On the Mechanics of New-Keynesian Models*

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Abstract

The monetary transmission mechanism in New-Keynesian models is put to scrutiny. We show that, contrary to the conventional view, the transmission mechanism does not operate through the real interest rate channel. Instead, equilibrium inflation is approximately determined as in a flexible-price model; output is then pinned down by the New-Keynesian Phillips curve. The real rate only reflects the desire and feasibility to smooth consumption when income changes. Contractionary monetary policy shocks reducing output and inflation are consistent with an increase, decline, or no change in the real rate. Consistency with the real rate channel is purely observational.

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1 Introduction

The New-Keynesian model—a dynamic stochastic general equilibrium (DSGE) model with sticky prices—has become a workhorse in the analysis of monetary policy. It has grown in popularity at tremendous speed both in academia and at central banks around the world. From a basic framework, consisting of an Euler equation, a New-Keynesian Phillips curve, and a Taylor rule, it has quickly grown into a model with endogenous capital a lá Real Business Cycle (RBC) theory, a number of different frictions, adjustment costs, and other features required to "fit the data". The basic framework is typically used to illustrate optimal monetary policy (e.g., Clarida, Galí, and Gertler, 1999), whereas the extended model—often referred to as a medium-scale DSGE model—is used for practical monetary policy and forecasting (e.g., Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007).

Unfortunately, widespread understanding of the monetary transmission mechanism in New-Keynesian models—i.e., how unexpected changes in monetary policy transmit into the real economy—appears to have been lost along the fast track to popularity.[1] A description of the transmission mechanism in New-Keynesian models given by Ireland (2015) in the New Pelgrave Dictionary of Economics is representative of the typical exposition:

> A monetary tightening, in the form of a shock to the Taylor rule, that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their spending as summarized by the IS curve. Finally, through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock.

The standard description is thus based on the traditional real interest rate channel (e.g.,

[1] The responses of the real economy to unexpected changes in nominal variables controlled by the central bank are a subject of investigation by a large literature. Such interest stems from the desire to discriminate across potential channels of transmission when guiding monetary policy (Christiano, Eichenbaum, and Evans, 1999).

Bernanke and Gertler, 1995; Taylor, 1995; Mishkin, 1996). According to the real interest rate channel, the central bank—controlling the short-term nominal interest rate—has leverage over the ex-ante real interest rate because nominal prices are sticky. As a result, an increase in the nominal rate leads to an increase in the ex-ante real rate—an intertemporal price—that induces households and firms to cut consumption and investment, thus reducing aggregate demand and output. This puts pressure on firms to gradually adjust prices to a lower level. This channel is at the core of the textbook IS-LM (AS-AD) model and served as a motivation for the inception of New-Keynesian models in the wake of the rational expectations revolution (Ireland, 2015); see also Goodfriend and King (1997) for the connections between IS-LM and New-Keynesian models and RBC and New-Keynesian models. Indeed, like in the above quote, the real effects of monetary policy in New-Keynesian models are commonly portrayed as operating through this channel (e.g., Woodford, 2003; Galí, 2015).[2]

The main message of the paper is that the transmission mechanism of monetary policy in New-Keynesian models *does not* operate through the real interest rate channel. Any consistency with the real interest rate channel is purely observational, not structural, due to a specific parameterization. The paper then provides a more accurate description of the inner workings of the transmission mechanism in the model.

Of course, unlike in the above Old-Keynesian narrative of the real interest rate channel, in the general equilibrium of New-Keynesian models all variables are simultaneously determined in a dynamic setting. So any description of the model in terms of the traditional real rate channel may perhaps be understood to be only cursory. Nevertheless, we should observe declines in output and inflation, in response to a contractionary monetary policy shock, to always coincide in equilibrium with an increase in the ex-ante real interest rate, if real activity and prices indeed decline as a consequence of intertemporal substitution by households and firms in the face of higher real interest rates.

[2] The emphasis of the theoretical literature on the real interest rate channel has been reinforced by a large empirical literature documenting that, broadly speaking, in response to a positive innovation in a short-term nominal interest rate in a VAR model: (i) the nominal interest rate increases, (ii) output declines, and (iii) inflation (persistently) declines, but less than output; e.g., Christiano et al. (1999). The ex-ante real interest rate increases as a result of (i) and (iii).

We demonstrate that while this outcome always holds in the basic three-equation framework under standard preferences, it is not the case once endogenous capital is introduced. As a first pass, the presence of endogenous capital makes the ex-ante real interest rate *approximately* constant. In the actual solution of the model, the declines in output and inflation, in response to a contractionary monetary policy shock, are consistent with an increase, decline, or no change in the ex-ante real interest rate, depending on parameterization (this applies to both short and long rates). In most cases, in fact, the ex-ante real interest rate *declines*.[3]

It is important to point out the mechanism in the presence of endogenous capital for at least two reasons. First, it is investment, rather than consumption, that plays a key role in the traditional real interest rate channel, which the New-Keynesian models are supposed to capture.[4] And second, endogenous capital is a key ingredient in the transition from the basic three-equation framework to the medium-scale DSGE models, containing both New-Keynesian and RBC features.

If not through the real interest rate channel, how does monetary policy transmit into output and inflation in the New-Keynesian model? We demonstrate that equilibrium inflation is essentially determined as in a flexible-price model; output is then pinned down by the New-Keynesian Phillips curve (that is, each individual firm that cannot optimally adjust prices to keep up with the equilibrium path of the aggregate inflation rate adjusts output). In the model with endogenous capital, when output temporarily drops, as a result of temporarily low inflation, households can keep consumption relatively smooth by reducing investment. The ex-ante real rate only reflects their desire and ability to do so by adjusting the capital stock. *As a first pass*, they can adjust capital with almost no effect on the real rate, given the large size of the capital stock, relative to investment. In contrast, in the model without capital, consumption smoothing in the aggregate is not possible (effectively,

[3]It is well known that even in the basic model without capital the *nominal* interest rate can decline in response to a contractionary monetary policy shock, if the shock is sufficiently persistent (Woodford, 2003; Galí, 2015). This is due to a persistent decline in inflation and thus inflation expectations. The ex-ante real rate in the basic model, however, always increases (under standard preferences), irrespective of the shock persistence.

[4]Woodford (2003), for instance, regards the basic model as a shortcut to capturing the effects of monetary policy on aggregate expenditures working through investment.

the costs of adjusting capital are infinite). Therefore, under standard preferences, as everyone wants to keep consumption smooth by borrowing, the real interest rate has to increase to restore equilibrium. The equilibrium outcome of the basic model thus makes it appear *as if* monetary policy affected output and inflation through the real interest rate channel. Indeed, moving from zero to infinite adjustment costs in the model with capital makes the model gradually behave in the "standard" way.

There are at least two reasons to be aware of the findings in this paper. The first reason is purely academic. It is critical to understand how the model works when attempting to extend it in various directions. We hope that our exposition will be helpful in this respect. Second, there are policy implications. Misunderstanding the transmission mechanism can lead to policy mistakes. Essentially, while the model can be parameterized to be observationally equivalent with the real interest rate channel, this channel is not a structural relationship in the model. As such, it is subject to Lucas critique when policy parameters change. We provide an example to illustrate this point.

Our position that the basic model without capital can be misleading in understanding the monetary transmission mechanism in the presence of sticky prices is shared by Barsky, House, and Kimball (2007). Their point, however, is different. Barsky et al. (2007) show that if long-lived and non-durable goods are produced by separate sectors, prices must be sticky in the long-lived goods sector, in order for monetary policy to have aggregate real effects. We carry out the analysis in a more common setup where all goods are produced by a single sector. Furthermore, monetary policy shocks in the model studied here always have real effects, as long as prices are sticky. The point is that such real effects are consistent with the behavior of the ex-ante real rate that is at odds with the real rate channel of monetary transmission.

The New-Keynesian model is usually studied in its log-linear form in the neighborhood of a steady state, under the assumption that the nominal interest rate can increase or decrease without any constraint. Recent studies started to explore the model's behavior at the zero lower bound (e.g., Cochrane, 2016; Kocharlakota, 2016). Among policymakers, Bullard (2015) builds on Cochrane's analysis in his description of the recent nominal environment in G7 countries. We abstract from the issues occurring due to the zero lower bound.

The paper proceeds as follows. Section 2 deals with the basic model without capital. Section 3 covers the model with endogenous capital, either without or with capital adjustment costs. Section 4 briefly relates our findings to recent critiques of New-Keynesian models. Section 5 concludes. Secondary material is contained in an Appendix.

2 The basic model without capital

In the interest of a clear, self-contained exposition, the paper proceeds in a step-by-step fashion, starting with the basic framework. In order not to duplicate well-known derivations from first principles, the starting point of our analysis is the set of conditions characterizing the general equilibrium. The basic model serves the purpose of setting up notation and establishing standard results that are used as a basis for the analysis. Given our interest in the monetary transmission mechanism, we focus only on the responses of the model to monetary policy shocks.

The exposition is based on a standard per-period utility function

$$u = \log c - \frac{l^{1+\eta}}{1+\eta}, \quad \eta \geq 0,$$

and an intermediate goods aggregator

$$y = \left[\int y(j)^{\varepsilon} dj \right]^{\frac{1}{\varepsilon}}, \quad \varepsilon \in (0,1),$$

of the typical intermediate-final good producer setup of the model. The starting point of our analysis is the system of equations describing the general equilibrium, with the New-Keynesian Phillips curve (NKPC) already in its linearized form, around the zero steady-state inflation rate, the usual approximation point in the literature.[5] The derivation of this system from first principles can be found, for instance, in Woodford (2003), Walsh (2010), and Galí (2015). In the spirit of much of the literature, the economy is cashless and monetary policy is formulated as a Taylor-type rule.

The general equilibrium of the basic model is characterized by the following system

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{1}$$

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \right), \tag{2}$$

$$y_t = l_t, \tag{3}$$

$$\chi_t = w_t, \tag{4}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{5}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{6}$$

$$y_t = c_t. \tag{7}$$

Here, c_t is consumption, w_t is a real wage rate, l_t is labor, η is a parameter governing the labor supply elasticity, i_t is a one-period nominal interest rate, π_t is the inflation rate between periods $t-1$ and t, y_t is output, χ_t is the marginal cost, and ξ_t is a standard mean-zero monetary policy shock. Equation (1) is the consumer's first-order condition for labor, equation (2) is the Euler equation for a one-period nominal bond, that is in zero net supply, equation (3) is a production function, equation (4) gives the marginal cost, equation (5) is the NKPC (for the Rotemberg, 1982, quadratic price adjustment cost specification), equation (6) is the Taylor rule, and equation (7) is the goods market clearing condition. In the NKPC, $\phi \geq 0$ is the Rotemberg cost parameter. Further, $\beta \in (0,1)$ is a discount factor

[5] Most of the literature works with approximation around zero inflation steady state as this yields a simple-looking NKPC allowing a straightforward interpretation. Throughout the paper, we therefore proceed in this tradition. Nevertheless, all the results presented in this paper were cross-checked against results obtained under a nonzero inflation steady-state, without detecting any significant differences.

and $\nu > 1$ is the weight on inflation in the Taylor rule. Variables without a time subscript denote steady-state values (the steady-state value of the inflation rate is equal to zero). As in Cochrane (2011) or Galí (2015) the exposition is cleaner when the weight on output in the Taylor rule is set equal to zero, as implicitly assumed in equation (6).

