Time Orientation and Asset Prices

Per Krusell,
Burhanettin Kuruşçu,
and
Anthony A. Smith, Jr.

PRELIMINARY

April 2001

Abstract

We analyze a general-equilibrium asset pricing model where a small subset of the consumers/investors have a short-run "urge to save". That is, their attitudes toward consumption in the long run is a standard one—they do place zero weight on consumption far enough out in the future—but their short-run effective rates of discount may be negative. Our model, which is an elaboration on the framework proposed by Faruk Gul and Wolfgang Pesendorfer, does not feature time inconsistencies. Thus, we view consumers as fully rational, but subject to specific "internal frictions" in the form of temptation. The model nests the Mehra-Prescott model and we use it as a way of interpreting the wealth and asset pricing data. Some aspects of these data, we argue, can possibly be better understood using our model than the standard one.

1 Introduction

Self-control problems that take the form of "short-run urges" have recently received significant attention in the savings literature. We consider such problems, but we focus on the opposite of what has typically been studied: rather than emphasizing a present-bias—an urge for immediate consumption—we hypothesize that an agent may have an urge to save. We present no independent evidence for this hypothesis, but we think it is not an implausible one for a subset of the population. A focus on wealth accumulation alongside frugal attitudes toward consumption is if not a typical so at least a not uncommon description of the behavior of many households. On the one hand, one could simply think of such behavior as originating from a standard utility function with a low discount rate. On the other hand, it is conceivable too that such consumers do see themselves consuming their wealth, but only "just not yet", and that they do not change these attitudes over time. For example, a hard-to-explain feature of the savings data is the reluctance of many old-age consumers to dissave in cases where there are no natural heirs. That is, the interpretation would be that continued wealth accumulation—perhaps a focus on "not running down one's assets"—also can be regarded as an urge, or a temptation.

On a general level, we consider preference heterogeneity among consumers as rather natural; in fact, although most of the macroeconomic literature works with a representative agent, we view this practice as more of a modelling short-cut than motivated by realism. In this paper, we additionally ask the reader to consider the possibility that *at least some* consumers have "urges", and that these urges may take different expressions in different consumers. We also think such a supposition is quite natural, despite its possibly radical tone (at least in the context of the macroeconomic literature). Our basic argument on the level of motivation, then, continues by pointing out that it would tend to follow from these assumptions that those consumers with a future-bias—an urge to save—will accumulate more wealth than the average consumer and thus become dominant asset holders in the economy. That is, those consumers who have a future-bias would turn out to be of particular interest for studies of the aggregate economy, both in terms of aggregate savings (and other macroeconomic variables) and in terms of the influence on asset prices that these agents would have.

The purpose of this paper is to formulate a simple equilibrium model with these features: we model an urge to save, and we embed consumers with such an urge in a general-equilibrium asset-pricing model. More precisely, we look at a model where a small set of consumers have a future-bias and where the remainder of the population have no future-bias (are standard, unbiased consumers or have a present-bias). The model allows an alternative interpretation of the puzzles in the asset pricing and wealth data. For example, we illustrate how a low risk-free rate is not surprising in our economy, and we explore the conditions on preferences under which both the risk-free rate and the equity premium take on realistic values. These illustrations are carried out in a model parameterized so that most of the wealth is held by very few investors—as in the data—and where the aggregate consumption movements nevertheless are significantly influenced also by the middle- and low-wealth households—as in the data.

Although the setup we use is in the spirit of the incomplete-markets models in Krusell and Smith (1998) and Guvenen (2001) we simplify quite a bit with respect to the former by assuming two types of agents, and with respect to the latter by assuming complete markets; we maintain the presence of a borrowing constraint. We show that under certain assumptions on parameters, the borrowing constraint will bind for the poorer agents on the entire equilibrium path. This implies that this agent's consumption will be given "exogenously"—consumption will equal labor income—and that the rich agent can be studied in isolation. Thus, the rich agent determines the interest rate and all the asset prices.

An important aspect of our approach is our emphasis on "rationality": following important recent work by Faruk Gul and Wolfgang Pesendorfer (2000a and 2000b), we have learned that behavior with "urges" can be rationalized within the standard economic theory paradigm. These authors have demonstrated that it is not necessary to interpret "preference reversals"—as documented in the experimental literature—as expressions of irrationality or of "time inconsistency". Instead, one can think of a consumer's choices as depending not only on what they consume, but also on the set from which they make their choices. For example, a large set of possibilities may be strictly worse than a strict subset of the large set because the large set allows temptation—it contains tempting elements. The result of the temptation may either be that the consumer succumbs to the temptation or that he exercises self-control, but the consumer would be better off with the smaller set in either of these cases. Our contribution in the present paper is unimportant on a formal level; the framework we use is borrowed from our earlier paper (Krusell, Kuruscu, and Smith (2001)) where we extend and specialize Gul and Pesendorfer's work. This framework is attractive because of its ease of interpretation and its tractability. Moreover it is formally simply a generalization of the standard Lucas (1978) and Mehra and Prescott (1985) asset pricing model.

