three lines are extremely close: nearly all the risk is assumed by the risk-averse borrower. That is the insurance comes in the form of access to borrowing.

Panel C plots the optimal weight on the high output outcome $\lambda(I(n)) = [I(n)]^{0.9}$. For high levels of net worth investment is more affordable and slightly exceeds\(^{20}\) the unconstrained optimal level $I^*$ denoted by the dotted

\(^{20}\)It is logical to expect that the optimal investment never exceeds the unconstrained optimal investment. But when the borrower is less patient it is less costly to provide incentives for investment. Yet the unconstrained optimal level of investment remains unchanged.
Panel D plots histogram of the country’s net worth. The lowest net worth corresponds to a debt of 49.9% of output. At the highest level of net worth the country’s debt equals 21.1% of output. The whole distribution is skewed towards the endogenous lower limit of net worth. Near the lower bound, $n \leq 0.54$, investment is very sensitive to changes in the net worth. The weight on the high output outcome fluctuates between 0.30 and 0.60. The economy spends 39% of the time in this “high sensitivity” region.\(^{21}\)

\subsection*{5.3 Interest rate spread}

One of the main objects of interest in this work is an interest rate spread. I propose the following reinterpretation of the current setup. Consider a government-borrower that instead of a contract faces an interest rate schedule $R(n)$. That is the government of the economy with net worth $n$ promises to repay $R(n)$ for each unit borrowed. The government “taxes away” the excess return (capital outflows) from the risk-neutral creditors to finance future consumption and stimulate investment. The implied interest rate schedule

\[^{21}\text{By assuming a larger discount factor differential it is possible to force the economy completely into the “high-sensitivity” region. However, such “behavioral” parameterizations are not realistic.}\]
is:

\[ R(n) = \frac{u'(c(n))}{\beta \sum_j g_j(I(n)) u'(c(Y_j - d_j(n)))}, \]  

(20)

where \( I(n), d_j(n), c(n) \) are the policy functions under the optimal contract. Figure 5 plots the implied (annualized) interest rate schedule \([R(n)]^4\). Interest rate and spread increases significantly for low levels of net worth reaching in equilibrium as high as 20% above the world interest rate. For high levels of net worth the implied interest rate coincides with the world level.

5.4 Near crisis dynamics

Figure 6: Interest rate schedule
I now describe how the economy evolves in a crisis-like situation. Figure 6 plots the path of the economy for a constant sequence of low output realizations. Panel A shows that starting with zero debt the economy steadily accumulates obligations. After four quarters debt/output ratio reaches about 1/3. After six quarters the economy nearly exhausts its borrowing capacity. That is when the interest rate jumps 16% as shown in panel B. Panel C shows current account that first improves gradually (that is capital inflows gradually shrink) and then in quarter six it jumps to zero and later increases slightly. In this economy the interest rate is not a good indicator of the economy’s health; debt level and current account are. While such the path of output is unlikely (it’s probability is only 0.0024) it is 5.17 times more likely to occur in this model, that is under moral hazard, than in the model without frictions.

5.5 Simulation results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>LE+MH</th>
<th>MH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r)$</td>
<td>8.1833</td>
<td>4.5017</td>
<td>4.5464</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
<td>4.7342</td>
<td>7.3437</td>
<td>8.0856</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.1135</td>
<td>0.6555</td>
<td>0.6472</td>
</tr>
</tbody>
</table>

Cross-correlations

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>LE+MH</th>
<th>MH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(c, y)$</td>
<td>0.9710</td>
<td>0.7850</td>
<td>0.7801</td>
</tr>
<tr>
<td>$\rho(r, y)$</td>
<td>-0.5803</td>
<td>-0.6294</td>
<td>-0.5988</td>
</tr>
<tr>
<td>$\rho(tb, y)$</td>
<td>-0.8077</td>
<td>0.7603</td>
<td>0.7676</td>
</tr>
</tbody>
</table>