The linearized NKPC is derived for the Rotemberg specification. It is, however, well-known that the same form is obtained also for the Calvo (1983) specification.[6] Namely, under the Calvo specification,

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{8}$$

where $\theta \in [0, 1]$ is the fraction of producers not adjusting prices in a given period. The mapping between Rotemberg and Calvo NKPC is thus

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = -\frac{1}{\phi(\varepsilon - 1)} > 0.$$

The endogenous variables in the system (1)-(7) are c_t, w_t, l_t, i_t, π_t, y_t, and χ_t. The exogenous variable is the monetary policy shock ξ_t. In the model without capital, the shock is the only state variable. In a linear solution, the dynamics of the endogenous variables are thus fully governed by the exogenous process for the shock; the model parameters affect only the sign and size of the responses of the endogenous variables to the shock.

Eliminating equations (1), (3), (4), and (7) by substitution for c_t, l_t, w_t, and χ_t, the system can be reduced to a three-equation system, that when log-linearized around a steady state (with $y = 1$) becomes

$$-\widehat{y}_t = -E_t \widehat{y}_{t+1} + \widehat{i}_t - E_t \pi_{t+1}, \tag{9}$$

[6] The Calvo specification leads to an aggregation bias that shows up as total factor productivity in the production function (3). This bias, however, disappears once the model is linearized around the zero inflation steady state. The Rotemberg specification, on the other hand, leads to a resource loss that shows up in the goods market clearing condition (7). Again, it disappears in a linearized version of the model. For these reasons, the above general equilibrium system abstracts from these two details.

$$\pi_t = \Omega \widehat{y}_t + \beta E_t \pi_{t+1}, \tag{10}$$

$$\widehat{i}_t = \nu \pi_t + \xi_t. \tag{11}$$

Here, $\widehat{i}_t \equiv i_t - i$ and $\widehat{y}_t \equiv (y_t - y)/y$. Further, $\Omega \equiv -(1+\eta)/[\phi(\varepsilon - 1)] = (1+\eta)(1-\theta)(1-\theta\beta)/\theta > 0$, depending on whether Rotemberg or Calvo NKPC is used. The ex-ante real interest rate is defined as $\widehat{R}_t \equiv \widehat{i}_t - E_t\pi_{t+1}$, which from equation (9) implies $\widehat{R}_t = E_t\widehat{y}_{t+1} - \widehat{y}_t$. The system (9)-(11) is the usual three-equation representation of the basic New-Keynesian model.[7]

2.1 Equilibrium output and inflation

It is convenient to reduce the above system further by substituting out i_t from equation (11) to get two first-order difference equations in two endogenous variables, π_t and y_t,

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1} \tag{12}$$

$$\pi_t = \Omega \widehat{y}_t + \beta E_t \pi_{t+1}. \tag{13}$$

The system (12)-(13) is solved here by the method of undetermined coefficients. Assume that the equilibrium decision rule and pricing function are linear functions of the state variable

$$\widehat{y}_t = a\xi_t \quad \text{and} \quad \pi_t = b\xi_t,$$

where a and b are unknown. The guesses are linear, rather than affine, functions of the state as all variables are expressed as deviations from steady state, and thus are equal to zero when $\xi_t = 0$. Suppose that the monetary policy shock follows a stationary AR(1) process

$$\xi_{t+1} = \rho\xi_t + \epsilon_{t+1}, \quad \rho \in [0, 1),$$

[7] As we are concerned only with monetary policy shocks, it is not necessary to further normalize the variables as deviations from flexible-price levels, as the nominal shock does not affect the flexible-price equilibrium. Deviations from steady state and output/inflation gaps are thus the same thing.

where ϵ_{t+1} is an innovation. Substituting the guesses into the system (12)-(13), evaluating the expectations using the AR(1) process, and aligning terms gives unique equilibrium coefficients[8]

$$a = -\frac{1-\beta\rho}{(1-\rho)(1-\beta\rho)+\Omega(\nu-\rho)} < 0, \tag{14}$$

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega}+(\nu-\rho)} < 0. \tag{15}$$

2.1.1 Flexible prices

It is illustrative to consider two extreme cases of price stickiness. First, suppose that prices are fully flexible ($\theta \to 0$ or $\phi \to 0 \Rightarrow \Omega \to \infty$). The reason for considering this case is that the solution for *inflation* under this assumption is the same as in the case of sticky prices and linear preferences and is approximately the same as in the case of sticky prices, standard preferences, and endogenous capital. In the present model, under flexible prices,

$$a \to 0 \quad \text{and} \quad b \to -\frac{1}{\nu-\rho} < 0.$$

Output in this case is unaffected by the monetary policy shock.

The equilibrium coefficient b can alternatively be obtained by solving forward equation (12), with $\widehat{y}_t = 0$, and excluding explosive paths for inflation (the 'bubble term')

$$\pi_t = -\frac{1}{\nu}\sum_{j=0}^{\infty}\left(\frac{1}{\nu}\right)^j E_t\xi_{t+j} = -\frac{1}{\nu-\rho}\xi_t. \tag{16}$$

This equation makes it clear that under flexible prices inflation is determined only by the expected path of the monetary policy shock, with the real rate playing no role in its determination.

Why is the response of inflation to a positive monetary policy shock negative under

[8]This solution implicitly excludes explosive paths of inflation, a common assumption in the literature under $\nu > 1$.

flexible prices? To understand this, it is helpful to rewrite the monetary policy rule (6) as

$$i_t = (i + \zeta_t) + \nu(\pi_t - \zeta_t), \tag{17}$$

where the new shock ζ_t is related to the original shock as $\zeta_t \equiv -(\nu - 1)^{-1}\xi_t$. The shock ζ_t thus inherits the persistence of the original shock but the two shocks are negatively related. When the policy rule is rewritten as equation (17), the shock ζ_t has an interpretation as an inflation target shock (e.g., Smets and Wouters, 2003; Ireland, 2007). This reformulation makes it easier to see why, when ξ_t increases (the inflation target declines), the equilibrium inflation rate declines.

2.1.2 Fixed prices

Second, suppose that prices are completely fixed ($\theta \to 1$ or $\phi \to \infty \Rightarrow \Omega \to 0$). This case is useful as it shows why the New-Keynesian model can be perceived as working through the real rate channel. When prices are completely fixed

$$a \to -\frac{1}{1-\rho} < 0 \quad \text{and} \quad b \to 0.$$

Now, inflation is unaffected by monetary policy and therefore monetary policy is completely in charge of the ex-ante real interest rate. Observe that output is fully determined by the Euler equation (9) and the monetary policy rule (11), both of which have $\pi_t = 0 \ \forall \ t$ (on the production side, as $\Omega \to 0$, producers become increasingly sensitive to any given change in inflation and, in the limit, find any output level optimal; see the NKPC). Combining equations (9) and (11) yields

$$E_t\widehat{y}_{t+1} - \widehat{y}_t = \xi_t, \tag{18}$$

where the monetary policy shock translates one-for-one to the ex-ante real interest rate, $\xi_t = \widehat{i}_t = \widehat{R}_t$. Monetary policy thus affects output through the real interest rate channel.

Why is the response of output to a positive ξ_t shock negative? According to equation

(18), output is expected to grow as long as ξ_t is positive (the ex-ante real interest rate is above steady state). Because the model is stationary—ξ_t is governed by a stationary AR(1) process—the only way output can grow is if it falls, on the impact of the shock, below its steady state level.

2.1.3 The intermediate case

In general, the combination of the Euler equation (9) and the Taylor rule (11) yields

$$\pi_t = -\frac{1}{\nu}\sum_{j=0}^{\infty}\left(\frac{1}{\nu}\right)^j E_t\xi_{t+j} + \frac{1}{\nu}\sum_{j=0}^{\infty}\left(\frac{1}{\nu}\right)^j E_t\widehat{R}_{t+j} \tag{19}$$

where $\widehat{R}_{t+j} = E_{t+j}\widehat{y}_{t+1+j} - \widehat{y}_{t+j}$. Under flexible prices, only the first infinite sum determines inflation, as in equation (16). Under fixed prices, the two infinite sums exactly cancel each other out. In general, however, the first infinite sum dominates and the response of inflation to a positive monetary policy shock is negative, as reflected by the equilibrium coefficient $b < 0$, given by (15). Observe that the parameter Ω in equations (14) and (15), governing price stickiness, works like a weight shifting the equilibrium coefficients a and b between the fully flexible and completely fixed price solutions.

2.2 Equilibrium nominal and real interest rates

The equilibrium functions for the nominal and the ex-ante real interest rates can be derived, respectively, as

$$\widehat{i}_t = \nu\pi_t + \xi_t = (1 + \nu b)\xi_t$$

and

$$\widehat{R}_t \equiv \widehat{i}_t - E_t\pi_{t+1} = (1 + \nu b - \rho b)\,\xi_t,$$

where $b < 0$ is given by (15). The equilibrium nominal interest rate consists of two terms: a direct effect of the shock in the Taylor rule and an indirect effect due to the response of monetary policy to the equilibrium inflation rate. While the first effect is positive, the second effect is negative and its absolute value increases with the persistence of the shock, as is apparent by inspecting equation (15). The sign of the effect of the monetary policy shock on the equilibrium nominal interest rate thus depends on the persistence of the shock, a well-known issue in the New-Keynesian literature (e.g., Woodford, 2003).

The equilibrium ex-ante real interest rate depends on three effects. In addition to the above two effects, it also depends on expected inflation, captured by the term $-\rho b$. Substituting in for b and rearranging terms gives

$$\widehat{R}_t = \left(1 - \frac{1}{1 + \frac{1-\rho}{\nu-\rho}\frac{1-\beta\rho}{\Omega}}\right)\xi_t, \tag{20}$$

where the expression in the brackets is positive, as the term in the denominator is greater than one. Alternatively, one can see that the ex-ante real interest rate always responds positively to the ξ_t shock by recalling that output always declines on impact of the shock (a is always negative) and converges back to its steady state from that point on (i.e., it is growing). This can only happen, according to the Euler equation, if the deviation from steady state of the ex-ante real interest rate is positive.