Our results are hard to judge quantitatively since we do not have any independent information regarding the strength or nature of the possible savings urges among investors. Clearly, if these urges are strong, the equilibrium risk-free rate will be very low. If we only introduce a savings urge, both the risk-free rate and the equity return will fall, so an increase in the equity premium cannot be obtained without other changes in the model. The natural way to obtain a high risk premium and a low risk-free rate, therefore, is to assume a high risk aversion and a strong savings urge. We show that this is possible in our model. Another result of interest is that our departure from the representative-agent setup pays off also quantitatively: with more wealth concentration—the dividend risk is borne by a small group—we need less extreme values of risk aversion in order to obtain a substantial equity premium. Moreover, we note that wealth concentration helps substantially in lowering the variability of the risk-free rate.

Our focus on discounting is related to the argument in Kocherlakota (1990). He argues that a negative rate of discount can help solve the risk premium. What we offer is an interpretation of Kocherlakota's point: it is possible to accept a negative rate of discount without having to accept a preference for consumption at any future date over current consumption (in his paper, utils t periods from now are δ^t times more valuable than utils now, and since $\delta > 1$ this amount increases without bound). In other words, our deviation from constant discounting allows a distinction that Kocherlakota could not make. We make a similar point to the one in this paper in our Krusell, Kuruşçu, and Smith (2000) piece. There, we study a "Laibson model" (see, e.g., Laibson (1897)) and extend it to also allow Epstein-Zin (1989) preferences, allowing to disentangle risk aversion from intertemporal elasticity of substitution. Finally, Luttmer and Mariotti (2000) also studies asset pricing in the Laibson model (in much more depth than in our earlier paper); they, however, focus exclusively on consumption urges and on how the stochastic properties of the pricing kernel changes as a function of this urge.

In Section 2 we illustrate how the basic temptation model works for a single agent living in a two-period world without uncertainty. That model can then be extended to more periods, uncertainty, and more agents. Section

3 describes the full model. Section 4 contains our quantitative results and Section 5 concludes.

2 A 2-Period Illustration

Our 2-period model is now described. It is designed to illustrate the workings of the model. We will then briefly mention the effect of adding periods before we proceed to the infinite-horizon model.

A consumer in the economy values consumption today (c_1) and tomorrow (c_2). To express the idea of temptation, however, it is actually necessary to think of preferences over a different domain than simply pairs of period 1 and period 2 consumption levels: we need to define preferences over *sets* of feasible intertemporal consumption bundles. The reason why sets are needed is, loosely speaking, that temptation has to do with not just what a consumer chooses, but what he could have chosen. Thus, in a utility function representation of preferences, the agent's utility function is defined over sets of those (c_1, c_2) pairs that the agent has available to him; in abstract, if A is an arbitrary set of consumption goods from which the consumer can make a choice, his utility is given by

$$w(A) = \max_{(c_1,c_2)\in A}\{\tilde{u}(c_1, c_2) + \tilde{v}(c_1, c_2)\} - \max_{(\tilde{c}_1,\tilde{c}_2)}\{\tilde{v}(\tilde{c}_1, \tilde{c}_2)\}.$$

That is, the agent's preferences can be described by two functions whose domain is pairs of consumption in periods 1 and 2: $\tilde{u}$ and $\tilde{v}$. The former is referred to as the "commitment utility function" and the latter the "temptation utility function". To see why these terms are used, notice that if the set A is a singleton $(\bar{c}_1, \bar{c}_2)$, we have $w(A) = \tilde{u}(\bar{c}_1, \bar{c}_2)$: if the agent has no choice—has committed to consumption—his preferences are given by $\tilde{v}$. If, on the other hand, A is not a singleton, then the utility is influenced by both functions.

The agent's choice out of the given set is that given by the argument that maximizes $\tilde{u} + \tilde{v}$. The solution to the second maximization problem—with $\tilde{v}$ being the objective—has no interpretation in terms of observed choice. The second maximization problem, however serves to emphasize a "disutility of self-control": $\tilde{v}(c_1, c_2) - \max_{(\tilde{c}_1,\tilde{c}_2)}\{\tilde{v}(\tilde{c}_1, \tilde{c}_2)\}$ has to be nonpositive, and it is negative whenever $\tilde{v}$ dictates a different choice than $\tilde{u}$.