Output process

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>LE+MH</th>
<th>MH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(y)$</td>
<td>0.9365</td>
<td>0.1825</td>
<td>0.1645</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>0.0540</td>
<td>0.0540</td>
<td>0.0540</td>
</tr>
<tr>
<td>$\mu(d/Y)$</td>
<td>0.4175</td>
<td>0.4175</td>
<td>0.4175</td>
</tr>
</tbody>
</table>

I now turn to the model moments that are reported in table 3. The reported results were obtained by simulating time series of length 20,000 starting from $n_0$ that is the mid-point of the net worth’ support. The first
half of the sample was discarded.

Before I start discussing the results note that the model with limited enforcement and moral hazard ("LE+MH") and the model with moral hazard ("MH") are nearly identical. Limited enforcement simply imposes bounds on how much insurance can be provided to the borrower. But moral hazard curtails insurance motives even more and, so, limited enforcement has nearly no effect. So, in what follows I describe the "MH" results.

First, note that the model reproduces exactly moments of the output process and the average debt/output ratio. Both are endogenous. While output persistence stands at only 0.18 it is worth noting that the shocks in the model are i.i.d. That is the model has a strong propagation mechanism.

Second, interest rate spread in the model matches well the data. It is substantial, 4.54%, and it is very volatile, 7.34%. Moreover, the spread is strongly counter-cyclical. After a bad realization of output net worth declines. The economy moves closer to the borrowing limit and interest rate increases.

Third, consumption volatility and its correlation with output are significantly increased relative to the frictionless model. But the model success ends here. The two statistics are significantly smaller than in the data. It is also less correlated with output than in the data. That is the model allows “too much” insurance. As a consequence current account is not sufficiently volatile and is pro- rather than counter-cyclical. This failure is, however, expected. In any exchange economy current account is the difference between output and consumption: \( ca_t = Y_t - c_t \). Then there is the following trade-off between matching the consumption and the current account data moments:\[\rho(ca_t, Y_t) \sigma(ca_t) = 1 - \rho(c_t, Y_t) \sigma(c_t) = 1 - 0.9710 \cdot 1.1135 = -0.0812. \] (21)

If the model is parametrized to match the moments for consumption and output well it cannot match the data on current account. On one hand, to match the significant and negative correlation between current account and output (-0.77) current account’s volatility must be counter-factually low. On

\[22\] For example, a workhorse RBC model offers no internal propagation.

\[23\] Using the definition of correlation we get \( \rho(ca_t, Y_t) = [\sigma(Y_t) - \rho(c_t, Y_t) \sigma(c_t)]/\sigma(ca_t) \).
the other hand, to match the standard deviation of current account \((\sigma(ca) = 2.65\sigma(y))\) correlation between the latter and output must be close to zero; again, contradicting the data. Given this trade-off a model’s test is to fit the consumption data moments well. This can be achieved by reducing the borrower’s discount factor from 0.98 to 0.95. Appendix C presents moments for this alternative specification.

6 Conclusions

This work analyzes a model with moral hazard and limited enforcement in a small open exchange economy. I find that when state contingent contracting is allowed adding the moral hazard friction improves the model’s predictions along several dimensions. First, it justifies why non-contingent debt is an optimal way to finance an emerging economy. Second, it explains the limited consumption risk-sharing and high, volatile and counter-cyclical interest rates. Third, it generates realistic crisis like dynamics in which capital inflows are brought to a halt and interest rates sky-rocket. The model also has a strong internal propagation mechanism.

Limited enforcement friction, alone or together with moral hazard has nearly no effect on the model’s performance.

The model’s big short coming is inability to explain the current account dynamics. As I explained above there a trade-off between matching consumption and current account data. To break this constraint one has to introduce capital into the model. A model that merges the setting in Clementi et al. (2010) and the one presented here could be a good start.