3 The model with capital

We now introduce endogenous capital into the above framework with the standard preferences. Existing literature offers limited help in isolating the effect of endogenous capital on the properties of New-Keynesian models. Textbooks (e.g., Walsh, 2010; Galí, 2015) stop at the basic model, while research based on the medium-scale DSGE models (e.g., Christiano et al., 2005; Smets and Wouters, 2007) starts straight away with the full-blown version containing many additional features.[9]

[9]McCandless (2008) comes closest to bridging the gap between the basic model and the medium-scale DSGE models, but his treatment is carried out in the context of a model with money and a monetary policy rule formulated as a money growth rule (as in, e.g., Hairault and Portier, 1993; Yun, 1996; Ellison and Scott, 2000). The New-Keynesian literature, instead, follows Woodford (2003) by abstracting from money and

When endogenous capital is introduced into the model, it allows households to adjust their saving in the aggregate (recall that, in contrast, bonds are in zero net supply). The general equilibrium becomes characterized by the following system

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{21}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \right], \tag{22}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right], \tag{23}$$

$$y_t = k_t^{\alpha} l_t^{1-\alpha}, \tag{24}$$

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha} \left(\frac{k_t}{l_t} \right), \tag{25}$$

$$\chi_t = \left(\frac{r_t}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}, \tag{26}$$

$$\pi_t = \Psi \widehat{\chi}_t + \beta E_t \pi_{t+1}, \tag{27}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{28}$$

$$y_t = c_t + k_{t+1} - (1-\delta) k_t. \tag{29}$$

Here, k_t is capital, r_t is the capital rental rate, and $\delta \in (0,1)$ is a depreciation rate; investment can be defined residually as $x_t \equiv k_{t+1} - (1-\delta)k_t$. Further, $\widehat{\chi}_t \equiv (\chi_t - \chi)/\chi$ and $\Psi \equiv -\chi/[\phi(\varepsilon - 1)] = [(1-\theta)(1-\theta\beta)/\theta] > 0$ is the elasticity of inflation to the marginal cost, expressed for the Rotemberg and Calvo specifications, respectively. The endogenous variables are c_t, w_t, l_t, i_t, π_t, y_t, χ_t, r_t, k_{t+1}; the exogenous variables are ξ_t and k_0.[10]

Notice that (21), (22), and (28) are the same as before. Further, (24), (26), and (27) are

formulates monetary policy as a nominal interest rate (Taylor) rule. Woodford (2003), Chapter 5, extends the basic model to include endogenous capital, but presents results for parameterizations leading to only a subset of the possible outcomes documented here, thus obscuring the mechanism at work (Woodford, 2005, corrects some mistakes contained in that chapter). Galí and Gertler (2007) also extend the basic model to include capital, but do not study the transmission mechanism.

[10] As in the baseline model, for the reasons discussed earlier, the exposition abstracts from the aggregation bias in the case of Calvo pricing and the resource loss in the case of Rotemberg pricing.

the same as before for $\alpha = 0$. The truly new equations are equations (23) and (25), that add the two new endogenous variables, k_{t+1} and r_t. Equation (23) is the Euler equation for capital and equation (25) is a condition for the optimal mix of capital and labor in production; it equates the marginal rate of technological substitution with the relative factor prices (a first-order condition of a cost minimization problem of each firm j).[11]

This model contains the key element of a prototypical RBC model: capital accumulation as a means for the economy as a whole to smooth out consumption in the presence of fluctuations in income (output). In fact, under flexible prices, the model collapses into a RBC model with two additions, the Euler equation for bonds and the Taylor rule. To see this, note that under flexible prices, $\Psi \to \infty$. The NKPC (27) then implies $\widehat{\chi}_t = 0$, or $\chi_t = \chi$. If, in addition, $\varepsilon = 1$ (perfect competition), $\chi = 1$; see Galí (2015), Chapter 3. This is a standard profit maximization condition under perfect competition, stating that the marginal cost is equal to the good's relative price, that is equal to one, as all goods are perfect substitutes. When this condition is used in equation (26), and the resulting equation is combined with the cost minimization condition (25), we get the standard RBC conditions equalizing marginal products to factor prices: $r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha}$ and $w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha}$. Under flexible prices, these two conditions replace equations (25) and (26) in the above system. Notice that the system becomes recursive (a classical dichotomy): equations (21), (23), (24), (29), and the above two marginal product conditions—the standard RBC system—determine c_t, w_t, l_t, y_t, r_t, and k_{t+1}, given k_0, independently of ξ_t (in addition, $\chi_t = 1$ from the NKPC). Equations (22) and (28) then pin down i_t and π_t. The NKPC (27), with $\Psi < \infty$, is what breaks the classical dichotomy under sticky prices.

[11] We follow the simpler setup in which capital can be rented each period on an economy-wide rental market. Woodford (2005) considers the opposite case in which capital is firm specific. In equilibrium, the two environments differ only in the form of the reduced-form parameter Ψ. The Woodford (2005) setup implies that a given value of Ψ is consistent with prices being sticky for a shorter period of time than in the model with common rental market. While this implication has a clear empirical appeal, the distinction between the two environments is unimportant for the points made in this paper.

3.1 The log-linear system

In what follows it is convenient to substitute in for r_t in the expression for the marginal cost (26) from the cost minimization condition (25). The marginal cost then becomes

$$\chi_t = \frac{w_t}{1-\alpha}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}.$$

Observe that for $\alpha = 0$ this expression becomes the same as in the model without capital. Further, substitute in for l_t in the first-order condition for labor (21) from the production function (24). This gives the first-order condition for labor as

$$\frac{w_t}{c_t} = \left(\frac{y_t}{k_t^{\alpha}}\right)^{\frac{\eta}{1-\alpha}}.$$

Again, for $\alpha = 0$, this condition is the same as in the model without capital.

With the above two substitutions, we can log-linearize the general equilibrium system to get

$$-\widehat{c}_t + \widehat{w}_t = \frac{\eta}{1-\alpha}\widehat{y}_t - \frac{\alpha\eta}{1-\alpha}\widehat{k}_t$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{r}_{t+1},$$

$$\widehat{l}_t = \frac{1}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\widehat{r}_t = r(\widehat{l}_t - \widehat{k}_t + \widehat{w}_t),$$

$$\widehat{\chi}_t = \widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\pi_t = \Psi\widehat{\chi}_t + \beta E_t\pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t.$$

Here, variables without time subscripts are steady-state values, interest rates are expressed as percentage point deviations from steady state, $\widehat{r}_t \equiv r_t - r$, $\widehat{i}_t \equiv i_t - i$, and all other variables are expressed as percentage deviations from steady state, e.g., $\widehat{c}_t \equiv (c_t - c)/c$. Eliminating $\widehat{r}_t$, $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{i}_t$, and $\widehat{l}_t$ we get a final system of four equilibrium first-order difference equations in four endogenous variables $\widehat{c}_t$, $\widehat{y}_t$, $\widehat{k}_{t+1}$, and $\widehat{\pi}_t$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{30}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{31}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{32}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t. \tag{33}$$

Relative to the basic model, the number of equations has increased to four. This is because of the presence of the extra variable $\widehat{k}_{t+1}$ and because $\widehat{c}_t \neq \widehat{y}_t$ due to investment. Here, (30) is the same as in the model without capital, (33) is the same as in the model without capital for $k = 0$, and (32) is the same as in the model without capital for $\widehat{k}_t = 0$ and $\alpha = 0$. Only equation (31), the Euler equation for capital, is genuinely new.

Observe that when prices are fully flexible ($\Psi \to \infty$), the NKPC (32) implies that, given a steady-state initial condition ($\widehat{k}_t = 0$), $\widehat{c}_t = \widehat{y}_t = 0$. Equation (33) then implies $\widehat{k}_{t+1} = 0$. Monetary policy is neutral and inflation is determined from equation (30) in exactly the same way as under flexible prices in the basic model. The addition of capital thus does not change consumption, output, or inflation responses to the monetary policy shock when prices are fully flexible.

Consider now the other extreme, when prices are completely fixed ($\Psi \to 0$). Now, like in the model without capital, the NKPC (32) implies that inflation is equal to zero. Further, equation (30) determines consumption as a function of the monetary policy shock in the same way as in the model without capital and a positive monetary policy shock reduces consumption. The presence of capital thus has no effect on equilibrium consumption or inflation when prices are completely fixed either (it, however, affects output differently than in the basic model due to the presence of investment). The interesting case is the case in-between the two extremes, to which we turn next.

3.2 The monetary transmission mechanism—a first look

Here we take a first look at the transmission mechanism in the presence of capital and revisit it in more detail in the sections that follow. As in equilibrium all variables are determined simultaneously by the system of the difference equations (30)-(33), it is difficult to gain insight into the exact inner workings of the model. However, an approximate description can be made once we realize that for any plausible calibration, the steady-state capital rental rate r is close to zero (under standard calibrations it is around 0.03, given by $1/\beta - 1 + \delta$). Let us therefore proceed under the assumption that the steady-state capital rental rate is in fact equal to zero. This greatly simplifies the analysis and provides a useful insight, as the system (30)-(33) becomes recursive.[12] Equation (31) then implies $\widehat{c}_t = E_t\widehat{c}_{t+1}$; that is, the presence of capital allows perfect consumption smoothing across time. Further, as the model is stationary (see below), it then has to be the case that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Otherwise, under $\widehat{c}_t = E_t\widehat{c}_{t+1}$, a given shock would lead to a permanent shift of consumption (in expectations) away from the steady state, violating stationarity. With consumption determined, equation (30) then determines the equilibrium inflation rate, that depends only on the monetary policy shock. The solution for the inflation rate is therefore the same as in the model without capital under flexible prices, $\pi_t = -[1/(\nu - \rho)]\xi_t$, even though prices here are sticky, though not completely fixed. The inflation rate falls on the impact of the shock and converges back to zero from below. Along this path, $\pi_t - E_t\pi_{t+1}$ is negative. Thus, for β close to one, equation (32) implies that on the impact of the shock output has to decline. This is because $\widehat{c}_t = 0$

[12] We check that the description of the mechanism that follows under this assumption is consistent with the actual workings of the model by computing, in the next section, impulse responses for the exact calibrated value of r.

for the above reasons and $\widehat{k}_t = 0$, as in the impact period the existing capital stock is in steady state. From equation (33) then, $\widehat{k}_{t+1}$ has to decline; i.e., the decline in output is fully absorbed by a decline in investment.[13]

What is going on? Essentially, the presence of endogenous capital, as summarized by equation (31), allows the economy as a whole to smooth out fluctuations in output (income) brought about by the monetary policy shock in the presence of sticky prices. Because investment makes up only a small fraction of the capital stock, consumption can be kept smooth by adjusting investment with a minimal effect on the expected rate of return on capital, and thus—through arbitrage—on the ex-ante real interest rate (the exact, though small, effects on the ex-ante real rate are derived below). Effectively, the presence of endogenous capital makes consumption sticky and *equilibrium* prices behave as in a flexible-price economy, being pinned down by the first term in equation (19). Given the equilibrium inflation, each individual firm that cannot optimally adjust prices to keep up with the equilibrium path of the aggregate inflation rate adjusts output. When equilibrium inflation temporarily drops, those firms that cannot adjust prices reduce output. A temporary drop in households' income brought about by the lower output is then smoothed out by lower investment.[14]

While this mechanism holds only approximately (we have assumed $r = 0$), the numerical experiments below show that it provides a good description of how the model works. It shows again that monetary policy affects output in New-Keynesian models even if the ex-ante real interest rate does not change. This is in a sharp contrast with the narrative of the real interest rate channel and the usual description of how the monetary transmission mechanism in New-Keynesian models works. And even though the model with capital may produce responses of consumption that are from empirical perspective too smooth, it uncovers that the mechanism through which monetary policy shocks have real effects does not operate through the real

[13] As will be demonstrated below, in the actual solution, the decline in next period's capital stock leads to a small decline in next period's consumption, relative to today's, and thus to a small decline in the ex-ante real interest rate.