For a brief (but not self-contained) background, Gul and Pesendorfer consider axioms over sets designed to (i) include the standard case without temptation but (ii) allow in addition a narrow set of circumstances that can be described by the idea of temptation and self-control. For this set of axioms, they provide the utility function characterization above: an agent satisfying the axioms can be uniquely identified with a pair of functions $\tilde{u}$ and $\tilde{v}$. From the point of view of applications, the important point here is that these functions are independent of the particular choice problem the agent is faced with. Aside from basic axioms such as completeness, transitivity, and continuity, the additional key axiom is set betweenness: for any two sets A and B where A is preferred to B, it must also be that A is preferred to the union of A and B, and that the union of A and B is preferred to B: the union is "in between". When A is strictly preferred to the union, we have a case of a "preference for commitment": you strictly prefer the smaller set. A standard consumer (i.e., one who does not display a preference for commitment) would in contrast always be indifferent between A and $A \bigcup B$ whenever A is preferred to B. Moreover, when there is a preference for commitment, the agent "succumbs" to temptation if he is also indifferent between $A \bigcup B$ and B; this is simply because when B is available, he would choose an element in B. On the other hand, the agent may not succumb to temptation but rather exercise self-control: he might strictly prefer $A \bigcup B$ over B, because he chooses an element in A when facing the set $A \bigcup B$.

2.1 Our parameterization

We specify the functions $\tilde{u}$ and $\tilde{v}$ as follows:

$$\tilde{u}(c_1, c_2) = u(c_1) + \delta u(c_2)$$

and

$$\tilde{v}(c_1, c_2) = \gamma \left(u(c_1) + \beta \delta u(c_2) \right),$$

where u has the usual properties. For a standard consumer—one without a preference for commitment in any choice situation—we have that $\beta = 1$. When $\beta < 1$, the temptation function gives a stronger preference for present consumption. The parameter γ allows us to regulate the strength of the temptation; as it increases toward infinity, the agent moves closer to succumbing to temptation and let his actual choice be governed by $\tilde{v}$ alone.

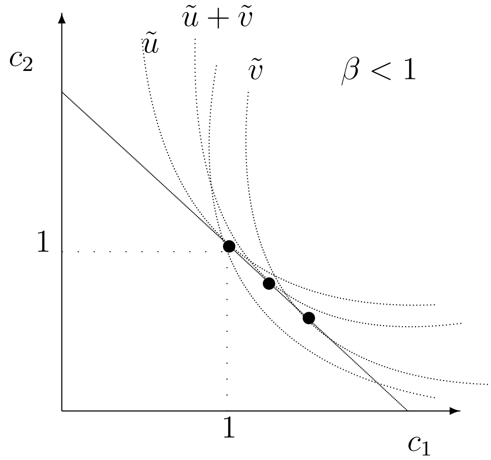

Figure 1

Figure 1 illustrates the "urge to consume" for the case where the set from which the consumer chooses is a standard budget set. Here, the commitment utility function would indicate that the best point in the budget set is where its indifference curve intersects the budget line. However, the temptation utility at this point has a slope greater than the interest rate. The maximization of $\tilde{u} + \tilde{v}$ would therefore lead to a higher level of consumption in period 1.

Figure 2 depicts the case we are primarily interested in here: it considers the possibility that $\beta > 1$. Now, the consumer will end up with a choice of consumption in period 1 below that maximizing $\tilde{u}$.

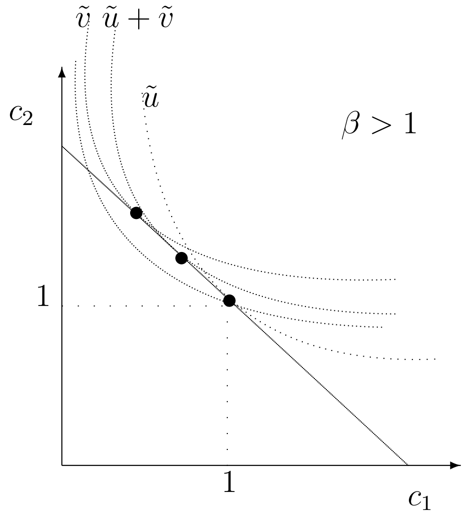

Figure 2

Formally, the utility of a typical consumer in this choice situation, then, is:

$$\max_{c_1,c_2} \{\tilde{u}(c_1, c_2) + \tilde{v}(c_1, c_2)\} - \max_{\tilde{c}_1,\tilde{c}_2} \tilde{v}(\tilde{c}_1, \tilde{c}_2), \tag{1}$$

where both maximizations are subject to the budget constraint.

In this two-period problem, the "temptation" part of the problem (i.e., the second maximization problem in the objective function) plays no role in determining the consumer's actions in period 1. As we describe just below, this is not true when the horizon is longer than two periods: then, the consumer's temptation in future periods will influence the choices in the earlier periods, since the savings decisions earlier on influence the sizes of the budget sets later on, thus affecting the extent of temptation, with the associated disutility of self-control, in the later periods.