References


A Euler Equations

To solve optimization problem (10) one has to maximize Lagrangean

\[ L_{Atk} = u(n + b - \theta I) + \beta \sum_{j=1}^{s} g(Y_j|I)V(Y_j - d_j) \]

\[ + \kappa \left( \beta c \sum_{j=1}^{s} g(Y_j|I)(d_j - b) \right) + \phi \cdot (M - b) \]

\[ + \mu \left( -\theta u'(n + b - \theta I) + \beta \chi(I) \sum_{j=1}^{s} \Delta g_j V(Y_j - d_j) \right) \]

\[ + \beta \sum_{j=1}^{s} g(Y_j|I) \gamma_j \left( V(Y_j - d_j) - V_{aut}(Y_j) \right) \]

with respect to current controls \((I, b, d_1, \ldots, d_n)\) and minimize with respect to Lagrange multipliers \(\kappa, \mu, \gamma, \phi \geq 0\). The first order necessary conditions and the envelope theorem imply

\[ V'(n) = u'(n + b - \theta I) - \mu \theta u''(n + b - \theta I) \]

\[ = V'(Y_j - d_j) \left( 1 + \gamma_j + \mu \frac{\chi(I) \Delta g_j}{g(Y_j|I)} \right) + \phi. \]  (22)

The optimality condition for the model with moral hazard only is:

\[ V'_{mh}(n) = V'_{mh}(Y_j - d_j) \left( 1 + \mu \frac{\chi(I) \Delta g_j}{g(Y_j|I)} \right) + \phi. \]  (23)

The optimality condition in the model with limited enforcement only is:

\[ V'_{le}(n) = V'_{le}(Y_j - d_j)(1 + \gamma_j) + \phi. \]  (24)
B Computational algorithm

Let $k$ denote the iteration number. Fix the borrowing limits $\bar{d}^k = (\bar{d}_1^k, ..., \bar{d}_S^k)$. The operator defined by 10 is a contraction. The value function can be, thus, computed iteratively. Once the value function is computed the borrowing limits can be updated: $\bar{d}_{j}^{k+1} = Y_j - (V^k)^{-1}(V_{aut}(Y_j)), \forall j$. Starting with $\bar{d}^0 = \infty$ so-constructed sequence of borrowing limits is monotone. Because the sequence is bounded below by a vector of zeros it must converge. This computational procedure while guaranteed to converge is time inefficient. I speed up the algorithm by updating the borrowing limits with each iteration on the value function using the following updating rule:

$$\bar{d}_{j}^{k+1} = 0.5\bar{d}_{j}^{k} + 0.5[Y_j - (V^k)^{-1}(V_{aut}(Y_j))].$$

C Alternative specification

In this section I demonstrate what happens when the borrower’s discount factor is lowered from 0.98 to 0.95. The remaining parameters are assigned the values reported in table 2.

Table 4: Moments for a model with low discount factor

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(c)/\sigma(y)$</th>
<th>$\rho(c, y)$</th>
<th>$\mu(r)$</th>
<th>$\sigma(r)$</th>
<th>$\rho(r, y)$</th>
<th>$\rho(y)$</th>
<th>$\mu(d/Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.11</td>
<td>0.95</td>
<td>8.18</td>
<td>4.73</td>
<td>-0.58</td>
<td>0.94</td>
<td>0.42</td>
</tr>
<tr>
<td>Model</td>
<td>0.97</td>
<td>1.00</td>
<td>19.0</td>
<td>15.4</td>
<td>-0.99</td>
<td>0.42</td>
<td>0.47</td>
</tr>
</tbody>
</table>

First, note that the model matches the consumption data well. Volatility of consumption is close to that of output, yet it is not larger. Second, note that interest rate spread becomes extremely large and volatile. Third, the model propagation mechanism becomes about twice stronger as measured by output persistence.

Yet, such a low discount factor implies that while the borrower’s debt/output ratio is on average 0.472 it is never below 0.425.