[14] Note that this description of the transmission mechanism is sensible only for the case in which prices are sticky, but not completely fixed. As equation (30) implies, it cannot be that, in response to the monetary policy shock, both consumption and prices are completely fixed.

interest rate channel.

3.3 Model solution and numerical impulse responses

The model can be again solved by the method of undetermined coefficients (this is done for the general non-zero value of r). There are two state variables, k_t and ξ_t. The solution thus has the form:

$$\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t, \quad \pi_t = b_0\widehat{k}_t + b_1\xi_t, \quad \widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t, \quad \text{and} \quad \widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t.$$

Substituting these functions into the system (30)-(33), evaluating expectations, and collecting terms yields a system of eight equations in eight unknowns, the coefficients of the above four linear functions. The resulting system is provided in the Appendix. It is recursive, whereby a_0, b_0, d_0, and f_0 can be solved for independently of a_1, b_1, d_1, and f_1. The persistence of the shock, ρ, thus has no effect on the equilibrium coefficients loading onto $\widehat{k}_t$. In other words, the internal dynamics of the model are unaffected by the dynamics of the shock. The equilibrium coefficients loading onto $\widehat{k}_t$, however, affect a_1, b_1, d_1, and f_1 and thus the responses of the endogenous variables to the monetary policy shock. In other words, the presence of capital affects the responses of the endogenous variables to the shock.

From here we proceed numerically, using the following standard parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$. The persistence of the monetary policy shock is treated as a free parameter and we consider five values, $\rho \in \{0, 0.1, 0.5, 0.95, 0.995\}$.[15]

Figures 1-5 display responses to a 1 percentage point increase in ξ_t in period $t = 1$ for the above five values of ρ. Interest and inflation rates are reported as percentage point deviations from steady state; all other variables as percentage deviations from steady state.

[15] Previous studies have investigated how endogenous capital affects the determinacy of equilibria (e.g., Dupor, 2001; Carlstrom and Fuerst, 2005; Kurozumi and Van Zandweghe, 2008). In our setup the findings of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) hold and the Taylor principle with respect to current inflation ensures a unique nonexplosive equilibrium.

The impulse-response functions confirm our conjecture from the previous subsection that the real effects of monetary policy shocks in the model with capital have nothing to do with the real interest rate channel. In all cases but $\rho = 0$, output and inflation fall, in response to the positive monetary policy shock, while the ex-ante real interest rate declines (the nominal interest rate declines as well in all but the case of $\rho = 0$). Experimentation reveals that the real interest rate increases only for $\rho \in [0, 0.04]$; output and inflation, however, fall for all values of $\rho \in [0, 1)$.[16]

Regarding consumption and investment, both variables fall, although consumption falls only a little, in line with our discussion in the previous subsection. In the cases of high persistence ($\rho = 0.95$ and $\rho = 0.995$), consumption initially somewhat increases, before declining below the steady state. Sometimes it is argued that it is the long-term real interest rate, rather than the one-period real interest rate, that is crucial for the workings of the real interest rate channel. The cases of high persistence, however, show that in the model with endogenous capital, output and inflation can decline even when both short- and long-term ex-ante real interest rates decline. We see in the figures that the short rate declines. To see that also the long rate declines, observe that by forward substitution of the log-linear Euler equation for bonds, and imposing stationarity,

$$\widehat{c}_t = -\sum_{j=0}^{\infty} \widehat{R}_{t+j}.$$

Here, the right-hand side can be interpreted as the ex-ante long-term real interest rate (as in the expectations hypothesis). Because in the cases of high persistence $\widehat{c}_t$ increases on the impact of the shock, this equation implies that the long-term real interest rate declines. Thus, in the model with endogenous capital, inflation and output decline, in response to the monetary policy shock, even though the ex-ante short- and long-term real interest rates

[16]In all cases, the real interest rate increases above its steady-state level several periods after the impact of the shock due to the decline in capital—once the effect of sticky prices (the NKPC) dies off, the dynamics of the real rate become governed by the marginal product of capital, as in a real business cycle model. The decline in capital increases its marginal product and thus the real rate.

decline.

3.4 Consumption smoothing and the real interest rate

How is the ex-ante real interest rate determined and why does the model have such a hard time generating its increase in response to the monetary policy shock? We offer explanations from two perspectives, using either the equilibrium consumption (a more straightforward explanation) or inflation functions.

First, use the log-linear Euler equation for bonds to write

$$\widehat{R}_t = E_t\widehat{c}_{t+1} - \widehat{c}_t =$$

$$= a_0(f_0\widehat{k}_t + f_1\xi_t) + a_1\rho\xi_t - (a_0\widehat{k}_t + a_1\xi_t)$$

$$= a_0(f_0 - 1)\widehat{k}_t + (\rho a_1 - a_1 + a_0 f_1)\xi_t. \tag{34}$$

Focus on the immediate response from steady state, thus setting $\widehat{k}_t = 0$. The coefficient loading onto ξ_t in equation (34) consists of three terms. The first two terms are the same as in the model without capital, although the value of a_1 may now be different. We have shown in Section 2 that these two terms in the basic model always lead to a positive response of $\widehat{R}_t$ to ξ_t. The third term is related to the direct role of endogenous capital. Here, f_1 gives the equilibrium response of $\widehat{k}_{t+1}$ to ξ_t. It is negative, reflecting a decline in investment—due to consumption smoothing—in response to a drop in output brought about by a positive ξ_t shock. The coefficient a_0 gives the equilibrium response of $\widehat{c}_{t+1}$ to $\widehat{k}_{t+1}$. This coefficient is positive, prescribing a lower consumption when capital is lower.[17] The two coefficients together thus prescribe a drop in tomorrow's consumption, relative to consumption today, in response to a positive ξ_t shock today. Declining consumption then implies a fall in the ex-ante real interest rate. The consumer is effectively trading off not having to adjust consumption in line with the fall in income on the impact of the shock, for a slightly lower consumption

[17] The signs of the coefficients are established numerically.

in the immediate future due to lower capital stock. If this direct effect of endogenous capital is strong enough, the real rate declines even if $\rho a_1 - a_1$ is positive as in the basic model.

Second, we can write the equilibrium function for the real rate as

$$\begin{aligned}
\widehat{R}_t &= \widehat{i}_t - E_t\pi_{t+1} \\
&= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\
&= \nu(b_0\widehat{k}_t + b_1\xi_t) + \xi_t - E_t(b_0\widehat{k}_{t+1} + b_1\xi_{t+1}) \\
&= b_0(\nu - f_0)\widehat{k}_t + (1 + \nu b_1 - \rho b_1 - b_0 f_1)\xi_t. \qquad (35)
\end{aligned}$$

Consider again the immediate response from steady state, thus setting $\widehat{k}_t = 0$. As an example, focus on the mid-case of $\rho = 0.5$. This yields equilibrium functions $\pi_t = -0.057\widehat{k}_t - 1.44\xi_t$ and $\widehat{k}_{t+1} = 0.936\widehat{k}_t - 1.56\xi_t$. Observe that the equilibrium coefficient loading onto ξ_t in equation (35) consists of four terms. The first three terms, $1 + \nu b_1 - \rho b_1$, are the same as in the basic model, even though the value of b_1 may now be different. In the model without capital, for $\rho = 0.5$, the equilibrium function for inflation is $\pi_t = -0.35\xi_t$ (i.e., $b_1 = -0.35$), that results in the sum of the three terms being positive (again, we have shown that in the basic model the sum of the three terms is always positive). Now, however, $b_1 = -1.44$.[18] As a result, the sum of the three terms is now negative. Furthermore, there is a fourth term in equation (35), $-b_0 f_1$. This term is related to the direct role of endogenous capital. Specifically, f_1 is the equilibrium response of $\widehat{k}_{t+1}$ to ξ_t and b_0 is the equilibrium response of π_{t+1} to $\widehat{k}_{t+1}$. The product of f_1 and b_0 thus captures the equilibrium response of expected inflation to today's monetary policy shock working directly through a change in the capital stock. As both f_1 and b_0 are negative ($f_1 = -1.56$ and $b_0 = -0.057$), the product is positive and the capital channel increases inflation expectations, thus contributing to a decline in the ex-ante real interest rate.

[18] The coefficient is now more negative due to the decline in the real rate, that—through the second term in equation (19)—pushes inflation down above and beyond the effect of the first term.

The reason for the negative response of k_{t+1} to ξ_t is consumption smoothing. The reason for the increase in π_{t+1} in response to a decline in $\widehat{k}_{t+1}$ is that—in equation (19)—inflation depends positively on the ex-ante real interest rate, that, in turn, depends negatively on the capital stock; as shown above, below-steady-state capital at $t+1$ reduces consumption at $t+1$ below its steady-state level, that increases the ex-ante real interest rate from $t+1$, as consumption from that point on is expected to grow along the convergence of path of capital back to steady state.

Observe that the direct effect of endogenous cspital plays a negative role even when the first three terms generate a positive response of the ex-ante real rate, as in the baseline model. For instance, the case of $\rho = 0.1$ yields $b_1 = -0.7$ and thus a positive sum of the first three terms. The fourth term, however, counterweights them and the real rate declines (Figure 2). Out of the five cases considered, only in the case of $\rho = 0$ is the sum of the first three terms positive and large enough to dominate the fourth term, thus producing an increase in the real rate (Figure 1).