The consumer's intertemporal first-order condition is:

$$\frac{1+\gamma}{(1+\beta\gamma)\delta}\,\frac{u'(c_1)}{u'(c_2)} = 1 + r,$$

where r is the interest rate. It is straightforward to see that the intertemporal consumption allocation (which, in effect, maximizes $\tilde{u} + \tilde{v}$) represents a compromise between maximizing $\tilde{u} = u(c_1) + \delta u(c_2)$ and maximizing $\tilde{v} = u(c_1) + \beta\delta u(c_2)$. In the former case, the first-order condition is:

$$\frac{1}{\delta}\,\frac{u'(c_1)}{u'(c_2)} = 1 + r,$$

whereas in the latter case, the first-order condition is:

$$\frac{1}{\beta\delta}\frac{u'(c_1)}{u'(c_2)} = 1 + r.$$

Since

$$\frac{1}{\beta\delta} \leq \frac{1+\gamma}{(1+\beta\gamma)\delta} \geq \frac{1}{\delta}$$

in the case we are interested in, the consumer's consumption allocation is tilted towards the future relative to maximizing $u(c_1) + \delta u(c_2)$ and is tilted towards the present relative to maximizing the temptation function $u(c_1) + \beta\delta u(c_2)$.

2.2 More than 2 periods

With more than two periods, we first need to specify what an agent is tempted by. We will assume that temptation only involves the immediate present. That is, we will assume that $\tilde{u}$ and $\tilde{v}$ agree on any future variables: holding current consumption constant, these functions are identical. Formally, this means that if A and B are sets of intertemporal consumption bundles and if A and B have identical possibilities for current consumption, then either $A \bigcup B$ is indifferent to A or it is indifferent to B.

These assumptions allow us to solve the problem backwards. One first analyzes the last two periods, $T-1$ and T, in a manner parallel to that in the previous subsection. In the context of budget sets again, this gives rise to a utility over sets $w_{T-1}(\omega_{T-1})$, where ω_{T-1} is incoming wealth in period $T-1$. The third period from the last, the utility of any ω_{T-2} can then be expressed using the indirect utility from the previous step. Our assumption will now be that the commitment utility attaches a weight of δ on $w_{T-1}(\omega_{T-1})$, where as the temptation utility attaches a weight $\beta\delta$. This is the natural generalization of the two-period case, and the formulation we have adopted in our earlier work (Krusell, Kuruşçu, and Smith (2001)). Our earlier paper also shows that this formulation—a "quasi-geometric" temptation—turns out to work as a generalization of sorts of the Laibson model.

To look in some more detail at the multi-period model, consider the three-period case: in period 2, the consumer will be given some initial wealth that was determined in period 1 and chooses over consumption levels in periods 2 and 3 as in the above subsection. However, how is the choice in period 1 determined? Here, since we are considering a choice of period-2 budget sets, the agent in period 1 must consider not only the consumption outcomes in periods 2 and 3 but also the possible disutility of temptation in period 2. (Period 3, of course, does not admit such a disutility, since the choice set in period 3 is a singleton.) It is straightforward to show that the first-order condition for savings in period 1 now becomes

$$u'(c_1) = \delta \frac{1+\beta\gamma}{1+\gamma} (1+r) \left[u'(c_2) + \gamma \left(u'(c_2) - u'(\tilde{c}_2)\right)\right],$$

where $\tilde{c}_2$ as before refers to temptation consumption in period 2. This expression looks like a standard Euler equation except in two places: (i) the discount factor is different; and (ii) there is an added term $\gamma(u'_{t+1} - \tilde{u}'_{t+1})$. The discount factor, which is the same as in the 2-period model, is greater than δ. The term $\gamma(u'_2 - \tilde{u}'_2)$ is the derivative of the disutility/cost of self-control, $\gamma(u_2 - \tilde{u}_2)$, with respect to wealth. This term is negative, since temptation consumption is lower than actual consumption and the utility function is strictly concave. The interpretation of these equations is that the marginal benefit from wealth tomorrow falls short of u'_2, because the self-control cost gets larger as wealth increases in this model!

Similarly, there will be an Euler equation for the maximization of temptation utility in period 1:

$$u'(\tilde{c}_1) = \beta\delta(1+r) \left[u'(\tilde{c}_2) + \gamma \left(u'(\tilde{c}_2) - u'(\tilde{\tilde{c}}_2)\right)\right],$$

where $\tilde{c}_2$ is period 1 temptation consumption, $\tilde{c}_2$ is period 2 actual consumption if you succumb in period 1, and $\tilde{\tilde{c}}_2$ is period 2 temptation consumption if you succumb in period 1.

We will now move on to the infinite-horizon case, where uncertainty and borrowing constraints will also be added.