3.5 Adjustment costs

One way to reduce consumption smoothing is to introduce into the model capital adjustment costs. Suppose that whenever the household changes the capital stock, it has to incur a cost in terms of foregone real income. The simplest form of capital adjustment costs is a quadratic cost function

$$-\frac{\kappa}{2}(k_{t+1} - k_t)^2, \quad \kappa \geq 0.$$

In steady state, the adjustment cost is equal to zero. Further, as the adjustment cost is quadratic, it does not affect the resource constraint of the economy in a log-linear approximation of the model around a steady state. The Euler equation for capital now becomes

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\frac{r_{t+1} - \delta}{q_t} + \frac{q_{t+1}}{q_t} \right) \right],$$

where $q_t \equiv 1+\kappa(k_{t+1}-k_t)$ is Tobin's q, the price of capital in terms of current consumption. Notice that for $\kappa = 0$, the Euler equation collapses into the Euler equation in the version without adjustment costs. The expression in the round brackets has an interpretation as a sum of a dividend yield and a capital gain. Denote the capital gain by $G_{t+1} \equiv q_{t+1}/q_t$.

The log-linearized system of the model now becomes

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{36}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{37}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{38}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t, \tag{39}$$

where $\widehat{G}_{t+1} = \widehat{q}_{t+1} - \widehat{q}_t = \overline{\kappa}(\widehat{k}_{t+2} - \widehat{k}_{t+1}) - \overline{\kappa}(\widehat{k}_{t+1} - \widehat{k}_t)$ is a percentage deviation of capital gains from steady state and $\overline{\kappa} \equiv \kappa k$. Only equation (37) in the above system is different, compared with the system of the previous subsection. Of course, it coincides with equation (31) if $\kappa = 0$. Combining equations (36) and (37), the ex-ante real interest rate can be written as a sum of capital gains and the expected dividend yield

$$\begin{aligned}\widehat{R}_t &\equiv i_t - E_t\pi_{t+1} \\ &= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\ &= E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right).\end{aligned} \tag{40}$$

The exposition proceeds again under the simplifying assumption that $r \approx 0$. Under this assumption, the dividend term in equations (37) and (40) drops out. Now, however, the capital gains term in equation (37) does not allow us to conclude that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Capital adjustment costs prevent perfect consumption smoothing, resulting in $\widehat{c}_t \neq E_t\widehat{c}_{t+1} \neq 0$. Any drop in output dictated by inflation dynamics and the NKPC has to be, at least partially, accommodated by a drop in consumption. The higher is κ, the more any given change in output is accounted for by a change in consumption, rather than investment. Increasing κ thus brings the response of consumption closer to the response of output and thus closer to the response of consumption in the basic model; i.e., consumption falls on impact and converges back to steady state from below. As a result, $\widehat{R}_t = E_t\widehat{c}_{t+1} - \widehat{c}_t > 0$. By bringing the consumption response closer to the basic model, capital adjustment costs also bring the response of inflation closer to the basic model. From (36), $\pi_t = \nu^{-1}\left(-\xi_t + E_t\pi_{t+1} + E_t\widehat{c}_{t+1} - \widehat{c}_t\right)$. This equation is the same as equation (12) and the response of consumption is similar to that in the basic model. When $\kappa = \infty$, the responses in the model with capital coincide with those in the basic model.

The model can again be solved by the method of undetermined coefficients, guessing $\widehat{c}_t$, π_t, $\widehat{y}_t$, and $\widehat{k}_{t+1}$ as linear functions of $\widehat{k}_t$ and ξ_t. Relative to the system of restrictions in the model without adjustment costs, only the restrictions resulting from equation (37) are different. These are contained in the Appendix.

Figures 6-8 show the responses of the model under $\rho = 0.5$ and $\kappa \in \{0.1, 0.2, 0.5\}$. The rest of the parameterization is as before. The figures show that as κ increases, the model starts to produce responses consistent with the real interest rate channel. Specifically, at $\kappa = 0.1$ the model still suffers from producing a decline in the nominal interest rate and only a gradual increase in the ex-ante real interest rate. At $\kappa = 0.2$, the ex-ante real interest rate increases on impact, but the nominal interest rate still falls. At $\kappa = 0.5$, finally, both the ex-ante real interest rate and the nominal interest rate increase on impact. Throughout these experiments, the increase in the ex-ante real rate occurs due to expected capital gains.

Sufficiently high capital adjustment costs, as in Figure 8, thus make the model consistent with the real interest rate channel. As before, consumers want to smooth consumption when income declines, as producers cut output in the face of a drop in inflation, as dictated by the NKPC. To prevent consumption smoothing in equilibrium, expected capital gains—and thus the ex-ante real interest rate—have to sufficiently increase.

The consistency with the real interest rate channel, however, is only observational and thus subject to change when policy parameters change. To illustrate this point, we contrast Figure 8, which is observationally equivalent to the real interest rate channel, with Figure 9. Figure 9 plots again the responses for $\kappa = 0.5$, but under a shock persistence $\rho = 0.85$, instead of $\rho = 0.5$. Under $\rho = 0.85$, both inflation and output again decline, yet the ex-ante real interest rate declines as well (the decline in output is in fact as large as in Figure 8, but much more persistent). An econometrician estimating a VAR on data generated by the model with $\rho = 0.5$ would conclude a presence of the real interest rate channel. A policy advice based on such evidence would then be that the central bank needs to increase the ex-ante real interest rate in order to reduce inflation and output. Such advice, however, would be misguided—the same model, but with a different persistence parameter for the policy shock, predicts that about the same effect on inflation and output can be achieved with a decline in the real rate. The reason for such a contrasting policy recommendation is that the whole focus on the ability to affect the ex-ante real interest rate in the conduct of monetary policy is misguided in the context of the New-Keynesian model. In order to affect output, all that is required of the central bank *in the model* is to affect inflation. This is achieved in the model by persistently changing the nominal interest rate and letting the bond market—as summarized by equation (19)—do its job, translating such a change into inflation.

4 Relating the findings to recent critiques

Although this paper is meant to be constructive, rather than critical, it is worth relating our findings to some recent critiques of New-Keynesian models. Specifically, some researchers have been skeptical about the real interest rate channel as it relies on a sensitivity of expenditures to real interest rates perceived to be unrealistic (Bernanke and Gertler, 1995; Taylor, 1995, contain references). By invoking this channel, the New-Keynesian literature exposes itself to the same criticism. A recent critique along these lines has been developed, for instance, by Kaplan, Moll, and Violante (2015). Our analysis, however, shows that the monetary transmission mechanism in New-Keynesian models does not operate through the real interest rate channel, even through the model's behavior can be observationally equivalent to that channel at a particular parameterization.[19]

New-Keynesian models have also been critiqued on a number of other grounds, that apply to the environments and solutions used in this paper. Cochrane (2011) attacks New-Keynesian models on the basis that the way inflation is determined under a Taylor rule is ad hoc and that Taylor rule parameters cannot be identified. One could also argue, along the lines of Gomme, Ravikumar, and Rupert (2011), that in the model with capital there is too close a connection between the short-term real interest rate and the return on capital. Nekarda and Ramey (2013) point out that New-Keynesian models exhibit counterfactual behavior of markups. Broer, Hansen, Krusell, and Oberg (2015) highlight the models' difficulties in a worker-capitalist setup. Finally, Chari, Kehoe, and McGrattan (2009) question a number of the additional features of the medium-scale models that we have abstracted from.

5 Conclusion

How does monetary policy affect inflation and output in the economy? A widely accepted view is that it is through its effect on the ex-ante real interest rate. In this paradigm, a common justification for the transmission from the nominal interest rate, the policy instrument, to the real interest rate, a price that ultimately affects decisions of the private sector, rests on nominal price rigidities. Introducing this channel into a modern dynamic stochastic general equilibrium environment was one of the motivations for the development of New-Keynesian models. These models, both in their basic and extended (medium-scale DSGE)

[19] The Kaplan et al. (2015) critique, however, still applies in the sense that the features they emphasize—household heterogeneity and illiquid assets—are important for generating empirically plausible responses of consumption, especially at the individual household level.

forms are routinely used at central banks around the world to guide monetary policy. This paper scrutinizes the inner workings of the transmission mechanism in this important class of models.

We demonstrate that the transmission mechanism does not operate through the real interest rate channel. Instead, as a first pass, inflation is determined as in a flexible price model, through current and expected future monetary policy shocks, while output is then pinned down by the New-Keynesian Phillips curve. The ex-ante real interest rate only reflects the desire and ability of households to smooth consumption in response to movements in output (income). An increase, decline, or no change in the ex-ante real interest rate are all consistent with declines in output and inflation in response to a standard contractionary monetary policy shock. High enough capital adjustment costs make the model appear as if it operated through the real interest rate channel. This relationship, however, is not structural and is thus subject to a change when policy parameters change.

The policy implications of our analysis are that (i) either monetary policy in actual economies does transmit through the real interest rate channel, but then the New-Keynesian model—in the form currently used in the literature—may be a misleading tool for its analysis or (ii) the New-Keynesian model—for its micro-foundations of the price-setting behavior and internal consistency—is a useful tool in its own right for the analysis of monetary policy, but then policy makers need to rethink the way monetary policy transmits into inflation and real activity. This paper provides a way how to think more accurately about the transmission mechanism in New Keynesian models.

Appendix

This Appendix contains the systems that determine the equilibrium coefficients in the version with capital, both without and with capital adjustment costs.

The equilibrium functions in the model with capital take the form: $\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t$, $\pi_t = b_0\widehat{k}_t + b_1\xi_t$, $\widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t$, and $\widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t$. In the version without adjustment costs, using these functions in the system (30)-(33) and aligning terms yields a system of eight equations in eight unknowns, a_0,a_1, b_0, b_1, d_0, d_1, f_0, f_1. From equation (30) we get:

$$-a_0 = -a_0 f_0 + \nu b_0 - b_0 f_0,$$

$$-a_1 = -a_0 f_1 - a_1 \rho + \nu b_1 - b_0 f_1 - b_1 \rho + 1.$$

From equation (31):

$$-a_0 = -(1-r)a_0 f_0 + \frac{r(1+\eta)}{1-\alpha} d_0 f_0 - \frac{r(1+\alpha\eta)}{1-\alpha} f_0,$$

$$-a_1 = -(1-r)a_0 f_1 - (1-r)a_1 \rho + \frac{r(1+\eta)}{1-\alpha} d_0 f_1 + \frac{r(1+\eta)}{1-\alpha} d_1 \rho - \frac{r(1+\alpha\eta)}{1-\alpha} f_1.$$

From equation (32):

$$b_0 = -\psi \frac{\eta+\alpha}{1-\alpha} d_0 + \psi \frac{\alpha\eta+\alpha}{1-\alpha} - \psi a_0 + \beta b_0 f_0,$$

$$b_1 = -\psi \frac{\eta+\alpha}{1-\alpha} d_1 - \psi a_1 + \beta b_0 f_1 + \beta b_1 \rho.$$

And from equation (33):

$$f_0 = \frac{y}{k} d_0 - \frac{c}{y} a_0 + (1-\delta),$$

$$f_1 = \frac{y}{k} d_1 - \frac{c}{y} a_1.$$

With capital adjustment costs, the second pair of equations becomes

$$-\overline{\kappa} - a_0 + (1-r)a_0 f_0 - \frac{r(1+\eta)}{1-\alpha} d_0 f_0 + \frac{r(1+\alpha\eta)}{1-\alpha} f_0 + 2\overline{\kappa} f_0 - \overline{\kappa} f_0 = 0,$$

$$-a_1 + (1-r)a_0 f_1 + (1-r)a_1 \rho - \frac{r(1+\eta)}{1-\alpha} d_0 f_1 - \frac{r(1+\eta)}{1-\alpha} d_1 \rho + \frac{r(1+\alpha\eta)}{1-\alpha} f_1 + 2\overline{\kappa} f_1 - \overline{\kappa} f_0 f_1 - \overline{\kappa} f_1 \rho = 0.$$

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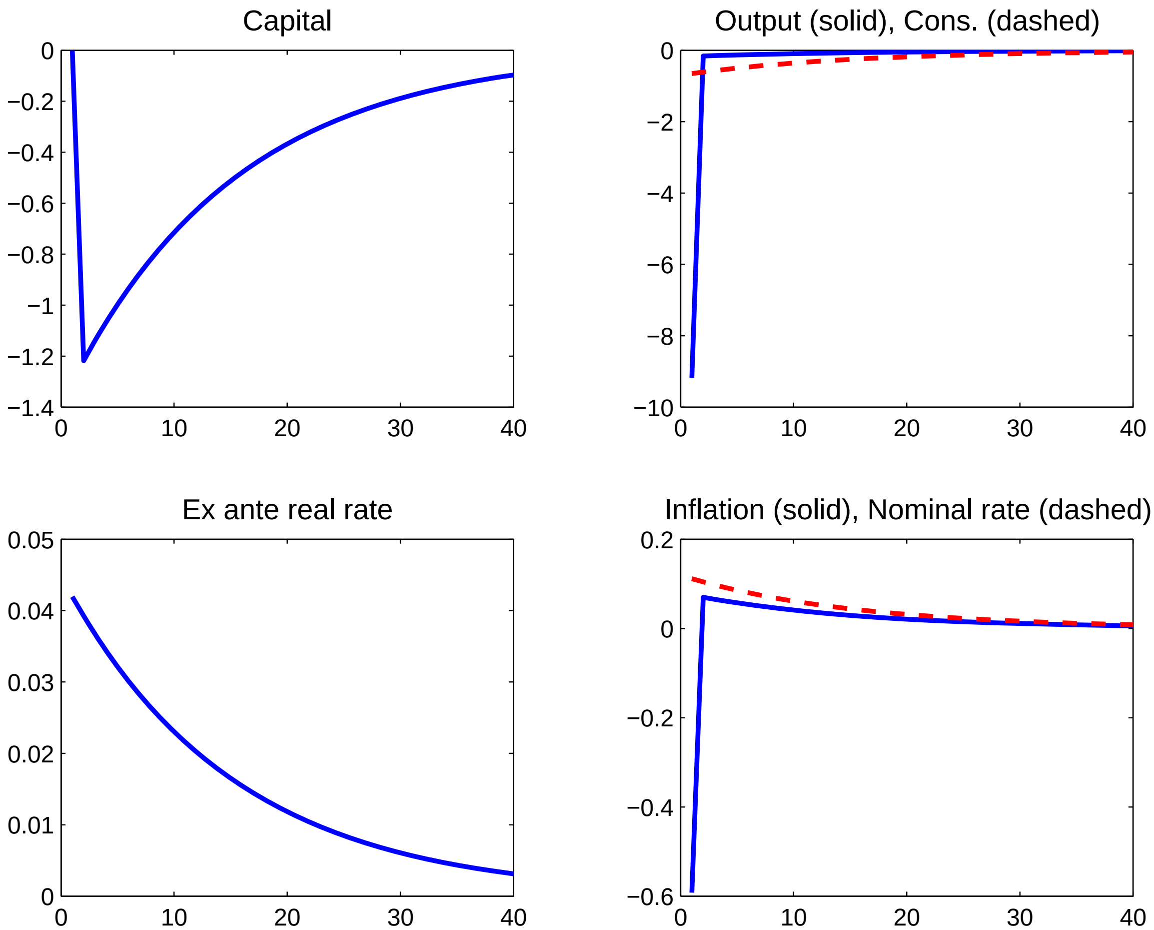

Figure 1: The model with capital, $\rho = 0$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

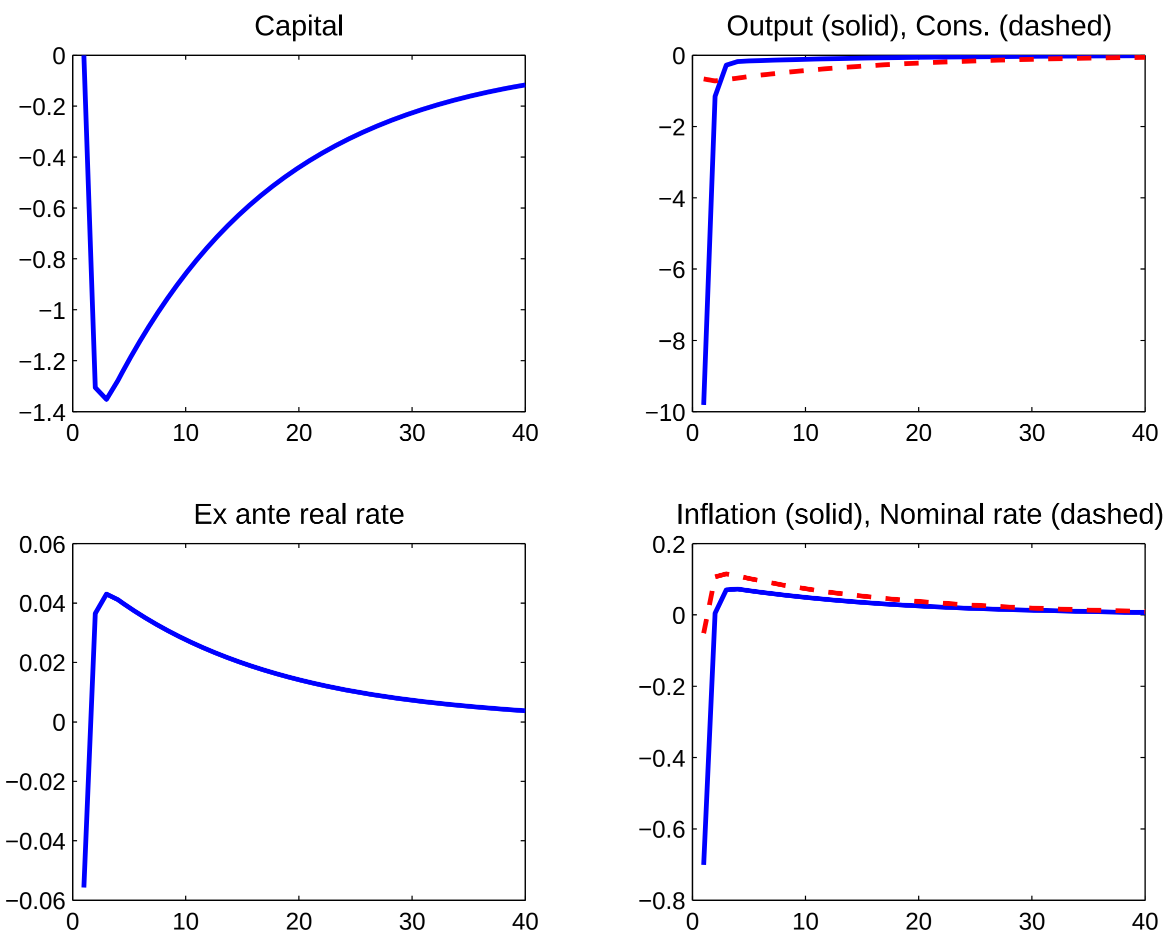

Figure 2: The model with capital, $\rho = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

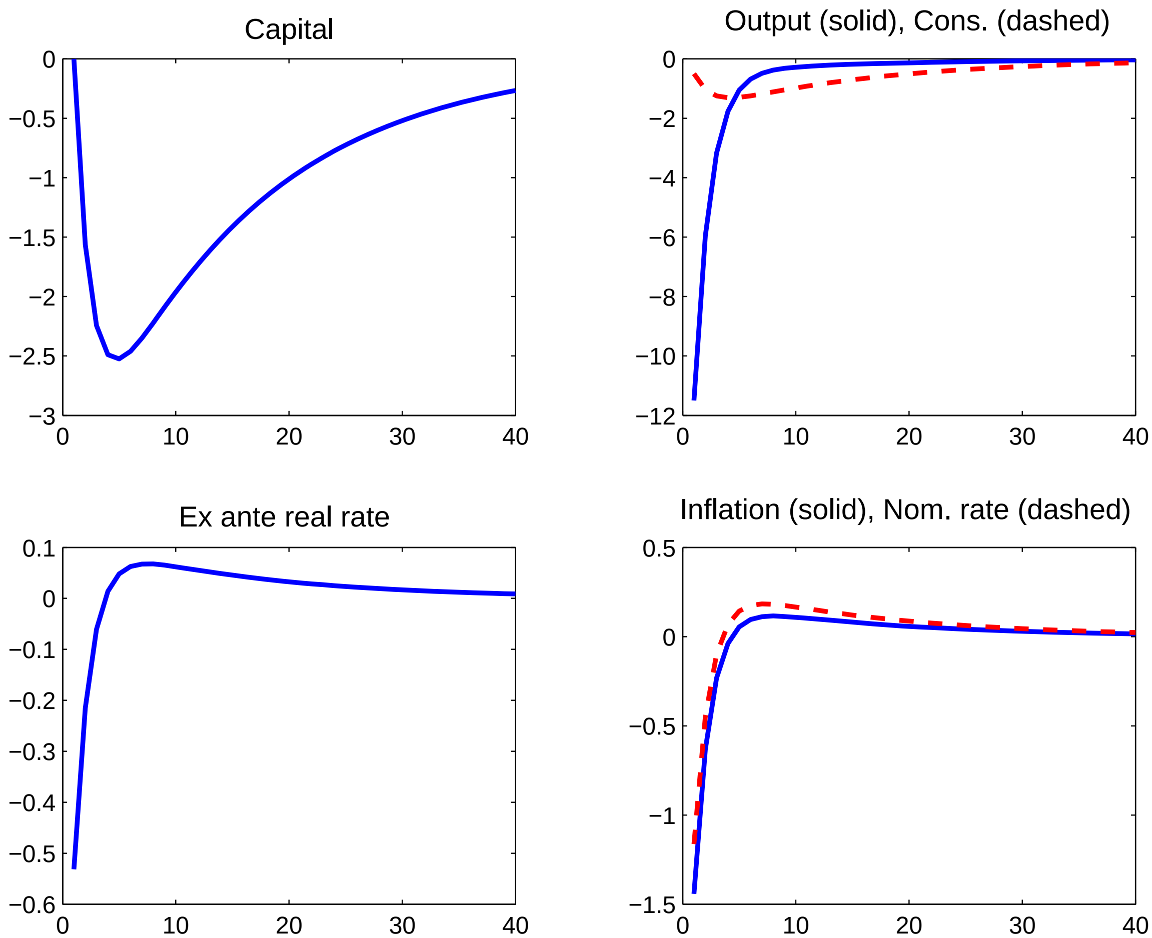

Figure 3: The model with capital, $\rho = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