3 The Infinite-Horizon Asset Pricing Model

3.1 Primitives

Consider now an infinite-horizon setup with two agents: the big guy (B) and the little guy (L). As in the Lucas asset pricing model, there is no physical capital accumulation nor production: total consumption will equal total endowments. The endowment process has two parts: on a per-agent basis, there is a "dividend from a tree", d as well as "labor income", l. The agents have clones: there is a measure θ of big guys and a measure $1-\theta$ of little guys. As explained in the introduction, we will consider parameter values allowing us to study the little guy residually: each little guy will consume l in equilibrium, and each big guy will therefore consume $d/\theta + l$. As in Mehra and Prescott (1985), there is dividend growth, and labor income will grow too. Total income, y, grows stochastically at one of two possible rates. The dividend is specified as a fraction η of total output: $d = \eta y$, with an implied $l = (1-\eta)y$. The dividend share of output is also stochastic but has a 4-state support. Therefore, our stochastic process for endowments has four possible states. We denote these by 1 through 4; states 1 and 3 have low and states 2 and 4 high output growth, and the dividend/output ratio is increasing from state 1 through state 4. Associated to each state i are four transition probabilities π_{ij} for $j = 1, 2, 3, 4$.

Our parameterization will allow us to make dividends more variable than labor income, so that the percentage risk born by the big guy is larger than that borne by the little guy. Moreover, the smaller is the group of big guys, the larger is this difference, since the dividend risk becomes more concentrated.

We also should describe the preferences here; essentially, they can be thought of as the extension of the finite-period case to an infinite horizon. However, we postpone this task as it is much easier to describe the preferences in the context of a specific choice problem, which requires a description of the asset structure.

3.2 Assets and asset prices

The asset structure is one with contingent claims: there are four such claims for each date and state, each delivering one unit of consumption in each of the consecutive four states. The price of a claim in state i delivering 1 unit of consumption in state j next period is denoted q_{ij}. The contingent claims allows us to easily compute the prices of stocks and bonds. The stock prices, p_i for states $i = 1, 2, 3, 4$, will be given by the solution the four equations

$$p_i = q_{i1}(d_1 + p_1) + q_{i2}(d_2 + p_2) + q_{i3}(d_3 + p_3) + q_{i4}(d_4 + p_4)$$

for $i = 1, 2, 3, 4$. Clearly, the contingent-claims prices will typically uniquely define the stock prices. Similarly, the bond prices, q_i, are given by

$$q_i = q_{i1} + q_{i2} + q_{i3} + q_{i4}$$

for $i = 1, 2, 3, 4$.

The agents have no endowments of assets; the market-clearing level of each contingent claim is zero. The agents' endowments simply consist of the dividend and labor income processes. We assume that the big guy's endowment equals the dividend plus the labor income, and that the little guy's endowment is the labor income only.[1]

3.3 Borrowing constraints

We assume that agents face borrowing constraints. The brutal honest truth about these constraints is that we impose them in order to facilitate the solution of the model. With our approach, as we have already indicated, one of the two kinds of agents will be de facto more impatient than the other, and with a borrowing constraint the more impatient of the two—the little guy–will end up always consuming his labor endowment. This modelling approach really is a shortcut for something which we believe is more reasonable—to have an incomplete set of asset markets. Krusell and Smith (1997 and 1998) specify such a model; there, no agent faces a binding constraint eternally, since discount factors are assumed to be stochastic, and solution of that model is therefore much more involved. However, the paper shows that small differences in discount factors lead to large differences in wealth holdings and savings behavior, and our present approach can merely be viewed as a simple way to mimic these results.

The implementation of borrowing constraints with contingent claims requires a specification of several constraints. The simplest way to do this is to require that all holdings of contingent claims be nonnegative; this is also what we do.[2] In the formulation of the big guy's problem below, we will for simplicity omit the borrowing constraints; however, once a solution is found one has to show that the solution to the agent's problem indeed is interior.

[1] We could have chosen another endowment pattern. However, what is important for our quantitative results is how much of total income we let the little guy consume. How much he is allowed to consume is a function both of his endowment and of the borrowing constraint he is faced with: for any assumption on the endowment level, there is a set of borrowing constraints that let us vary his consumption level, and vice versa.

[2] There are alternatives as well, including constraints on total wealth, margin constraints, and so on.

3.4 The big guy's problem

We now specify the big guy's problem. We use recursive methods to get at the limit of the finite-horizon problems discussed above.[3]

It is given by the following recursive functional equation:

$$W_i(\omega) = \max_{\{s'_j\}_{j=1}^4} \left\{ (1+\gamma)u(\omega - \sum_{j=1}^4 q_{ij}s'_j) + \delta(1+\beta\gamma)E(W(\omega')) \right\} -$$

$$\gamma \max_{\{\tilde{s}'_j\}_{j=1}^4} \left\{ u(\omega - \sum_{j=1}^4 q_{ij}\tilde{s}'_j) + \beta\delta E(W(\tilde{\omega}')) \right\},$$

where

$$\omega'_j = s'_j + d_j + l_j$$

and

$$\tilde{\omega}'_j = \tilde{s}'_j + d_j + l_j.$$

This problem delivers decision rules for actual behavior, $\{s'_{ij}(\omega)\}_{j=1}^4$, and temptation behavior, $\{\tilde{s}'_{ij}(\omega)\}_{j=1}^4$.