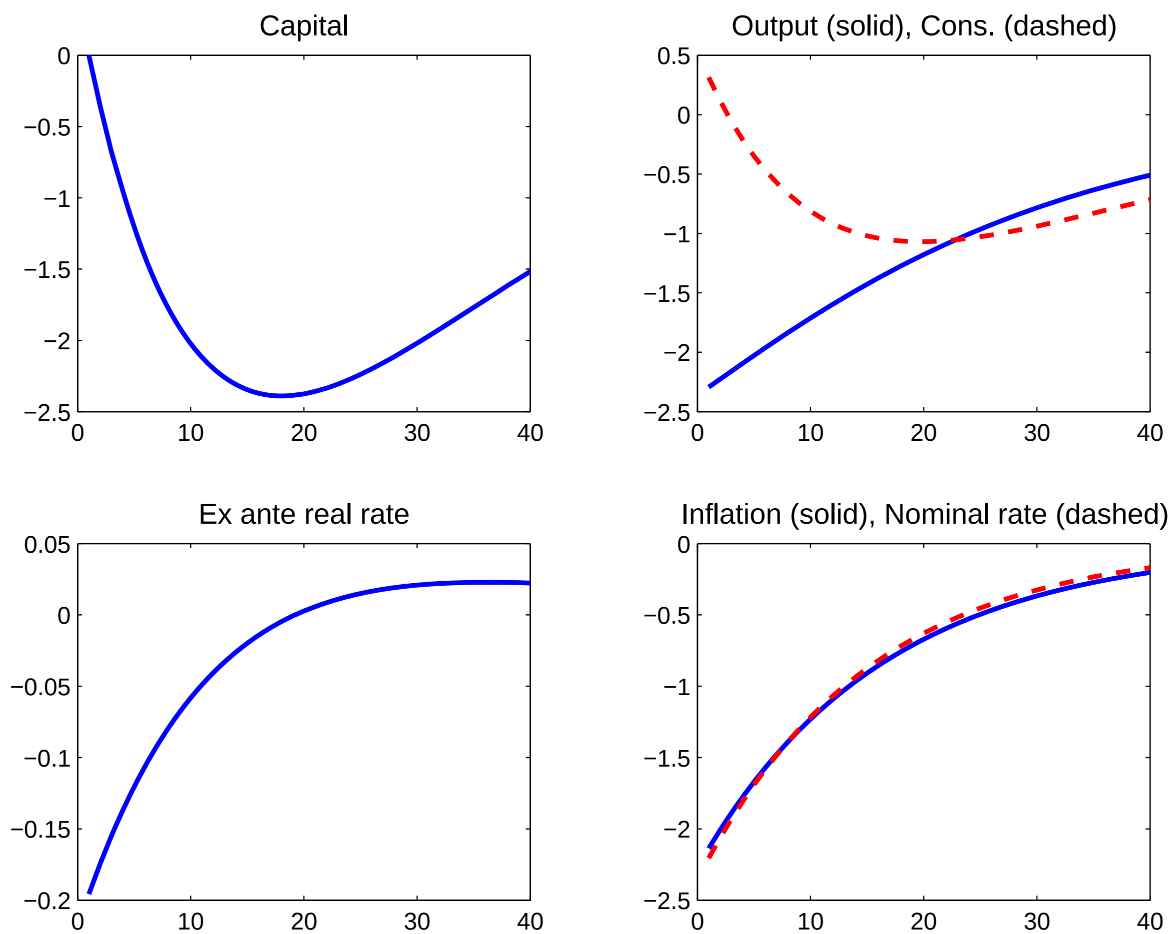

Figure 4: The model with capital, $\rho = 0.95$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

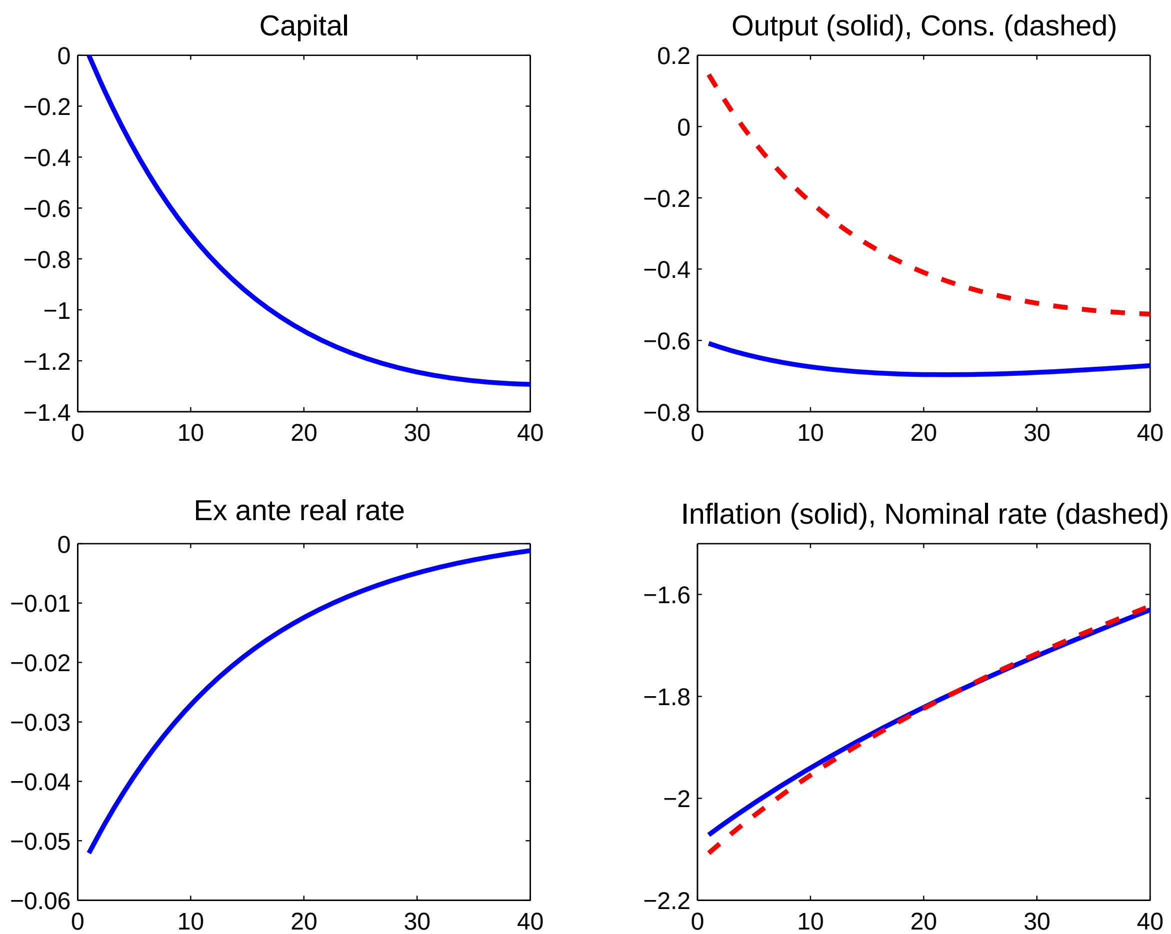

Figure 5: The model with capital, $\rho = 0.995$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

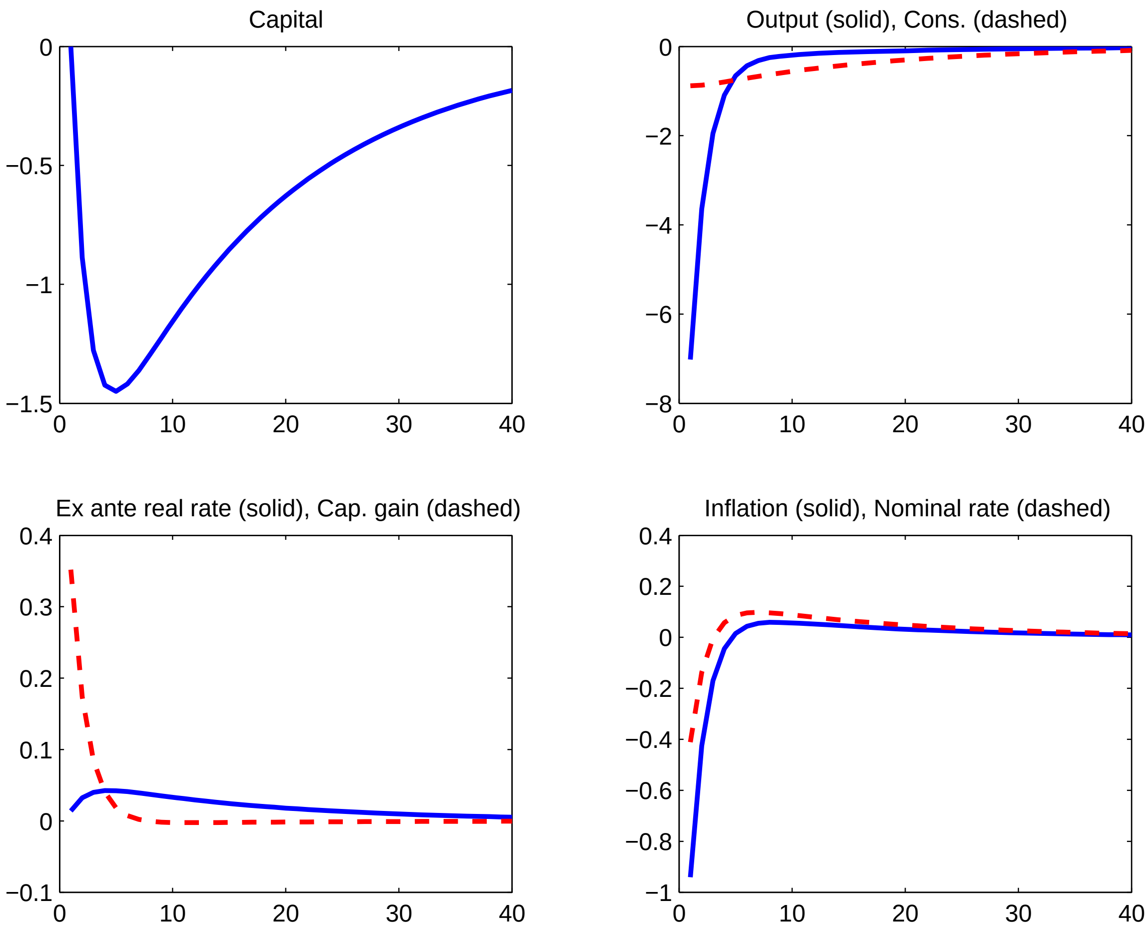

Figure 6: The model with capital adjustment costs, $\kappa = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