3.5 Finding an equilibrium

Algorithmically, it is clear how to find an equilibrium for our economy. We first solve the problem of the big guy in isolation to obtain market clearing prices and behavior that is optimal given these prices. We then verify that, given these prices, it is indeed optimal for the little guy to have a binding borrowing constraint; this is established using the appropriate Euler inequalities for all four states.

To find the market-clearing prices in this economy is significantly more complicated, unfortunately, than in the Mehra and Prescott economy. In the latter, contingent claims prices can be obtained immediately from the first-order conditions for each contingent claim, since consumption has to equal endowments. In the present economy too, actual consumption has to equal endowment, but it is also necessary to find the temptation consumption levels, since these appear in the first-order conditions. Finding the temptation

[3]Axiomatization leading to the recursive version of the Gul-Pesendorfer preferences with quasi-geometric temptation that we consider here is not fully established. See Krusell, Kuruşçu, and Smith (2001) for details.

consumption levels is what is hard. To see this, consider the first-order conditions; there are such conditions both for actual decision rules and for the temptation rules, as we saw in the deterministic case above. In the case of uncertainty, the conditions read as follows:

$$q_{ij}u'\left(\omega - \sum_{l=1}^{4} q_{il}s'_{il}(\omega)\right) = \delta\frac{1+\gamma\beta}{1+\gamma}\pi_{ij}\cdot$$

$$\left[u'\left(s'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}s'_{jl'}(s'_{ij}(\omega) + d_j/\theta_j + l_j)\right) + \right.$$

$$\gamma\left(u'\left(s'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}s'_{jl'}(s'_{ij}(\omega) + d_j/\theta_j + l_j)\right) - \right.$$

$$\left.\left. u'\left(s'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}\tilde{s}'_{jl'}(s'_{ij}(\omega) + d_j/\theta_j + l_j)\right)\right)\right]$$

for actual behavior and

$$q_{ij}u'\left(\omega - \sum_{l=1}^{4} q_{il}\tilde{s}'_{il}(\omega)\right) = \delta\frac{1+\gamma\beta}{1+\gamma}\pi_{ij}\cdot$$

$$\left[u'\left(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}s'_{jl'}(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j)\right) + \right.$$

$$\gamma\left(u'\left(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}s'_{jl'}(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j)\right) - \right.$$

$$\left.\left. u'\left(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j - \sum_{j=1}^{4} q_{jl'}\tilde{s}'_{jl'}(\tilde{s}'_{ij}(\omega) + d_j/\theta_j + l_j)\right)\right)\right]$$

for temptation behavior; each of these equations have to hold for all ω, i, and j.

These equations look complicated because they are expressed as functional equations in the decision rules s' and $\tilde{s}'$: these decision rules are evaluated explicitly at the relevant values. For a full understanding of the conditions describing fully rational behavior, the reader is invited to carefully distinguish how these two functions appear in the equations.

On a conceptual level, what is important about the first-order equations is that they have to hold for all ω and not just for equilibrium wealth $d/\theta + l$. In the temptation behavior, the consumer is choosing to save a different amount than that actually saved. A consequence of such savings would be that next period's wealth differ from that of next period's actual wealth and, as a result, all consecutive periods would produce non-equilibrium wealth levels. To verify optimality of temptation behavior, therefore, it is necessary to know what consumption levels result from each possible wealth level: it is necessary to solve for entire (actual and temptation) decision rules. This is what makes the determination of asset prices in this economy a level more difficult than in the Mehra and Prescott economy.

Fortunately, it is possible to show that under constant relative risk aversion, the decision rules s' and $\tilde{s}'$ are all affine: they are of the kind $s'_{ij}(\omega) = a_{ij} + b_{ij}\omega$ and $\tilde{s}'_{ij}(\omega) = \tilde{a}_{ij} + \tilde{b}_{ij}\omega$. This means that each of the 4×4 first-order conditions for actual behavior produces two equations: one requiring that the intercept on the left-hand side equals that on the right-hand side, and one requiring that the slope on wealth on the left-hand side equals that on the right-hand side.[4] Similarly, the temptation first-order conditions produce 32 equations. With an additional 4×4 market-clearing conditions we have a total of 80 equations. These 80 equations have to be solved for $4 \times 4 \times 2$ decision rule coefficients for actual behavior, $4 \times 4 \times 2$ decision rule coefficients for temptation behavior, and 4×4 prices: 80 unknowns. The system is nonlinear. For our actual computational techniques, see the Appendix.

Finally, for the little guy the above Euler conditions have hold with inequalities: the left-hand side has to exceed the right-hand side when evaluated at $s' = 0$. We assume that the little guy is has no urge to save (that his β is less than or equal to one) so that $\tilde{s}'$ is less than or equal to s': thus, if the borrowing constraint binds for actual behavior, it will for temptation behavior.[5]

[4]The first-order conditions produce linear equations once both sides are raised to the appropriate power.

[5]This point was made by Gul and Pesendorfer in their (2000b) paper: when faced with a binding borrowing constraint, an agent with an urge to consume does not suffer from a disutility of self-control, and can thus be viewed as observationally equivalent to one without an urge to consume.

4 Quantitative Results

Although a key purpose of our model is to examine the role of the different parameters for asset prices—comparative-statics exercises—some parameters will be calibrated and remain unchanged throughout the analysis. We therefore first describe this calibration. Thereafter we show our main results.

4.1 Calibration

We took the dividend/labor income process from Heaton and Lucas (1996): they consider a 4-state process, which was matched to U.S. data. The transition probability matrix we thus use (it is derived from Table 1 in Heaton and Lucas's paper) is as follows:

$$\begin{array}{cccc} 0.5297 & 0.3024 & 0.1068 & 0.0611 \\ 0.4101 & 0.4675 & 0.0572 & 0.0652 \\ 0.0652 & 0.0572 & 0.4675 & 0.4101 \\ 0.0611 & 0.1068 & 0.3024 & 0.5297. \end{array}$$

In this array, the (i, j) element is the probability of moving from state i to state j. The four states are $(0.9904, 0.1402)$, $(1.047, 0.1437)$, $(0.9904, 0.1561)$, and $(1.047, 0.1599)$, where the first number in each ordered pair is the growth rate of income and the second number in each ordered pair is the share of dividends in income.

As for the remaining parameters, we vary them around a baseline configuration: $\delta = 0.95$, $\theta = 1$ (so that there is no little guy—the representative-agent version of the model), $\beta = 1$ and/or $\gamma = 0$ (the standard model), and $\alpha = -3$ (the coefficient of risk aversion being $1 - \alpha$).

We should also show, for comparison, what the data says about our variables of interest. Our source here is a paper by Cecchetti and Mark (1990), which uses annual data. Let r_e be the return on equity and let r_f be the return on the risk-free asset; similarly let σ_e and σ_r denote the standard deviations in these returns, respectively. The facts for 1889–1985 reported by Cechetti, Lam, and Mark are: $E(r_e) = 8.42$, $E(r_f) = 2.66$, $\sigma_e = 18.47$, $\sigma_f = 5.13$, $\text{corr}(r_e, r_f) = 0.03$, market price of risk $= E(r_e - r_f)/\sigma_{r_e - r_f} = 0.302$.

4.2 Findings

4.2.1 The effects of risk aversion with one vs. with two agents

We first set $\delta = 0.95$, $\theta = 1$, and $\beta = 1$ to obtain a set of results regarding the effect of risk aversion in for the "standard model":

α	μ_e	μ_f	μ_{e-f}	σ_e	σ_f	mpr
-2	10.96	10.80	0.163	3.09	1.89	0.067
-3	12.72	12.55	0.168	3.61	2.56	0.066
-4	14.39	14.25	0.147	4.30	3.24	0.052
-5	15.99	15.89	0.102	5.10	3.92	0.032

Note here that the equity premium increases and then *decreases* as risk aversion increases, and the market price of risk decreases as risk aversion increases! This is a result of the properties of the particular driving process.

Moving to our two-agent model, we notice that the decreases in the equity premium and the market price of risk go away as we lower θ (the fraction of big guys). When $\theta = 0.3$ (recall that in the data the top 1% control roughly 30% of wealth, while the top 10% control roughly half and the top 30% control 90%) we obtain:

α	μ_e	μ_f	μ_{e-f}	σ_e	σ_f	mpr
-2	10.96	10.65	0.312	3.33	0.53	0.095
-3	12.68	12.28	0.406	3.33	0.72	0.125
-4	14.31	13.82	0.495	3.33	0.91	0.154
-5	15.84	15.26	0.581	3.34	1.10	0.182

These results are dramatically different. First, the risk premium goes up significantly; the return on equity does not change much, but the return on bonds goes down. Secondly, the variability of the bond return falls quite dramatically. Third, and in conclusion, we see that the effect of increased risk aversion on the price of risk is now monotone (increasing) and strong: it is thus the labor income process and low variability that makes the risk premium decreasing.

4.2.2 The effects of wealth concentration

Motivated by the findings above, let us display the effects of changing θ when $\delta = 0.95$, $\alpha = -3$, $\gamma = 3$, and $\beta = 1$ (i.e., still the standard model except for the heterogeneity):

θ	μ_e	μ_f	μ_{e-f}	σ_e	σ_f	mpr
1	12.72	12.55	0.168	3.61	2.56	0.066
0.3	12.68	12.28	0.406	3.33	0.72	0.125
0.1	12.89	11.76	1.111	7.87	2.74	0.151
0.01	13.39	10.90	2.485	14.08	5.91	0.195

We see that $\theta = 0.01$ delivers a risk premium of about two and a half percent, and a market price of risk of about 0.2!