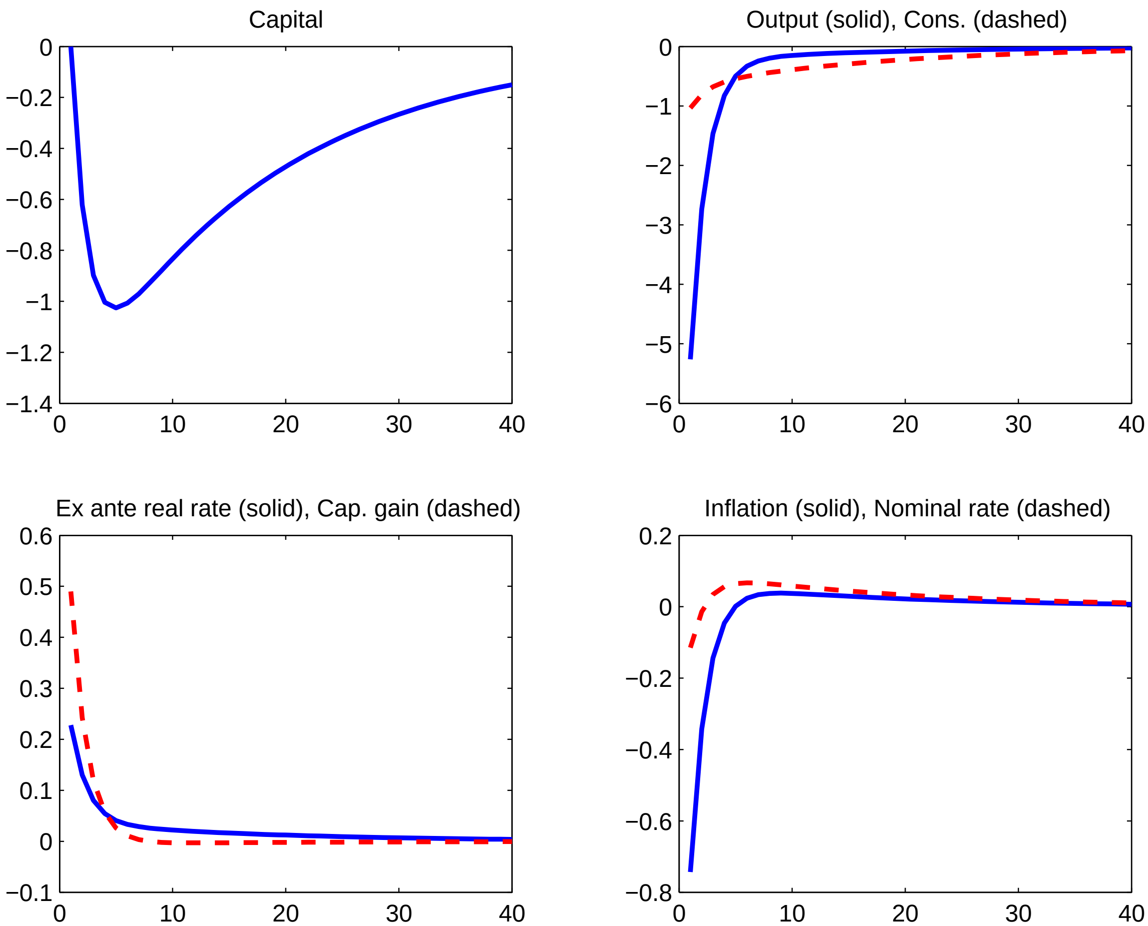

Figure 7: The model with capital adjustment costs, $\kappa = 0.2$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

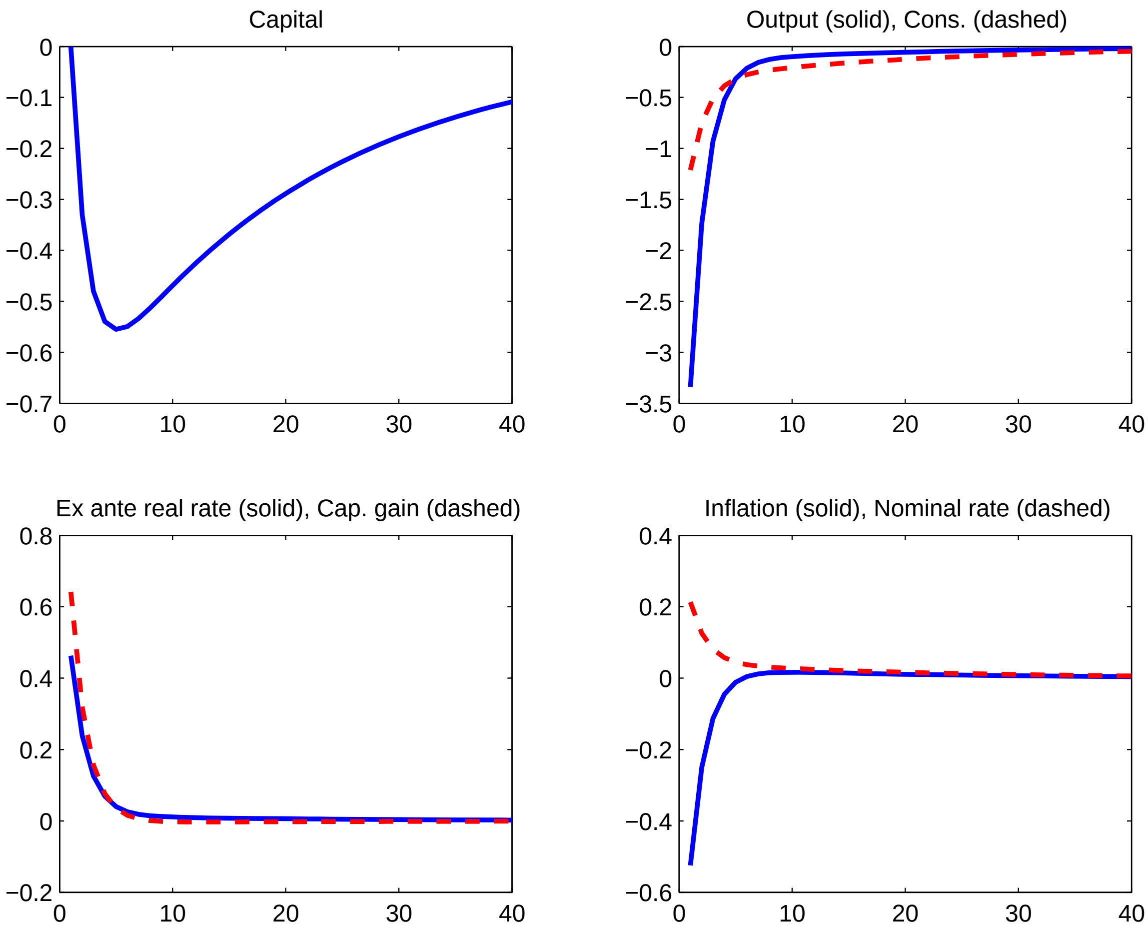

Figure 8: The model with capital adjustment costs, $\kappa = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

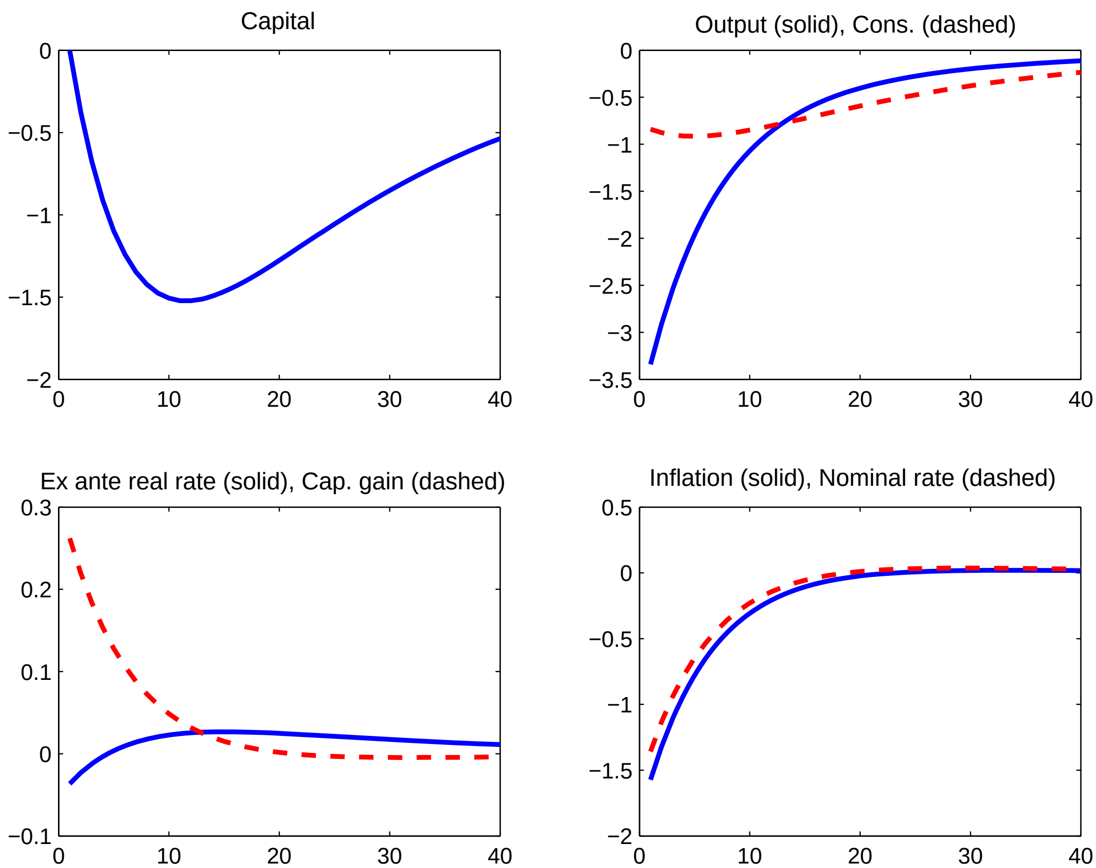

Figure 9: The model with capital adjustment costs, $\kappa = 0.5$, but higher shock persistence, $\rho = 0.85$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.