4.2.3 The urge to save

We now set $\delta = 0.95$, $\theta = 1$, $\gamma = 3$, and $\alpha = -3$ and vary β (recall that a higher β increases the urge to save):

β	μ_e	μ_f	μ_{e-f}	σ_e	σ_f	mpr
1	12.72	12.55	0.168	3.61	2.56	0.066
2	7.95	7.79	0.154	3.44	2.39	0.063
4	5.05	4.90	0.146	3.34	2.29	0.060
8	3.49	3.35	0.142	3.29	2.24	0.059

Naturally, an increase in β decreases the returns. In particular, in this table, we are keeping the degree of risk aversion constant, and we see a sharp fall in the return on stock as well. The equity premium, as well as the market price of risk, both fall as well. This effect is very similar to that obtained from a decrease in the discount rate in the standard model. See Kocherlakota (1990) for details.

The previous table assumed a representative agent. In order to consider wealth concentration, let us set $\delta = 0.95$, $\gamma = 3$, $\alpha = -3$, and use $\theta = 0.3$: more concentrated wealth. This gives:

β	μ_e	μ_f	μ_{e-f}	σ_e	σ_f	mpr
1	12.68	12.28	0.406	3.33	0.72	0.125
2	7.93	7.54	0.386	3.17	0.68	0.125
4	5.03	4.66	0.374	3.08	0.65	0.125
8	3.48	3.11	0.368	3.03	0.64	0.124

We see that the equity return goes down slightly. The return on bonds, however, falls more, resulting in a higher risk premium. This is in line with the change from a single-agent to a two-agent setup observed above.

5 Conclusions

We have solved a Gul-Pesendorfer asset-pricing model where the temptation is one of savings urges among some investors. This model allows us to depict the world of consumers/investors in a not altogether unrealistic fashion: there is a small group of investors who are very rich and a large group of consumers who are poor and, as we model it here in order to simplify, borrowing-constrained. The rich investors are rich precisely because of their attitudes toward savings: with a single-minded focus on wealth accumulation, they end up dominating the economy in asset holdings. As such, they bear more risk, and their intertemporal preferences will be those determining asset prices, and not those of the "average consumer". The average consumer, in contrast, will feature "hand-to-mouth" behavior. One implication that our setup has is that the risk-free rate becomes naturally lower. Secondly, with a reasonably high degree of risk aversion and concentrated risk, the equity premium can also be made much larger than in the standard model.

Obviously, one needs to systematically learn more about the actual behavior of the large investors. We know of no empirical study that is helpful in this regard. Our approach here can perhaps best be characterized as overly bold theorizing, but we do believe that moving in the direction of understanding the "psychology" of large investors is very important both for understanding asset pricing and macroeconomics.

REFERENCES

Cecchetti, Stephen G. and Nelson C. Mark (1990) "Evaluating Empirical Tests of Asset Pricing Models: Alternative Interpretations," *American Economic Review*, 80(2), pp. 48-51.

Epstein, Larry G. and Stanley E. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns: A Theoretical Framework," *Econometrica*, 57(4), pp. 937-969.

Gul, Faruk and Wolfgang Pesendorfer (2000a), "Temptation and Self-Control," forthcoming in *Econometrica*.

Gul, Faruk and Wolfgang Pesendorfer (2000b), "Self-Control and the Theory of Consumption," manuscript.

Guvenen, Muhammet F. (2001), "Mismeasurement of the Elasticity of Intertemporal Substitution: The Role of Limited Stock Market Participation," Working Paper, Carnegie-Mellon University.

Heaton, John and Deborah J. Lucas (1996), "Evaluating the Effects of Incomplete Markets on Risk sharing and Asset Prices," *The Journal of Political Economy*, 104(3), pp. 443-487.

Kocherlakota, Narayana (1990), "On the Discount Factor in Growth Economies," *Journal of Monetary Economics*, 25(1), pp. 43-47.

Krusell, Per, Burhanettin Kuruşçu, and Anthony A. Smith, Jr. (2001), "Temptation and Taxation," manuscript.

Krusell, Per, Burhanettin Kuruşçu, and Anthony A. Smith, Jr. (2000), "Asset Pricing with Laibson-Epstein-Zin Preferences," manuscript.

Krusell, Per and Anthony A. Smith, Jr. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *The Journal of Political Economy*, Vol 106, pp. 867-896.

Krusell, Per and Anthony A. Smith, Jr. (1997), "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, Vol 1, pp. 387-422.

Laibson, David (1997), "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, pp. 443-477.

Lucas, Robert E., Jr. (1978), "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), pp. 1429-1445.

Luttmer, Erzo G.J. and Thomas, Mariotti (2000), "Subjective Discounting in an Exchange Economy," manuscript.

Mehra, Rajnish and Edward C. Prescott (1985), "The Equity Premium:

A Puzzle," *Journal of Monetary Economics*, 15(2), pp. 145-161.